C. DUMITRESCU V. SELEACU

SMARANDACHE FUNCTION

(book series)

Vol. 2-3

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Number Theory Publishing Company

1993

A BRIEF HISTORY OF THE "SMARANDACHE FUNCTION"

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This function is originated from the exiled Romanian professor Florentin Smarandache. It is defined as follows:

For any non-null integers n, S(n) is the smallest integer such that (S(n))! is divisible by n.

The importance of the notion is that it characterizes a prime number, i.e.:

Let $p > 4$, then: p is prime if and only if $S(p) = p$.

Another properties:

If $(a,b) = 1$, then $S(ab) = \max \{ S(a), S(b) \}$;

and

For any non-null integers, $S(ab) \le S(a) + S(b)$.

{All three found and proved by the author in 1979 (see [3], 15, 12-13, 65).}

If $n > 1$, then S(n) and n have a proper common divisor.

{Found and proved by student Prodănescu in 1993: as a lemma needed to solve the conjecture formulated by the author in 1979 that:

the equation $S(n) = S(n + 1)$ has no solutions

(see [3], 37, and [30]).}

Etc.

Also, an infinity of open/unsolved problems, involving this function, provoked mathematicians around the world to study it and its applications (computational mathematics, simulation, quantum theory, etc.).

Thus, the unsolved question:

$$\text{Calculate } \lim_{n\to\infty} \left[1 + \sum_{k=2}^{n} \frac{1}{S(k)} - \log S(n) \right], \quad \text{(see [3], 29)}$$

made by the author in 1979, has been separately proved by J. Thompson from USA in 1992 (see [18], 1), by Nigel Backhouse from United Kingdom in 1993 (see [25]), and by Pål Grønås from Norway in 1993 (see [51]) that this limit is equal to ∞.

The author wonderred if it's posible to approach the function (see [3], 1979, 25-6), but Ian Parberry expressed that one can immediately find an algorithm that computes S(n) in $O(n\log n/\log\log n)$ time (see [38], 1993).

Some unsolved (by now!) other problems stated by the author in 1979 (see [3], 27-30):

a) To find a general form of the continued fraction expansion of S(n)/n, for all $n \ge 2$.

b) What is the smallest k such that for any integer n at least one of the numbers S(n), S(n+1), ..., S(n+k-1) is a perfect square?

c) To build the largest arithmetical progression $a_1, a_2, \ldots, a_r$ for which their images by the function are also an arithmetical progression.

Etc.

In 1975 Smarandache was a student at the University of Craiova, and he was attracted by the Number Theory. He created and published a lot of proposed problems of mathematics in various scientific journals. He liked to play with the numbers ...

Thus, in 1980 his research paper "A Function in the Number Theory", based on a special representation of integers, was published (for the first time) in <Analele Universităţii Timişoara>, Seria Ştiinţe Matematice, Vol. 18, pp. 79-88,

and was reviewed in <Zentralblatt fur Mathematik>, 471.10004, 1982, by P. Kiss, and in the <Mathematical Reviews>, 83c:10008, 1983, by R. Meyer.

In 1988 he escaped from the Ceauşescu's dictatorship, spent almost two years in a political refugee camp in Turkey (Istambul and Ankara), and finally emigrated to the United States.

Articles, notes, quickies, comments, proposals related to the Smarandache Function were presented to international conferences within the Mathematical Association of America or the American Mathematical Society at the New Mexico State University (Las Cruces), New Mexico Tech. (Socorro), University of Arizona (Tucson), University of San Antonio, University of Victoria (Canada) etc. or published in <Octogon> (Sacele), <Gazeta Matematică> (Bucharest), <The Mathematical Spectrum> (UK), <Elemente der Mathematik> (Switzerland), <The Fibonacci Quarterly> (USA) etc.

In 1992 Dr. J. R. Sutton from United Kingdom designed a BASIC PROCedure to calculate S(n) for all powers of a prime number up to a maximum. (see [26])

Jim Duncan from United Kingdom computed up to S(1499999), the first million taking 50 hours in Lattice C on an Atari 1040ST. (see [17])

Also, John McCarthy from United Kingdom estimated that his machine would take several years to just calculate and store S(n) to disk for the entire range of n it can handle (0<n<2^32), and using the compression detailed in ncld9207.c at least 12 Gigabytes of disk space would be needed. It took about 3 hours for his program to work out that 3,303,302 pages (!) would be needed to list the full range of n and S(n). (see [15])

In 1993 Henry Ibstedt from Sweden used a dtk-computer with 486/33MHz processor in Borland's Turbo Basic and calculated S(n) for n upto 10^6 which took 2 hours and 50 minutes! (see [52])

A group of professors (V. Seleacu, C. Dumitrescu, L. Tuţescu, I. Pătrascu, M. Mocanu) and scientific students from the University of Craiova, having a weekly meeting, are doing research on the function and its applicability.

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The Smarandache Function, together with a sample of The Infinity of Unsolved Problems associated with it, presented by Mike Mudge.[1]

The Smarandache Function, S(n) (originated by Florentin Smarandache — *Smarandache Function Journal*, vol 1, no 1, December 1990. ISSN 1053-4792) is defined for all non-null integers, n, to be the smallest integer such that (S(n))! is divisible by n.

Note N! denotes the factorial function, N!=1x2x3x...xN: for all positive integer N. In addition 0! = 1 by definition.

S(n) is an even function. That is, S(n) = S(-n) since if (S(n))! is divisible by n it is also divisible by -n.

S(p) = p when p is a prime number, since no factorial less than p! has a factor p in this case where p is prime.

The values of S(n) in *Fig 1* are easily verified. For example, S(14) = 7 because 7 is the smallest number such that 7! is divisible by 14.

Problem (i) Design and implement an algorithm to generate and store/tabulate S(n) as a function of n.

Hint It may be advantageous to consider the STANDARD FORM of n, *viz* $n = e p_1^{a_1} p_2^{a_2} \ldots\ldots p_r^{a_r}$ where $e = \pm 1$, $p_1, p_2, \ldots p_r$ denote the distinct prime factors of n and $a_1, a_2, \ldots a_r$ are their respective multiplicities.

Problem (ii) Investigate those sets of consecutive integers (i,i+1,i+2,...i+x) for which S generates a monotonic increasing (or indeed monotonic decreasing) sequence.

Note For (1,2,3,4,5) S generates the monotonic increasing sequence 0,2,3,4,5; here i = 1 & x = 4.

If possible estimate the largest value of x.

Problem (iii) Investigate the existence of integers m,n,p,q & k with n≠m and p≠q for which:

either **(A):** S(m) + S(m+1) +....S(m+p) = S(n) + S(m+1) +....S(n+q) or **(B):**

$$\frac{S(m)^2 + S(m+1)^2 + \ldots S(m+p)^2}{S(n)^2 + S(n+1)^2 + \ldots S(n+p)^2} = k$$

Problem (iv) Find the smallest integer k for which it is true that <u>for all n less than some given n_0</u> at least one of:

S(n), S(n+1)....S(n+k-1) is:

A) a perfect square

B) a divisor of k^n

C) a factorial of a positive integer.

Conjecture what happens to k as n_0 tends to infinity: i.e. becomes larger and larger.

Problem (v) <u>Construct prime numbers of the form</u> $\overline{S(n)\,S(n+1)\ldots S(n+k)}$: where $\overline{abcdefg}$ denotes the integer formed by the concatenation <u>of a,b,c,d</u>,e,f & g. For example, trivially $\overline{S(2)\,S(3)}$ = 23 which <u>is prime, but no so trivially</u> $\overline{S(14)S(15)S(16)S(17)}$ = 75617, also prime!

Definition An A-SEQUENCE is an integer sequence $a_1, a_2, \ldots$ with $1 \le a_1 < a_2 < \ldots$ such that no a_i is the sum of distinct members of the sequence (other than a_i).

Problem (vi) Investigate the construction of A-SEQUENCES $a_1, a_2, \ldots$ such that the associated sequences $S(a_1), S(a_2), \ldots$ are also A-SEQUENCES.

Definition The k^{th} order forward finite differences of the Smarandache function are defined thus:

$D_s(x)$ = *modulus(S(x+1) - S(x)),

$D_s^{(k)}(x) = D(D(\ldots k\text{-times} D_s(x)\ldots))$

Problem (vii) Investigate the conjecture that $D_s^{(k)}(1)$ = 1 or 0 for all k greater than or equal to 2.

c.f. Gilbreath's conjecture on prime numbers, discussed in 'Numbers Count' *PCW* Dec 1983. * Here modulus is taken to mean the absolute value of (ABS.), modulus (y) = y if y is positive and modulus (y) = -y if y is negative.

The following selection of Diophantine Equations (i.e. solutions are sought in integer values of x) are taken from the Smarandache Journal and make up:

Problem (viii) If m & n are given integers, solve each of:

a) S(x) = S(x+1), conjectured to have no solution

b) S(mx+n) = x

c) S(mx+n) = m+nx

d) S(mx+n) = x!

e) $S(x^m) = x^n$

f) $S(x)^m = S(x^n)$

g) S(x) + y = x + S(y), x&y not prime

h) S(x) + S(y) = S (x+y)

i) S(x+y) = S(x)S(y)

j) S(xy) = S(x)S(y)

Review, July 1992

The Smarandache Function: a first visit? [2]

This topic is certain to be revisited in the near future, and the lack of space available here will certainly be remedied on that occasion. Suffice it to report that Jim Duncan computed up to S(1499999), the first million taking 50 hours in Lattice C on an Atari 1040ST. In Problem (ii), no evidence for a largest value of x was found, while in Problem (vii) the conjecture was verified for the first 32,000 values of S(n). The very worthy prizewinner is John McCarthy of 17 Mount Street, Mansfield, Notts NG19 7AT, who has extensively *investigated* the computation of S(n) up to 2^{32}; arriving at conclusions such as: 'several years of computing', 'at least 12Gb of disk space' and '3,303,302 pages of output'. John's concluding comment, 'Am I mad?', is clearly answered NO! by examining his specimen pages of output including those relating to 10-digit values of n. Listings supplied. Details from John directly upon request.

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1 *Republished from <Personal Computer World>, No.112, 420, July 1992 (with the author permission), because some of the following research papers are referring to these open problems.*

2 *Republished from <Personal Computer World>, No.117, 412, December 1992 (with the author permission).*

ALGORITHM IN LATTICE C TO GENERATE S(n)

by Jim Duncan
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Formby
Liverpool
L37 7EG ENGLAND

Computer: Atari 1040ST
Run time: ca. 50 hrs to generate S(n) for 1 000 000 numbers

```
#include <stdio.h>
#include <math.h>

unsigned long int pst,ast,s,t;


main()
{
        long int n;
        FILE *fp;

        printf("input n (1 <= n < 2 147 483 647)\n");
        scanf("%ld", &n);
        fp = fopen("PRN:", "w");
        fprintf(fp, "n = %ld  S(n) = %ld\n", n, smaran(n));
        fclose(fp);

}
smaran(m)
unsigned long int m;
{
                unsigned long int mst,p,a,fact;
                double r;

                if (m == 1)
                        return(0);
                else {                  /* STANDARD FORM of m */
                r = mst = m;
                p = 1;
                fact = 1;
                while ( ++p <= sqrt(r)) {
                        a = 0;
                        while (mst % p == 0) {
                                 mst = mst/p;
                                 a++;
                                }
                        r = mst;
                        if (a > 0) {
                                pst = p;
                                ast = a;
                                t = s = 0;
```

```
                        tors3(); /* find smallest factorial (t) with */
                                 /* p^a divsor */
                        if (t > fact)
                                fact = t;
                         }
                }
                if (mst > fact)
                        fact = mst;
                return(fact);
                }
}
tors1()
{
        unsigned long int i;

    /* test  number is pst^ast */
    /* s is the difference between a factorial number t  and ast*(pst-1) */
    /* s forms a pattern which determines the smallest value of t for which */
    /* the test number is a divisor */

        i = 0;
        while (++i < pst*pst && t-s < ast*(pst-1)) {
                if (i % pst == 0)
                        s = s-pst+2;
                else
                        s++;
                t += pst;
                }
  }


  tors2()
  {
        unsigned long int i;

        tors1();
        i = 0;
        while (++i < pst*pst && t-s < ast*(pst-1)) {
                if (i % pst == 0)
                        s = s-3*pst+4;
                else
                        s = s-2*pst+3;
                t += pst;
                tors1();
                }
  }


  tors3()
  {
        unsigned long int i;

        tors2();
        i = 0;
        while (++i < pst*pst && t-s < ast*(pst-1)) {
                if (i % pst == 0)
                        s = s-5*pst+6;
                else
                        s = s-4*pst+5;
                t += pst;
                tors2();
                }
  }
```

MONOTONIC INCREASING AND DECREASING SEQUENCES OF S(n)

by Jim Duncan

Problem (11)

Monotonic increasing and monotonic decreasing sequences of S(n) were investigated for x >= 6.

First number (i) in range	number of sequences S							
	x = 6		x = 7		x = 8		x = 9	
	inc	dec	inc	dec	inc	dec	inc	dec
1 - 499 999	75	83	7	10	0	2	0	0
500 000 - 999 999	80	76	14	18	1	3	1	1
1 000 000 - 1 499 999	75	63	8	10	1	2	1	1

There appears to be no evidence for a largest value for x. The sequences for x = 9 are shown in Results Table 1.

The existence of sequences with the same first order finite differences was then considered eg:

i = 440	S(i) = 11;	i = 5073	S(i) = 89
i+1 = 441	S(i+1) = 14;	i+1 = 5074	S(i+1) = 59
i+2 = 442	S(i+2) = 17;	i+2 = 5075	S(i+2) = 29

Apart from the initial quartet 2,3,4,5 all such sequences with i < 1 000 000 are triplets. If the first order finite differences are multiples of 6 then the S(n) values appear to be prime numbers. The values are shown in Results Table 2.

RESULTS TABLE 1

Sequences of S(n)

x = 9

n = 586951	S(n) = 586951
n = 586952	S(n) = 73369
n = 586953	S(n) = 21739
n = 586954	S(n) = 9467
n = 586955	S(n) = 1319
n = 586956	S(n) = 1193
n = 586957	S(n) = 1181
n = 586958	S(n) = 1091
n = 586959	S(n) = 677
n = 586960	S(n) = 29

x = 9

n = 721970	S(n) = 73
n = 721971	S(n) = 827
n = 721972	S(n) = 907
n = 721973	S(n) = 6067
n = 721974	S(n) = 10939
n = 721975	S(n) = 28879
n = 721976	S(n) = 90247
n = 721977	S(n) = 240659
n = 721978	S(n) = 360989
n = 721979	S(n) = 721979

x = 9

n = 1091150	S(n) = 157
n = 1091151	S(n) = 709
n = 1091152	S(n) = 1451
n = 1091153	S(n) = 1607
n = 1091154	S(n) = 6271
n = 1091155	S(n) = 16787
n = 1091156	S(n) = 24799
n = 1091157	S(n) = 363719
n = 1091158	S(n) = 545579
n = 1091159	S(n) = 1091159

x = 9

n = 1473257	S(n) = 1473257
n = 1473258	S(n) = 8467
n = 1473259	S(n) = 6323
n = 1473260	S(n) = 3877
n = 1473261	S(n) = 3533
n = 1473262	S(n) = 2239
n = 1473263	S(n) = 1999
n = 1473264	S(n) = 787
n = 1473265	S(n) = 557
n = 1473266	S(n) = 463

RESULTS TABLE 2

same difference = 1 n = 4 S(n-2) = 2 S(n-1) = 3 S(n) = 4

same difference = 1 n = 5 S(n-2) = 3 S(n-1) = 4 S(n) = 5

same difference = 11 n = 18 S(n-2) = 6 S(n-1) = 17 S(n) = 6

same difference = 3 n = 442 S(n-2) = 11 S(n-1) = 14 S(n) = 17

same difference = 30 n = 5075 S(n-2) = 89 S(n-1) = 59 S(n) = 29

same difference = 60 n = 6409 S(n-2) = 149 S(n-1) = 89 S(n) = 29

same difference = 48 n = 6479 S(n-2) = 127 S(n-1) = 79 S(n) = 31

same difference = 36 n = 8177 S(n-2) = 109 S(n-1) = 73 S(n) = 37

same difference = 84 n = 13717 S(n-2) = 211 S(n-1) = 127 S(n) = 43

same difference = 168 n = 20468 S(n-2) = 379 S(n-1) = 211 S(n) = 43

same difference = 210 n = 22591 S(n-2) = 461 S(n-1) = 251 S(n) = 41

same difference = 120 n = 35145 S(n-2) = 311 S(n-1) = 191 S(n) = 71

same difference = 180 n = 59719 S(n-2) = 449 S(n-1) = 269 S(n) = 89

same difference = 150 n = 67771 S(n-2) = 401 S(n-1) = 251 S(n) = 101

same difference = 264 n = 73425 S(n-2) = 617 S(n-1) = 353 S(n) = 89

same difference = 840 n = 74005 S(n-2) = 1721 S(n-1) = 881 S(n) = 41

same difference = 24 n = 82297 S(n-2) = 151 S(n-1) = 127 S(n) = 103

same difference = 60 n = 104669 S(n-2) = 251 S(n-1) = 191 S(n) = 131

same difference = 330 n = 111507 S(n-2) = 769 S(n-1) = 439 S(n) = 109

same difference = 36 n = 114427 S(n-2) = 199 S(n-1) = 163 S(n) = 127

same difference = 252 n = 120523 S(n-2) = 631 S(n-1) = 379 S(n) = 127

same difference = 120 n = 129928 S(n-2) = 389 S(n-1) = 269 S(n) = 149

same difference = 952 n = 146004 S(n-2) = 1973 S(n-1) = 1021 S(n) = 69

same difference = 600 n = 153520 S(n-2) = 1301 S(n-1) = 701 S(n) = 101

same difference = 12 n = 180482 S(n-2) = 47 S(n-1) = 59 S(n) = 71

same difference = 660 n = 181485 S(n-2) = 1429 S(n-1) = 769 S(n) = 109

same difference = 60 n = 189954 S(n-2) = 53 S(n-1) = 113 S(n) = 173

same difference = 90 n = 192067 S(n-2) = 359 S(n-1) = 269 S(n) = 179

same difference = 324 n = 198697 S(n-2) = 811 S(n-1) = 487 S(n) = 163

same difference = 336 n = 209752 S(n-2) = 839 S(n-1) = 503 S(n) = 167

same difference = 228 n = 227099 S(n-2) = 647 S(n-1) = 419 S(n) = 191

same difference = 150 n = 231039 S(n-2) = 499 S(n-1) = 349 S(n) = 199

same difference = 264 n = 253725 S(n-2) = 727 S(n-1) = 463 S(n) = 199

same difference = 210 n = 266915 S(n-2) = 631 S(n-1) = 421 S(n) = 211

same difference = 648 n = 297638 S(n-2) = 1459 S(n-1) = 811 S(n) = 163

same difference = 2808 n = 306128 S(n-2) = 5669 S(n-1) = 2861 S(n) = 53

same difference = 1320 n = 324384 S(n-2) = 2749 S(n-1) = 1429 S(n) = 109

same difference = 18 n = 326163 S(n-2) = 199 S(n-1) = 181 S(n) = 163

same difference = 240 n = 342965 S(n-2) = 719 S(n-1) = 479 S(n) = 239

same difference = 36 n = 346390 S(n-2) = 139 S(n-1) = 103 S(n) = 67

same difference = 300 n = 386906 S(n-2) = 47 S(n-1) = 347 S(n) = 647

same difference = 840 n = 409422 S(n-2) = 1861 S(n-1) = 1021 S(n) = 181

same difference = 270 n = 440375 S(n-2) = 811 S(n-1) = 541 S(n) = 271

same difference = 936 n = 443450 S(n-2) = 2053 S(n-1) = 1117 S(n) = 181

same difference = 120 n = 443850 S(n-2) = 509 S(n-1) = 389 S(n) = 269

same difference = 792 n = 443969 S(n-2) = 1783 S(n-1) = 991 S(n) = 199

same difference = 450 n = 450043 S(n-2) = 1151 S(n-1) = 701 S(n) = 251

same difference = 306 n = 451215 S(n-2) = 883 S(n-1) = 577 S(n) = 271

same difference = 210 n = 460559 S(n-2) = 701 S(n-1) = 491 S(n) = 281

same difference = 240 n = 464212 S(n-2) = 761 S(n-1) = 521 S(n) = 281

same difference = 360 n = 470727 S(n-2) = 991 S(n-1) = 631 S(n) = 271

same difference = 624 n = 473922 S(n-2) = 1481 S(n-1) = 857 S(n) = 233

same difference = 90 n = 481779 S(n-2) = 449 S(n-1) = 359 S(n) = 269

same difference = 126 n = 511688 S(n-2) = 131 S(n-1) = 257 S(n) = 383

same difference = 672 n = 512894 S(n-2) = 1583 S(n-1) = 911 S(n) = 239

same difference = 480 n = 521946 S(n-2) = 1231 S(n-1) = 751 S(n) = 271

same difference = 714 n = 531775 S(n-2) = 1667 S(n-1) = 953 S(n) = 239

same difference = 726 n = 543455 S(n-2) = 1693 S(n-1) = 967 S(n) = 241

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same difference = 306  n = 565187 S(n-2) = 919 S(n-1) = 613 S(n) = 307
same difference = 552  n = 574498 S(n-2) = 1381 S(n-1) = 829 S(n) = 277
same difference = 1176  n = 586272 S(n-2) = 2549 S(n-1) = 1373 S(n) = 197
same difference = 3444  n = 592537 S(n-2) = 6971 S(n-1) = 3527 S(n) = 83
same difference = 390  n = 609871 S(n-2) = 1091 S(n-1) = 701 S(n) = 311
same difference = 840  n = 629508 S(n-2) = 1931 S(n-1) = 1091 S(n) = 251
same difference = 336  n = 681077 S(n-2) = 1009 S(n-1) = 673 S(n) = 337
same difference = 612  n = 705793 S(n-2) = 1531 S(n-1) = 919 S(n) = 307
same difference = 78  n = 724319 S(n-2) = 467 S(n-1) = 389 S(n) = 311
same difference = 264  n = 726827 S(n-2) = 881 S(n-1) = 617 S(n) = 353
same difference = 498  n = 731179 S(n-2) = 1327 S(n-1) = 829 S(n) = 331
same difference = 240  n = 751746 S(n-2) = 839 S(n-1) = 599 S(n) = 359
same difference = 1356  n = 778837 S(n-2) = 2939 S(n-1) = 1583 S(n) = 227
same difference = 1020  n = 792675 S(n-2) = 2311 S(n-1) = 1291 S(n) = 271
same difference = 2214  n = 803427 S(n-2) = 4591 S(n-1) = 2377 S(n) = 163
same difference = 1590  n = 810451 S(n-2) = 3391 S(n-1) = 1801 S(n) = 211
same difference = 252  n = 837969 S(n-2) = 883 S(n-1) = 631 S(n) = 379
same difference = 552  n = 898783 S(n-2) = 1471 S(n-1) = 919 S(n) = 367
same difference = 2850  n = 930311 S(n-2) = 5851 S(n-1) = 3001 S(n) = 151
same difference = 540  n = 941057 S(n-2) = 1459 S(n-1) = 919 S(n) = 379
same difference = 240  n = 943553 S(n-2) = 881 S(n-1) = 641 S(n) = 401
same difference = 120  n = 975546 S(n-2) = 619 S(n-1) = 499 S(n) = 379
same difference = 1122  n = 997443 S(n-2) = 2551 S(n-1) = 1429 S(n) = 307
same difference = 264  n = 1026550 S(n-2) = 947 S(n-1) = 683 S(n) = 419
same difference = 684  n = 1028985 S(n-2) = 1747 S(n-1) = 1063 S(n) = 379
same difference = 744  n = 1042162 S(n-2) = 1861 S(n-1) = 1117 S(n) = 373
same difference = 510  n = 1053175 S(n-2) = 1429 S(n-1) = 919 S(n) = 409
```

ON THE CONJECTURE $D_s^{(k)}(1)=1$ or 0 for k>=2

by Jim Duncan

Problem(vii)

For the first 32 000 S(n)'s the conjecture that $D_s^{(k)}(1) = 1$ or 0 for $k >= 2$ is true. The ratio of the number of ones to the number of zeros appears to be approximately 1 for large values of k. The results are shown in Results Table 3.
The true differences S(x-1) - S(x) were calculated and the k^{th} order differences $D_s^{*(k)}(1)$ were found to increase rapidly with increasing k. For large values of k (> 100) the ratio $D_s^{*(k)}(1)/D_s^{*(k-1)}(1)$ is approximately equal to -2. Some values are shown in Results Table 4.

RESULTS TABLE 3

k, $D_s^{(k)}(1)$	Ratio: total 1's / total 0's
D1 = 2	
D1000 = 0	st1/st0 = 1.112051
D2000 = 0	st1/st0 = 1.056584
D3000 = 0	st1/st0 = 1.075433
D4000 = 0	st1/st0 = 1.015625
D5000 = 0	st1/st0 = 1.010861
D6000 = 1	st1/st0 = 0.991039
D7000 = 0	st1/st0 = 0.990048
D8000 = 0	st1/st0 = 0.977014
D9000 = 0	st1/st0 = 0.987412
D10000 = 0	st1/st0 = 0.998201
D11000 = 1	st1/st0 = 1.011154
D12000 = 1	st1/st0 = 1.010556
D13000 = 1	st1/st0 = 1.015036
D14000 = 0	st1/st0 = 1.018601
D15000 = 1	st1/st0 = 1.012748
D16000 = 0	st1/st0 = 1.004134
D17000 = 0	st1/st0 = 1.005308
D18000 = 0	st1/st0 = 1.004789
D19000 = 1	st1/st0 = 1.003903
D20000 = 1	st1/st0 = 1.004711
D21000 = 0	st1/st0 = 1.008129
D22000 = 0	st1/st0 = 1.004830
D23000 = 0	st1/st0 = 1.004620
D24000 = 0	st1/st0 = 1.003590
D25000 = 1	st1/st0 = 1.004571
D26000 = 1	st1/st0 = 1.001001
D27000 = 0	st1/st0 = 1.001260
D28000 = 1	st1/st0 = 1.004080
D29000 = 0	st1/st0 = 1.006018
D30000 = 1	st1/st0 = 1.005415
D31000 = 0	st1/st0 = 1.006408
D32000 = 1	st1/st0 = 1.004699

RESULTS TABLE 4

k	$D_s^{*(k)}(1)$	$\frac{D_s^{*(k)}(1)}{D_s^{*(k-1)}(1)}$
D951	= 2.244421E+288	ratio = -2.002974
D952	= -4.496445E+288	ratio = -2.003387
D953	= 9.009916E+288	ratio = -2.003787
D954	= -1.805740E+289	ratio = -2.004170
D955	= 3.619671E+289	ratio = -2.004535
D956	= -7.257009E+289	ratio = -2.004881
D957	= 1.455178E+290	ratio = -2.005204
D958	= -2.918366E+290	ratio = -2.005504
D959	= 5.853595E+290	ratio = -2.005778
D960	= -1.174247E+291	ratio = -2.006026
D961	= 2.355827E+291	ratio = -2.006246
D962	= -4.726819E+291	ratio = -2.006437
D963	= 9.484823E+291	ratio = -2.006598
D964	= -1.903346E+292	ratio = -2.006728
D965	= 3.819685E+292	ratio = -2.006827
D966	= -7.665702E+292	ratio = -2.006894
D967	= 1.538452E+293	ratio = -2.006929
D968	= -3.087569E+293	ratio = -2.006932
D969	= 6.196454E+293	ratio = -2.006904
D970	= -1.243531E+294	ratio = -2.006843
D971	= 2.495458E+294	ratio = -2.006751
D972	= -5.007457E+294	ratio = -2.006629
D973	= 1.004734E+295	ratio = -2.006476
D974	= -2.015791E+295	ratio = -2.006293
D975	= 4.043841E+295	ratio = -2.006081
D976	= -8.111306E+295	ratio = -2.005842
D977	= 1.626784E+296	ratio = -2.005576
D978	= -3.262162E+296	ratio = -2.005283
D979	= 6.540525E+296	ratio = -2.004966
D980	= -1.311130E+297	ratio = -2.004625
D981	= 2.627848E+297	ratio = -2.004262
D982	= -5.265885E+297	ratio = -2.003877
D983	= 1.055006E+298	ratio = -2.003473
D984	= -2.113228E+298	ratio = -2.003049
D985	= 4.231969E+298	ratio = -2.002608
D986	= -8.473041E+298	ratio = -2.002151
D987	= 1.696031E+299	ratio = -2.001679
D988	= -3.394086E+299	ratio = -2.001193
D989	= 6.790531E+299	ratio = -2.000695
D990	= -1.358233E+300	ratio = -2.000186
D991	= 2.716013E+300	ratio = -1.999667
D992	= -5.429688E+300	ratio = -1.999139
D993	= 1.085180E+301	ratio = -1.998604
D994	= -2.168257E+301	ratio = -1.998063
D995	= 4.331129E+301	ratio = -1.997516
D996	= -8.649119E+301	ratio = -1.996966
D997	= 1.726721E+302	ratio = -1.996413
D998	= -3.446291E+302	ratio = -1.995858
D999	= 6.876396E+302	ratio = -1.995303
D1000	= -1.371668E+303	ratio = -1.994748

A Simple Algorithm to Calculate S(n)

by John C. McCarthy

Introduction

This short paper first outlines an "obvious" algorithm for calculating S(n) (the smallest integer m such that m! is divisible by n). Doubtless, there exist more subtle and efficient algorithms. I hope some readers will devise these and enlighten me concerning them through this journal.

This is followed by a small scale investigation of the efficiency of the algorithm.

Then there is a short discussion of a simple way of reducing the space required for storage of all S(n) for ranges of n. The storage space required for S(n) for all n which my routines can handle is considered.

Heavily commented listings of an implementation of the algorithm in "C", sample output and timing data are included to help illustrate the algorithm.

The Algorithm

The algorithm is described in detail at the start of the header file "S(n).H". Together with "S(n).C", this forms all the code necessary to implement the algorithm. Note that, for the S(n) function to work correctly, the function make_primes() must first be called from the main program.

The code for printing S(n) and timing the routines has been omitted. These activities are both implementation specific and easily done. They are therefore left as an exercise for the interested reader.

The algorithm hinges on finding the prime factors of n. Improvements on how this is done will most benefit its efficiency.

To be practical, the given implementation of the algorithm only works for $0<n<2^{32}$. However, the algorithm is generally applicable to any non-null integer.

Tables of S(n), constructed using the routines of "S(n).C", for the largest 2000 permitted n are included.[1] My paging routines are rather elaborate. Using them (without printing!), it took 2.4 hours to discover that 3,745,708 pages, as tightly packed as those shown, would be required to print S(n) for all $0<n<2^{32}$.

Efficiency of the Algorithm

In a letter to R. Muller (about computing the Smarandache Function, July 19, 1993), Ian Parberry (editor of <SIGACT News>,

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1 *For the smallest 4800 numbers, see Ibstedt's table (pp. 43-50) of this current journal.*

Denton, Texas) expressed that one can immediately find an algorithm that computes S(n) in O(nlogn/loglogn) time ('A Brief History of the "Smarandache Function"' by Dr. Constantin Dumitrescu, Department of Mathematics, University of Craiova, Romania). Disappointingly, a little analysis of the accompanying timing data on my TI85 advanced scientific calculator reveals that my algorithm is somewhat worse than this.

Trying to fit the version 2 timing data to various O(f(n)), I obtained the following results (x=3355443200 and 10(O(x+99)-O(x-100)) is calculated for comparison with the last entry of the version 1 timing data):

O(f(n))	Correlation Coefficient	$O(2^{32}-1)$ (years)	10(O(x+99)-O(x-100)) (milliseconds)
O(n)	0.9928879	0.6092	8909
O(nlogn/loglogn)	0.9944006	0.7906	11827
O(n√n)	0.9997756	24.2	469178

O(n√n) fits the version 2 timing data best, although the time it predicts for the last entry of the version 1 timing data is almost 3 times too large. Hence, I assume the time complexity of my algorithm is a little better than O(n√n).

As a rough upper limit on the time my program (on my 20MHz 368DX PC) would take to calculate S(n) for all $0<n<2^{32}$, let us assume that every value of n requires as much time as each n in the range of the last entry of the version 1 timing data (= 159111/199/10 = 79.9553 ms). In this "worst case", it would take 10.882 years. O(n√n) time complexity predicts more than twice this value, which is a measure of how pessimistic it is.

I would welcome a more rigorous analysis of the time complexity of my algorithm as I presently lack the necessary expertise.

Simple Compression of Stored S(n)

Without compression, each S(n) would be stored as a 32-bit (= 4 bytes) value. Hence 2^{34} bytes (= 16 Gigabytes) would be required to store S(n) for all $0<n<2^{32}$.

This requirement can be reduced considerably if we use the high bit of each each byte of each value to indicate if it is the last byte of the value. If the bit is set it means that further byte(s) are required and if it is reset it means that the byte is the last byte of the current value. This means that only 7 bits of each byte are used to form the numerical part of the value. Assuming that, as with Intel format, the values are stored low-'byte' (actually 7 bits) first, here are some examples:

i) 127 requires seven bits and so just one byte (with high bit reset to indicate no further bytes).
ii) 16,000 requires 14 bits. So it is stored as two bytes. The

first is 0 (16,000 mod 128) + 128 (to set the high bit indicating there is more to come). The second is 125 (16,000 div 128) (with high bit reset to indicate no further bytes). This reads simply as 0 (with more to follow) + 128*125 (no more to follow).

iii) A number stored as the three bytes 57+128, 93+128 and 125+0 would similarly represent:
57 + 93*128 + 125*128*128 = 2,059,961.

The largest numbers that can be represented by a given number of bytes is thus as follows:

1 byte can code up to 2^7-1 = 127.
2 bytes can code up to $2^{14}-1$ = 16,383.
3 bytes can code up to $2^{21}-1$ = 2,097,151.
4 bytes can code up to $2^{28}-1$ = 268,435,455.
5 bytes can code up to $2^{35}-1$ = 34,359,738,355 (or 8 times the largest unsigned long).

For small values of n, the savings are considerable (400%). However, even large n often have small S(n).

Using this technique to compress all S(n) calculated for some ranges of n (each range was also stored), I obtained the following results:

range of n	compression	time taken (seconds)	size	size after pkzip
1	without	4.5	40,008	19,836
-10,000	with	4.7	15,749	15,267
2,147,478,648	without	827.3	40,008	33,729
-2,147,488,647	with	842.4	33,541	30,836
4,294,957,296	without	1,066.2	40,008	34,320
-4,294,967,295	with	1,085.1	34,330	31,634

The results indicate that this compression is a little better than pkzip's (a commercial file compression utility). Application of pkzip to a pre-compressed file also gives a slight improvement.

Assuming that the savings shown for the middle range of 10,000 n are the average of all ranges of 10,000 n, using my compression together with that of pkzip would permit storage of S(n) for all $0<n<2^{32}$ in about $3.0836*2^{32}$ = 12.3344 Gigabytes. So look out for sets of 19 CD-ROMs with all your favourite numbers on them!

21st November 1993

S(n).H

```
/* (c).1993.11.13.John.C.McCarthy
   "S(n).h"

   Example Implementation of A Simple Algorithm to Calculate S(n),
   The Smarandache Function:

   Because there are more people familiar with C than with C++, this module
   has been written entirely in C (apart from "//" style comments).  The
   module was compiled using Borland C++ version 3.1.

   For efficiency, n is constrained to the limits of an unsigned long.  Hence,
   0 <= n <= 2^32 - 1 (= 4,294,967,295).  ("^" represents exponentiation).
   Although catering for n of vast magnitude is possible, it imposes heavy
   storage and processing overheads.  The range of an unsigned long therefore
   seems a reasonable compromise.

   The algorithm depends on the most elementary properties of S(n):
   1) Calculate the STANDARD FORM (SF) of n:
      In SF: n = +/-(p1^a1)*(p2^a2)*...*(pr^ar) where p1, p2,...pr denote the
      distinct prime factors of n and a1, a2,...ar are their respective
      multiplicities.
   2) S(n) = max[ S(p1^a1),...,S(pr^ar)].
   3) S(p^a), where p is prime, is given by:
      3.1) a<=p  ==>  S(p^a) = p*a.
      3.2) a>p   ==>  S(p^a) = x < p*a.  In this case, fortunately rare, x
           is the smallest integer such that p appears as a factor in the
           list of all integers > 1 and <= x at least a times.  Let the no.
           of times p appears as a factor in the list of all integers > 1
           and <= y be f(y, p).  Then:
           f(y, p) = Σ[int(y/(p^i))] for i>0 while y>=(p^i).
           Hence, x is the smallest integer such that f(x, p)>=a.
           Note that between succesive integer multiples of p there are no
           integers which have p as a factor.  The trick here is to look for
           the largest multiple of p (call it c), such that f(p*c, p)<=a
           (so that x = p*c, if f(p*c, p) = a, else x = p*(c+1)):
           3.2.1) c = a-2  (largest possibilty for c since f(p*(a-1), p)>=a
                  when a>p (Note: f(p*(a-1), p)=a is not sought for slight
                  performance gain)).
           3.2.2) z = f(p*c, p).
           3.2.3) While(z>a):
                  3.2.3.1) d = no. of times p appears as a factor of p*c
                             = (no. of times p appears as a factor of c) + 1.
                  3.2.3.2) c = c-1  (next largest possibility for c).
                  3.2.3.3) z = z-d  (= f(p*c, p)).
           3.2.4) If(z<a), x = p*(c+1).
           3.2.5) Else     x = p*c.

   To calculate the prime factors of all 32-bit n requires use only of primes
   < (2^16) (i.e. all primes expressible as an unsigned short integer).  This
   is because any factor of n remaining after division of n by all its prime
   factors < (2^16) is simply a prime.  Since there are only 6542 16-bit
   primes, the program first creates a list of these (which only takes about
   4 seconds on my 20 MHz 386DX PC) so that they never have to be
   recalculated, thus saving much time.
*/
```

```
#define PRIMES16 6542     // The number of 16-bit primes
#define MAX_SFK 9  /* max. distinct primes in the SF of n.  The smallest
    number with more than 9 distinct primes is the product of the 10 smallest
    primes (= 6,469,693,230), which is substantially more than the largest
    integer expressible as an unsigned long.  Hence, 9 distinct primes are
    more than ample.
*/

typedef unsigned long u_long;
typedef unsigned int  u_int;
typedef enum {false, true} boolean;

struct SF_struct {
  int    sfk;                 // no. of distinct primes
  u_long sfp[MAX_SFK];        // the distinct primes
  int    sfa[MAX_SFK];        // respective multiplicities
};

extern u_int prime[PRIMES16+1];  // list of all 16-bit primes
                                 // plus terminating zero.

void make_primes(void);  // construct list of all 16-bit primes (prime[]).
                         // Must be called before calls to getSF() or S().
void getSF(u_long n, struct SF_struct *SF); // calc. SF of n and store in SF
u_long S(u_long n);      // calc. S(n)
u_long Spa(u_long p, int a);  // calc. S(p^a) where p is prime
int f(int x, int p);  /* the number of times the prime p appears as a factor
    in the integers from 1 to x inclusive.  This function is only called from
    Spa(p, a) when a>p with x=p*(a-2) (refer to item (3) of algorithm outline
    above).  Max value of (a) occurs when p is a minimum, n is a maximum and
    (p^a)=n.  So, (2^max(a))=max(n)=(2^32)-1.  Hence max(a)<32.  So, x<60
    when (a) is at its max.  Max value of p (and x) occurs when a=p+1 and
    (p^a)=max(n).  So, max(p)^(max(p)+1)=(2^32)-1.  The upshot is that
    max(p)=9 when a=10.  Hence, max(x)=72.  This explains why it is safe for
    x, p and the return value of f(x,p) to be passed as ints.
*/
```

```
/* (c).1993.11.13.John.C.McCarthy
   "S(n).c"

   Example Implementation of A Simple Algorithm to Calculate S(n),
   The Smarandache Function:

   This is the code for the module.  Refer to "S(n).h" for details.
*/
#include "S(n).h"

u_int prime[PRIMES16+1]; // allocate storage for list of all 16-bit primes
                         // plus terminating zero.

void make_primes(void)
{
  u_int *pp;  // ptr to last prime so far of prime list
  u_int *tp;  // ptr to current test prime
  u_int p;    // number being tested for primality

  pp=prime;  // point to start of prime list
  *pp=2;     // set first prime to 2
  *++pp=3;   // set second prime to 3
  p=5;       // next possible prime.  N.B. p is kept odd so that trial
             // division by 2 is unnecessary.
  while(true) { // infinite loop!:
    tp=prime+1;    // point to first odd test prime
    // whilst test prime <= √p:
    while(((long) *tp)*(*tp)<=p) {
      if(!(p%*tp)) {  // If current test prime divides (is factor of) p:
        p+=2;         // try next odd number
        if(p<*pp) {   // done when p overflows:
          *++pp=0;       // terminate list
          return;
        }
        tp=prime+1;      // point to first odd test prime
      }
      else ++tp;      // Else point to next test prime
    }
    // no prime <= √p divides p so p must be prime:
    *++pp=p;      // so store it next in the list
    p+=2;         // try next odd number
    if(p<*pp) {   // done when p overflows:
      *++pp=0;       // terminate list
      return;
    }
  }
}
```

```
void getSF(u_long n, struct SF_struct *SF)
{
  u_int *pp;  // ptr to current prime
  u_long r;   // 'residue' of n remaining for factoring

  SF->sfk=0;  // no. of distinct prime factors discovered
  r=n;
  pp=prime;   // point to start of prime list

  // whilst current prime <= √r and prime list not exhausted:
  while(((long) *pp)*(*pp)<=r && *pp) {
    if(!(r%*pp)) {               // if current prime is a factor of r:
      SF->sfp[SF->sfk]=*pp;        // store current prime as next prime of SF
      SF->sfa[SF->sfk]=1;          // set its multiplicity to 1
      r/=*pp;                      // 'divide out' current prime
      while(!(r%*pp)) {            // while current prime factors r:
        SF->sfa[SF->sfk]++;          // increment multiplicity
        r/=*pp;                      // 'divide out' current prime
      }
      SF->sfk++;                   // increment count of distinct prime factors
    }
    ++pp;                        // next prime
  }

  if(r>1) {                   // If n contains prime > 2^16:
    SF->sfp[SF->sfk]=r;         // store it as last prime of SF
    SF->sfa[SF->sfk]=1;         // set its multiplicity to 1
    SF->sfk++;                  // increment count of distinct prime factors
  }
}
```

```
u_long S(u_long n)
{
  struct SF_struct SF;  // to store SF of n
  int sfi;              // index of current term of SF of n
  u_long Sn;            // current guess at S(n)
  u_long x;             // S(current term of SF of n) where it might exceed
                        // current value of Sn.

  if(n==1) return 0;    // special case

  getSF(n, &SF);        // calc. and store SF of n

  // First guess at S(n) is S(p^a), where p is the largest prime in the SF
  // of n and a is its multiplicity.  This pre-empts the calculation of S(p^a)
  // for the remaining terms where, as is likely, p*a for these terms is <=
  // this initial guess (since S(p^a) <= p*a always):
  sfi=SF.sfk-1;
  Sn=Spa(SF.sfp[sfi],SF.sfa[sfi]);

  while(sfi>0) {  // while more term(s):
    sfi--;           // next term
    if(SF.sfp[sfi]*SF.sfa[sfi]>Sn) {  // if this term may have larger S(p^a):
      x=Spa(SF.sfp[sfi],SF.sfa[sfi]);    // calc. it
      if(x>Sn) Sn=x;                     // if new max., update Sn with it
    }
  }
  return Sn;  // That's all folks!
}

u_long Spa(u_long p, int a)
{
  // Refer to item 3) of the algorithm description in S(n).h.
  int c;  // largest multiple of p such that f(p*c, p) <= a (eventually!)
  int z;  // f(p*c, p)
  int m;  // used to calc. no. of times p appears as factor of c

  if(a<=p) return p*a;

  c=a-2;
  z=f(p*c, p);
  while(z>a) {
    // d in items 3.2.3.1) and 3.2.3.3) of algorithm description is implicit
    // here:
    z--;
    m=c--;
    while(!(m%p)) {  // while p divides m:
      z--;
      m/=p;             // 'divide out' factor of p from m
    }
  }
  if(z<a) return p*(c+1);
  else return p*c;
}
```

S(n).C

```
int f(int x, int p)
{
  int k=0;  // count of appearance of prime p as a factor in the integers
            // from 1 to x.
  int xdp;  // successive divisions of x by p

  xdp=x/p;
  while(xdp>0) {
    k+=xdp;
    xdp/=p;
  }
  return k;
}
```

SMARANDACHE FUNCTION, S(n), for n=4294965296 to n=4294967295

\S of n plus:

n\	0	1	2	3	4	5	6	7	8	9
4294965296	268435331	168887	2799847	60492469	14316551	3226871	49941457	231547	337867	2798023
4294965306	5531	4294965307	1194373	101977	5174657	66569	967	4294965313	113025403	286331021
4294965316	1073741329	4583741	102261079	27356467	1602599	4919777	2147482661	22967729	2117833	24542659
4294965326	2147482663	648101	7255009	2671	57427	4294965331	3741259	75577	85889	858993067
4294965336	21517	11392481	5520521	2199163	12632251	40139863	2286989	595613	979691	4621
4294965346	3919	4294965347	8911	90407	83641	1431655117	1228537	2545919	715827559	1493
4294965356	14969	158597	3109	108907	2339	4294965361	2147482681	23719	118423	38629
4294965366	419	30971	4925419	1431655123	195137	11897411	357913781	104755253	641	1307
4294965376	5527	830587	154573	279857	1199711	204522161	39107	4294965383	6628033	858993077
4294965386	2147482693	21367987	3567247	30034723	7535027	35449	268435337	5749619	2147482697	86357
4294965396	357913783	4040419	35461	27012361	1952257	377513	70867	26349481	511549	977239
4294965406	235289	2347	154807	529393	5167	14461163	672349	4294965413	34301	24683
4294965416	5179	625997	22138997	4013	23860919	15107	17747791	204522163	39727	10105801
4294965426	816223	9965117	12109	6040739	321239	28793	5772803	746561	5003	1453457
4294965436	65983	1530091	79536397	2473	13421767	1551089	308857	2617	28871	858993089
4294965446	178793	65579	36097	564607	28633103	11731	121313	13997	2465537	2207
4294965456	8329	4294965457	567067	423191	70019	4294965461	415937	138547273	536870683	168331
4294965466	10275037	14364433	7615187	1822217	40699	88873	169681	4001	5549051	6079
4294965476	157	1194041	2147482739	1935541	18233	22031	718943	84869	1073741371	8863
4294965486	102261083	4294965487	14639	973253	18673763	2754949	7253	30509	28661	137593
4294965496	50951	14387	715827583	390451409	1227133	835759	16909313	637519	294337	72679
4294965506	17903	72091	29483	3804221	255197	2017	103583	7039	306783251	188417
4294965516	357913793	99882919	4699087	12236369	53687089	832519	715827587	142799	1073741381	1847297
4294965526	2147482763	252645031	57193	4294965529	384509	103391	32789	6381821	12132671	122713301
4294965536	5835551	26881	88919	3296213	2468371	6529	2306641	11831861	64319	106747
4294965546	19979	107077	1073741387	29339	2321603	622549	89478449	179623	17599	891997
4294965556	4948117	20355287	1038937	12569	28057	2293	7561	77933	119304599	78090283
4294965566	13174741	1431655189	67108837	397057	20452217	19709	51829	20507	298801	1031
4294965576	16268809	1618909	3081037	1431655193	11302541	4294965581	10374813	12781	2949839	337
4294965586	19701677	139	2843	2549	429496559	9739151	8801159	72796027	2970239	50529007
4294965596	1073741399	218941	961703	226050821	198841	4294965601	2147482801	33294307	40459	5335361
4294965606	154373	118361	536870701	923053	9283	35527	307	4294965613	78487	21859
4294965616	515231	4294965617	2957	89611	5147	7495577	2147482811	330381971	4409	3061
4294965626	692513	1828423	46684409	2459	251609	9137	170327	1302689	2147482817	3193283
4294965636	9177277	682933	2147482819	1431655213	2551	4294965641	784039	7027	4549	40153
4294965646	2937733	209623	89478451	119747	85899313	82245879	100547	6400843	160033	40961
4294965656	20731	26371	147827	4294965659	1745921	16582879	118757	28071671	20929	858993133
4294965666	9061109	17111417	119557	4603393	429496567	4294965671	59652301	4294965673	93368819	3011
4294965676	1073741419	28825273	715827613	4294965679	101873	18013	505409	4294965683	1360889	12347
4294965686	195225713	1431655229	400949	2038427	47721841	4294965691	130261	45613	113025413	7880671
4294965696•	2729	58741	1753	477218411	42949657	3373	94399	13054607	201907	286331047
4294965706	1059439	594953	1151	75431	61356653	13267	72727	2238127	1831	858993143
4294965716	220889	1286303	2147482859	889411	1556147	4294965721	118273	249287	153391633	29153
4294965726	1716813	849311	601873	8783161	70099	494413	23057	4294965733	2147482867	859853
4294965736	536870717	4294965737	179	7517	16519099	130150477	1689601	7309	6701	1553333
4294965746	2147482873	887573	63161261	650851	121843	226050829	29173	142837	2147482877	6270023
4294965756	2818219	4294965757	7877	204522179	1677721	18755309	89669	82183	1073741441	4457
4294965766	1026031	4294965767	2311	24793	7229	477218419	1000691	19792469	65075239	65497
4294965776	268435361	1404961	1162687	603989	4933	12377423	836573	42223	536870723	16943
4294965786	4286393	244157	84407	1811	3539	41698897	7433	4294965793	306783271	286331053
4294965796	4337	55823	57649	919	21474829	7052489	102559	556559	1451	8753
4294965806	23831	41357	24317	116080157	143165527	38008547	10627	1431655271	17471	122713309
4294965816	6761	677119	349241	2165893	57527	4294965821	20789	81037091	20719	19088737
4294965826	40829	2129	357913819	4799	38669	651641	536870729	1395829	238609213	4999
4294965836	359231	7883	5355319	4294965839	1626881	4294965841	2147482921	48661	1757351	99409
4294965846	1018247	4294965847	36599	1431655283	721843	954649	32749	348117	4303573	733
4294965856	7129	237173	72211	148102271	214748293	11098103	309391	4864061	25565273	7933
4294965866	107069	1186127	1073741467	33353	407879	613566553	268435367	65173	2147482937	15959
4294965876	83113	36749	306783277	477218431	107374147	186737647	28031	70309	1091	5843491
4294965886	2147482943	4294965887	2179	2081903	2161	366997	6869	620929	6949783	78090289
4294965896	19289	84121	2147482949	613566557	14316553	252645053	2147482951	9633	1009	858993181
4294965906	6871	364691	1073741477	110127331	429496591	4294965911	87767	7829	7588279	944987
4294965916	12064511	75217	167759	18481	103643	405683	25799	125941	351931	171798637
4294965926	256049	3119	1574401	52181	181913	97553	831713	2243	43826183	3886847
4294965936	89478457	4294965937	1621	130150483	214748297	49803	639703	1290167	536870743	19927
4294965946	74051137	263	3933119	4294965949	4651	79259	3947579	25718359	1494421	122713313
4294965956	5417	38693387	6221	138547289	627919	30034727	306783283	1431655321	241453	858993193
4294965966	1191061	4294965967	2591	8699	991909	4294965971	13597	311749	1990253	457
4294965976	76695821	4294965977	68389	1321	690509	1431655327	911	74471	91867	80513
4294965986	7430737	36709111	26188817	15259	229	61333	117709	5282861	5276371	858993199
4294965996	6907	5956957	33331	4289	131	4294966001	715827667	1322749	38053	32497
4294966006	36398017	4294966007	178956917	60492479	13159	243769	1073741503	471301	1318283	1340083
4294966016	15101	14737	415133	2579	5939	40031	19701679	6537239	529981	19219
4294966026	279511	35495587	258173	254879	2225371	116080163	1439	3637	358691	11003
4294966036	56512711	2235797	65075243	47197429	110581	320927	93368827	4294966043	357913837	1128769
4294966046	238001	163003	134217689	273617	9544369	3256229	3343	238649	126322531	277363
4294966056	4519	2213	2147483029	59069	2788939	54366659	24683713	351269	28627	16231
4294966066	2147483033	613566581	3224449	9695183	10475527	3853	940229	4294966073	11987	171798643
4294966076	103613	477218453	182129	91382257	1033	12521767	303617	46182431	7110871	68791
4294966086	79536409	4294966087	3701	1431655363	4013987	11392483	357913841	1081583	8803	13634813
4294966096	20543	44278001	45137	4294966099	829	85733	306783293	340573	2657	1433
4294966106	2147483053	669937	1073741527	35083	233549	4554577	7064089	167269	69273647	78090293
4294966116	131441	3271109	2147483059	33294311	22171	4294966121	14035837	2633333	1823	11453243
4294966126	2749	73583	257863	4294966129	674249	1759	242873	665783	31122943	2340581
4294966136	536870767	521	2147483069	252645067	1213	1328477	58040083	792283	4793489	27709459
4294966146	1993949	340519	8873897	9697	40009	8641783	4153	4294966153	2147483077	286331077
4294966156	4858559	186737659	77647	390451469	53687077	5168431	288679	4294966163	578213	122713319
4294966166	15675059	155801	905347	251	13015049	82139	16087	6661	9629969	171798647
4294966176	481067	4294966177	74051141	2237	9336883	390451471	55063669	817933	97987	86531
4294966186	128613	4294966187	18837571	80737	119737	21367991	24403217	14965039	191	365063
4294966196	1012009	47279	2008871	7548271	1022611	352133	6317	18313	2562629	19976587
4294966206	1063637	2113	5182219	57847	1633067	2245147	525571	6719	21503	286331081
4294966216	14510021	4294966217	43481	237409	10853	1748053	30246241	1361	5263439	15618059
4294966226	16007	890887	9787	446323	199673	4294966231	536870779	1739557	4029049	812671
4294966236	1121987	4294966237	95287	1190071	246271	38239	23743	4294966243	1073741561	15070057
4294966246	2147483123	566693	462421	28676809	3307	1431655417	1109	116080169	763957	9323
4294966256	38347913	477218473	171401	12744707	13463	5501	16393001	29671	821	29620457
4294966266	163543	355573	1073741567	381673	61356661	96263	683	2217329	2147483137	179
4294966276	859681	87652373	715827713	2963	9761287	4073	6691	35537	58169	858993257
4294966286	165191011	15391	143779	16417	2663	311	63161269	6721387	74051143	37347533
4294966296	304867	4294966297	3883333	332711	26141	72796039	380557	35747	1254371	40904441
4294966306	251197	2458481	357913859	127663	919693	477218479	2753	390451483	15230377	31547
4294966316	26188819	5940479	108191	6074917	5051	226050859	139801	49367429	1600211	197243
4294966326	1571	116080171	536870791	159072827	430789	19489	174337	613566619	49941469	397
4294966336	17159	4294966337	1411889	325697	1614649	2706343	2147483171	252645079	178956931	858993269
4294966346	195225743	68174069	77813	39403361	28633109	135347	5209	54133	200381	858993271
4294966356	56893	8761	2147483179	75350287	107053	146681	23091217	1762399	1367	25733
4294966366	283571	4294966367	581029	401887	429496637	1431655457	117877	4294966373	170557	701219
4294966376	26249	1907	113025431	390451489	14251	908603	108827	159072829	268435399	73727
4294966386	3851	186737669	151637	2464123	273043	33818633	890333	1057613	126322541	19991
4294966396	32783	1013681	17459213	399941	1342177	5557	48761	109001	357913867	858993281
4294966406	116099	1431655469	6047	41698703	3407	22067	97612873	149333	1679033	858993283

\S of n plus:

n\	0	1	2	3	4	5	6	7	8	9
4294966416	1877	813566631	48677	8088449	251	30899039	4804213	390451493	7213	57266219
4294966426	1564081	4294966427	7017919	95569	2939	204522211	577	389849	12541	636763
4294966436	37571	17674759	809	15913	261251	4294966441	614093	1431655481	41539	656221
4294966446	3911627	4294966447	1172207	1137137	965161	2669339	78301	3929521	199	5039
4294966456	26861	288427	6567227	172787	202021	19611719	126322543	99882941	2485513	858993293
4294966466	306783319	130150499	2848121	17970571	143165549	12763	6468323	1949	2147483237	13381
4294966476	38891	4294966477	3308911	7001	661	330382037	321143	2176871	1073741621	2951867
4294966486	69273653	613566641	178956937	3455323	429496649	159072833	56512717	6806603	7866239	858993299
4294966496	7895159	49367431	2147483249	4999961	86767	813566643	5413	1431655501	536870813	6151
4294966506	234487	416623	1153	477218501	81083	1868189	443	390487	1256573	26407
4294966516	306871	49891	1951	124193	8259551	345727	3984199	9565627	10631	7469507
4294966526	2251	477218503	16777213	613566647	443237	1210873	4493	32611	29417579	40123
4294966536	8521759	482093	2147483269	16086017	396947	2307881	715827757	613566649	149713	56509
4294966546	165191021	252645091	16193	3023	12271333	1431655517	536870819	4294966553	238609253	32443
4294966556	73883	1129957	8910719	330382043	64373	119083	52377641	617359	530767	16207421
4294966566	34781	65563	463219	511123	201547	25577	5107	186163	648199	364753
4294966576	1405421	293	37693	138547309	214748329	21577	22139003	4294966583	6170929	9439487
4294966586	58040089	75350291	133169	12739	47721851	4294966591	222953	21367993	298303	179593
4294966596	383	4441	9717119	688627	32587	465781	20369	208261	1073741651	286331107
4294966606	15661	58835159	29826157	22253713	1523	110127349	5088823	7993	9061111	490573
4294966616	870131	105323	764501	4294966619	6551	1693	21262211	92347	11719	16901
4294966626	79536419	45751	12064513	23469763	1297573	312839	719	8597	306783331	2219621
4294966636	873671	330382049	7639	4294966639	5821	15671	74051149	14549	153151	4567
4294966646	2147483323	1281697	1080223	284831	2202547	4294966651	1073741663	799361	195225757	15877
4294966656	17449	4294966657	4121849	139361	86627	4294966661	20063	756023	63997	26030101
4294966666	556487	4294966667	152239	214309	429496667	1237	8659207	1033189	715827779	120223
4294966676	757	8251771	49941473	18979	6053	24826397	86477	12030719	1073741671	5867
4294966686	17959	390451517	2383	36709117	21401	109451	357913891	54366667	1028981	286331113
4294966696	536870837	16582883	21691751	109961	87119	75350293	38569	138547313	12782639	218629
4294966706	2147483353	975907	4492643	16976153	143165557	2280917	536870839	1431655571	701	2111
4294966716	119304631	3607	919	20164163	431	2194	482689	69371	20259277	4241
4294966726	52377643	64103981	13765919	208889	144563	84349	397	889043	4079	924643
4294966736	1020667	30460757	11864549	32292983	71582779	40129	195225761	559459	36209	858993349
4294966746	115811	18041	38653	1431655583	17179867	4141723	14913079	69463	79841	35671
4294966756	1073741689	5811863	1299143	198391	4789	89417	46411	15176561	756689	27709463
4294966766	321047	191	27751	4294966769	47721853	116080183	107021	24265349	306783341	15618061
4294966776	1844917	9133	99223	477218531	16519103	2387419	715827797	6936557	178481	154523
4294966786	1490273	148102303	34019	85199	7459	562979	536870849	330382061	10683997	78311
4294966796	1789	43383503	2147483399	17821439	3579139	93229	93503	1431655601	4481	317323
4294966806	761	99682949	32173	204522229	429496681	1072937	32779	4294966813	4021	88129
4294966816.	3257	1516049	359171	1766749	214748341	13899569	26561	2907899	25679	61291
4294966826	29417581	13229	9502139	4294966829	35081	91382273	349981	477218537	27059	858993367
4294966836	48623	613566691	15449521	88609	349753	38351	20347	19259941	275389	1579
4294966846	2147483423	186737689	14983	131437	2381	6299	983	782183	4137733	65557
4294966856	511793	82559	761249	3731509	23860927	902873	1030957	35983	747731	4517
4294966866	191449	2999	1073741717	2909	293	330382067	1345541	1327	611993	2290649
4294966876	1073741719	4294966877	5076793	911689	26843543	1431655627	58040093	138053	27531839	221219
4294966886	306783349	338213	23633	1931	3491843	8311	1073741723	911	50647	9651611
4294966896	1489	330382069	1246363	154423	6135667	113783	715827817	59791	22193	95443709
4294966906	5639	178207	357913909	4294966909	2029	1431655637	70051	13687	2797	858993383
4294966916	7307	10928669	82339	641327	1154561	36092159	8555711	453199	3271	36191
4294966926	715827821	4294966927	572357	8263	5436667	104755291	57943	12744709	35204647	21839
4294966936	18121	2403451	42107519	186737693	19522577	8297	306783353	4294966943	9151	858993389
4294966946	20069939	5990191	56512723	47197439	53923	9817	536870869	1431655651	2147483477	683
4294966956	1381907	353899	23087	397351	36497	447439	217643	87652387	1073741741	393853
4294966966	70067	226050893	59652319	97169	7621	1431655657	449	390451543	715827829	12113
4294966976	67108859	163487	2147483489	260791	947	4294966981	20297	1431655661	78341	41177
4294966986	12558383	569851	602887	1588963	98081	3427747	323027	18341	2147483497	193
4294966996	119159	4294966997	26107	16330673	4294967	110127359	4106087	554977	13256071	6458597
4294967006	11483869	65519	5835553	1597	143165567	22453	21913097	375467	12707003	858993403
4294967016	38149	795217	14412641	150053	180007	365747	5549053	17401	14128181	10141
4294967026	23917	18541	32537629	4294967029	11608019	5683	536870879	3764213	3392549	858993407
4294967036	2445881	1413283	8355967	20549	241	10710641	96703	9851	34636831	11767033
4294967046	18111	39163	141319	477218561	5009	21582749	357913921	37201	2879	1228889
4294967056	5347	252645121	83459	1755197	214748353	7121	4007	768193	564533	19976591
4294967066	15761	132967	2599859	2408843	143165569	65551	12201611	16086019	1779191	791699
4294967076	119304641	23761	5250571	110127361	107374177	417839	102281121	390451553	342283	12917
4294967086	2147483543	4294967087	1132639	613566727	429496709	21467	9209	11332367	21691753	395303
4294967096	76695841	1431655699	2147483549	96553	362	181967	66977	68174081	5197	9199
4294967106	1583	2135737	2609	1431655703	1252177	4294967111	182423	88463	675097	286331141
4294967116	63841	10058471	754297	15053	236507	102871	12859183	17321	81031	14753
4294967126	2147483563	21841	85639	161471	463319	12899	1073741783	80021	186689	129347
4294967136	13147	138547327	1467863	159072857	214748357	1163	2476913	4294967143	7321	576119
4294967146	93368851	13990121	2251031	2044249	85899343	77249	16381	33818639	70853	515293
4294967156	1073741789	1932059	2147483579	5156023	647	4294967161	34919	4886197	381707	858993433
4294967166	1262483	3229	1801	179383	184571	651839	357913931	9739	2147483587	7283
4294967176	87481	104755297	317159	62929	34981	3583	195225781	14767	6197	44171
4294967186	152293	204522247	5449	4294967189	143165573	82791	23342213	29333	7134497	45210181
4294967196	2521	4294967197	1296007	46182443	31033	11576731	238609289	26029	97612891	286331147
4294967206	1809169	91382281	18539	488009	25264513	477218579	292493	75167	65981	13109
4294967216	3780781	1431655739	52377649	11959	25411	39403369	813749	3089	1726273	231223
4294967226	22037	252645131	10631107	69779	13854733	4294967231	8191	3557	318949	22025473
4294967236	417961	825479	29389	70409299	23327	1431655747	145681	619139	45083	69001
4294967246	28111	65543	1877171	15966421	4111	1664071	1527371	1431655751	862097	4259
4294967256	59652323	82613	2147483629	4817	32771	281	19867	64103989	262657	95443717
4294967266	367531	37549	29587	35809	115861	3449	536870909	395959	6118187	171798691
4294967276	361651	1861711	18046081	4294967279	127	8951	795659	1226783	46684427	122713351
4294967286	715827881	65539	2089	1039	22605091	4294967291	331	464773	2147483647	65537

Timing of S(n).c Module for Calculation of Smarandache Function, version 1

Time taken to calculate S(n) depends on how easy it is to factor n. Less time is required if n has "small" prime factors. So, in the following table, the values of n shown are the mid-points of ranges (n-99 thru n+99). Times shown are for calculating S(n) for all integers in each range 10 times over:

n	time (ms)
100	268
200	308
400	345
800	387
1600	432
3200	490
6400	571
12800	661
25600	766
51200	919
102400	2450
204800	4036
409600	5670
819200	7977
1638400	10423
3276800	13004
6553600	16302
13107200	23438
26214400	29642
52428800	37011
104857600	50330
209715200	62363
419430400	77888
838860800	108179
1677721600	158480
3355443200	159111

Timing of S(n).c Module for Calculation of Smarandache Function, version 2

"Time to n" is the time taken to calculate S(n) for all n <= that shown. "Time add." is the time taken to calculate S(n) for all n > previous n and <= current n. All times are in milliseconds (as per version 1):

n	Time to n	Time add.
50000	18223	18223
100000	66763	48540
150000	139191	72428
200000	229634	90443
250000	335252	105618
300000	452539	117287
350000	579419	126880
400000	715146	135727
450000	859963	144816
500000	1012335	152372
550000	1171221	158886
600000	1336899	165678
650000	1508825	171927
700000	1686808	177983
750000	1870023	183215
800000	2058983	188961
850000	2252457	193473
900000	2450892	198435
950000	2653620	202728
1000000	2860734	207115
1050000	3072049	211314
1100000	3288502	216454
1150000	3509106	220603
1200000	3733965	224860
1250000	3962171	228206
1300000	4194158	231987
1350000	4429331	235173
1400000	4668560	239229
1450000	4910513	241953
1500000	5155601	245088
1550000	5404652	249051
1600000	5656512	251859
1650000	5911306	254794
1700000	6169686	258380
1750000	6431383	261697
1800000	6696172	264789
1850000	6963206	267034
1900000	7232974	269768
1950000	7505412	272438
2000000	7779763	274351
2050000	8056579	276816
2100000	8336442	279863
2150000	8620053	283611
2200000	8905641	285588
2250000	9194727	289086
2300000	9486449	291722
2350000	9780105	293655
2400000	10076920	296815
2450000	10375202	298282
2500000	10676383	301181

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Mike Mudge pays a return visit to the Florentin Smarandache Function.[1]

The originator of this function, Florentin Smarandache, an Eastern European mathematician, escaped from the country of his birth because the Communist authorities had prohibited the publication of his research papers and his participation in international congresses. After spending two years in a political refugee camp in Turkey, he emigrated to the United States.

Robert Muller of The Number Theory Publishing Company, PO Box 42561, Phoenix, Arizona 85080, USA, decided to publish a selection of his papers, mmencing with *The Smarandache Function Journal*, Vol 1, No 1, December 1990. ISSN 1053-4792.

PCW readers may have met this function before, in Numbers Count -112- July 1992, where a very encouraging response was generated. This article [February 1993] is complete in itself so don't worry if you have filed the July issue! It may be thought that those readers who attempted the previous problem-set will have an unfair advantage. However, it must be realised that no Numbers Count problems are completely original so previous work within a given subject area is always a possibility and the prize is awarded using 'suitable subjective criteria' anyway, *so please have a go and submit your results*, however trivial they may seem to yourself.

Definition For all non-null integers, n, the Smarandache Function, S(n), is defined to be the smallest integer such that (S(n))! (The Factorial Function with argument S(n),) is divisible by n. e.g. S(18) = 6 because 6! is divisible by 18 but 1!....5! are not.

Problem (0) Design and implement an algorithm to generate and store/tabulate S(n) as a function of n upto a given n_{max}.

Hint It may be advantageous to consider the STANDARD FORM of n, *viz* $n = e p_1^{a_1} p_2^{a_2} p_3^{a_3} \ldots\ldots p_r^{a_r}$ where $e=\pm1$, and $p_1, p_2, p_3 \ldots p_r$ denote the distinct prime factors of n and $a_1, a_2, a_3 \ldots a_r$ are their respective multiplicities.

NOTE S(n) is an even function, by which is meant S(-n) = S(n).

Problem (i) Using either graphical or finite difference technique (i.e. the construction of difference tables etc) or indeed anything else that comes to mind, address the following questions:

(a) Is there a closed expression (formula) for S(n)?

(b) Is there a good asymptotic expression for S(n)? (By which is meant a formula, which although never (in general) exact, becomes a better and better approximation to S(n) as n becomes larger and larger.)

Problem (ii) For a specified non-null integer m, under what conditions does S(n) divide the difference n - m?

Problem (iii) Investigate the possible integer solutions, (x,y,z) of $S(x^n) + S(y^n) = S(z^n)$ for any n greater than or equal to 1. e.g. examine the solution (5,7,2048) when n = 3.

(It can be proved that an infinity of solutions exist for any such n-value.) Compare with Fermat's Theorem re. $x^n+y^n = z^n$.

Problem (iv) Investigate the possibility of finding two integers n and k such that the LOGARITHM of $S(n^k)$ to the BASE $S(k^n)$ is an integer.

Problem (v) Recall that 'Gamma' defined as the limit as n tends to infinity of (1 + 1/2 + 1/3 + 1/4....+1/n - log(n)) exists, is known as Euler's Constant and is approximately 0.577.

Investigate the possible existence of 'Samma' defined as the limit as n tends to infinity of (1 + 1/S(2) + 1/S(3)....+1/S(n) - log(S(n)).

Problem (vi) Find the number of PARTITIONS of n as the sum of S(m) for $2 < m \le n$. See *PCW* August 1989 and February 1990 for other problems involving PARTITIONS of n.

Review of 'Numbers Count -118- February 1993: a revisit to The Florentin Smarandache Function[2]

This produced a number of 'quite powerful' responses. As a note of related interest, the latest publication of Fl.Smarandache is 'A Numerical Function in Congruence Theory', *Libertas Mathematica* (American Romanian Academy of Arts and Science) vol 12, 1992, pp 181-185, Arlington, Texas.

Pal Gronas of Norway submitted theoretical results on both problems 0 & (v). However, the clear winner this month is a former regular respondent, now retired, Henry Ibstedt, Glimminge 2036, 280 60 Broby, Sweden. Henry used a dtk-computer with 486/33MHz processor in Borland's Turbo Basic. S(n) upto 10^6 took 2hr 50min. He completed a great deal of work on all problems except (vi): details of numerical results and conclusions available from Henry or myself to interested readers. What about problem (vi)?

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1 *Republished from <Personal Computer World>, No.118, 403, February 1993 (with the author permission), because some of the following research papers are referring to these open problems.*

2 *Republished from <Personal Computer World>, No.124, 495, August 1993 (with the author permission).*

A NOTE ON S(p^r)

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<u>**Problem (0).**</u> If $\prod_{i=1}^{k} p_i^{r_i}$ is the prime factorization of n, then it is easy to verify that

$$S(n) = S(\prod_{i=1}^{k} p_i^{r_i}) = \max\{ S(p_i^{r_i}) \}_{i=1}^{k}.$$

From this formula we see that it is essensial to determine $S(p^r)$, where p is a prime and r is a natural number.

Legendres formula states that

$$n! = \prod_{i=1}^{k} p_i^{\sum_{m=1}^{\infty}[n/p_i^m]}.$$

This formula gives us a lower and an upper bound for $S(p^r)$, namely

(1) $$(p-1)r+1 \leq S(p^r) \leq pr.$$

It also implies that p divides $S(p^r)$, which means that

$$S(p^r) = p(r-i) \text{ for a particular } 0 \leq i \leq [\tfrac{r-1}{p}].$$

A proof of the non-existence of "Samma".

by Pål Grønås

Introduction: If $\prod_{i=1}^{k} p_i^{r_i}$ is the prime factorization of the natural number $n \geq 2$, then it is easy to verify that

$$S(n) = S(\prod_{i=1}^{k} p_i^{r_i}) = \max\{\ S(p_i^{r_i})\ \}_{i=1}^{k}.$$

From this formula we see that it is essensial to determine $S(p^r)$, where p is a prime and r is a natural number.

Legendres formula states that

(1) $$n! = \prod_{i=1}^{k} p_i^{\sum_{m=1}^{\infty}[n/p_i^m]}.$$

The definition of the Smarandache function tells us that $S(p^r)$ is the least natural number such that $p^r \mid (S(p^r))!$. Combining this definition with (1), it is obvious that $S(p^r)$ must satisfy the following two inequalities:

(2) $$\sum_{k=1}^{\infty} \left[\frac{S(p^r)-1}{p^k}\right] < \ r \ \leq \sum_{k=1}^{\infty} \left[\frac{S(p^r)}{p^k}\right].$$

This formula (2) gives us a lower and an upper bound for $S(p^r)$, namely

(3) $$(p-1)r+1 \ \leq \ S(p^r) \ \leq \ pr.$$

It also implies that p divides $S(p^r)$, which means that

$$S(p^r) = p(r-i) \text{ for a particular } 0 \ \leq \ i \ \leq \ [\tfrac{r-1}{p}].$$

"Samma": Let $T(n) = 1 - \log(S(n)) + \sum_{i=2}^{n} \frac{1}{S(i)}$ for $n \geq 2$. I intend to prove that $\lim_{n\to\infty} T(n) = \infty$, i.e. "Samma" does not exist .

First of all we define the sequence $p_1 = 2$, $p_2 = 3$, $p_3 = 5$ and $p_n =$ the nth prime.

Next we consider the natural number p_m^n. Now (3) gives us that

$$\begin{aligned} S(p_i^k) &\le p_i\, k \quad \forall i \in \{1,\ldots,m\} \text{ and } \forall k \in \{1,\ldots,n\} \\ &\Downarrow \\ \frac{1}{S(p_i^k)} &\ge \frac{1}{p_i\, k} \\ &\Downarrow \\ \sum_{i=1}^{m}\sum_{k=1}^{n}\frac{1}{S(p_i^k)} &\ge \sum_{i=1}^{m}\sum_{k=1}^{n}\frac{1}{p_i\, k} = \Big(\sum_{i=1}^{m}\frac{1}{p_i}\Big)\cdot\Big(\sum_{k=1}^{n}\frac{1}{k}\Big) \\ &\Downarrow \end{aligned}$$

(4)
$$\sum_{k=2}^{p_m^n}\frac{1}{S(k)} \ge \Big(\sum_{k=1}^{m}\frac{1}{p_k}\Big)\cdot\Big(\sum_{k=1}^{n}\frac{1}{k}\Big)$$

since $S(k) > 0$ for all $k \ge 2$, $p_a^b \le p_m^n$ whenever $a \le m$ and $b \le n$ and $p_a^b = p_c^d$ if and only if $a = c$ and $b = d$.

Futhermore $S(p_m^n) \le p_m\, n$, which implies that $-\log S(p_m^n) \ge -\log(p_m\, n)$ because $\log x$ is a strictly increasing function in the intervall $[2, \infty)$. By adding this last inequality and (4), we get

$$\begin{aligned} T(p_m^n) &= 1 - \log(S(p_m^n)) + \sum_{i=2}^{p_m^n}\frac{1}{S(i)} \ge 1 - \log(p_m\, n) + \Big(\sum_{k=1}^{m}\frac{1}{p_k}\Big)\cdot\Big(\sum_{k=1}^{n}\frac{1}{k}\Big) \\ &\Downarrow \\ T(p_m^{p_m}) &\ge 1 - \log(p_m^2) + \Big(\sum_{k=1}^{m}\frac{1}{p_k}\Big)\cdot\Big(\sum_{k=1}^{p_m}\frac{1}{k}\Big) \quad (n = p_m) \\ &\Downarrow \\ T(p_m^{p_m}) &\ge 1 + 2\,\Big(-\log p_m + \sum_{k=1}^{p_m}\frac{1}{k}\Big) + \Big(-2 + \sum_{k=1}^{m}\frac{1}{p_k}\Big)\cdot\Big(\sum_{k=1}^{p_m}\frac{1}{k}\Big) \\ &\Downarrow \\ \lim_{m\to\infty} T(p_m^{p_m}) &\ge 1 + 2\cdot\lim_{m\to\infty}\Big(-\log p_m + \sum_{k=1}^{p_m}\frac{1}{k}\Big) + \lim_{m\to\infty}\Big[\Big(-2 + \sum_{k=1}^{m}\frac{1}{p_k}\Big)\cdot\Big(\sum_{k=1}^{p_m}\frac{1}{k}\Big)\Big] \\ &= 1 + 2\cdot\lim_{p_m\to\infty}\Big(-\log p_m + \sum_{k=1}^{p_m}\frac{1}{k}\Big) + \lim_{m\to\infty}\Big[\Big(-2 + \sum_{k=1}^{m}\frac{1}{p_k}\Big)\cdot\Big(\sum_{k=1}^{p_m}\frac{1}{k}\Big)\Big] \\ &= 1 + 2\gamma + \lim_{m\to\infty}\Big(-2 + \sum_{k=1}^{m}\frac{1}{p_k}\Big)\cdot\lim_{p_m\to\infty}\Big(\sum_{k=1}^{p_m}\frac{1}{k}\Big) \quad (\gamma = \text{Euler's constant}) \\ &= \infty \end{aligned}$$

since both $\sum_{k=1}^{t}\frac{1}{k}$ and $\sum_{k=1}^{t}\frac{1}{p_k}$ diverges as $t \to \infty$. In other words, $\lim_{n\to\infty} T(n) = \infty$. □

A BASIC PROCedure to calculate S(p^i)

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Integer function of a single variable S%(N%)

S is the least integer such that S! is divisible by N.

Obviously for a prime S(p)=p since this is the least factorial to include p.

It is easy to see that for two primes p1>p2 S(p1*p2)=p1 since this factorial is necessary to include p1 and already includes p2. This generalizes to the product of any number of primes.

In fact it generalizes to the product of relatively prime numbers n1 and n2. S(n1*n2)=Max(s(n1),S(n2)).

Therefore we can simplify the general case to:

S(∏ni^pi)=Max(S(ni^pi))

All we need now is a way of calculating S for powers of primes.

Start with the inverse problem: for a given factorial and a given prime what is the maximum power of the prime included ?

Consider p=2. All even numbers contribute a factor, all multiples of 4 contribute another, all multiples of 8 contribute yet another ...etc. So the answer is got by summing succesive DIV 2 results (DIV p in general).

Returning to the calculation of S. To do this for a single N would require factorisation of N first. A program to calculate S for all integers up to N can avoid this by doing powers of 2, then powers of 3 and their products with powers of 2 then powers of 5 etc. Calculating S for all powers of a prime up to a maximum is straightforward. A BASIC PROCedure is attached. The main program requires some care and I have not been able to finish in time.

```
 10REM TEST PROC TOCALC S(P^I) FOR VALUES UPTO N
 20:
 30:
 40:
 50:
 60INPUT"UP TO",N%
 70DIM SPP%(100)
 80DIM NPP%(100)
 90INPUT"WHICH PRIME",P%
100PROCSpp(P%,N%)
110FOR I%=0 TO 100
120PRINT SPP%(I%),NPP%(I%)
130NEXT I%
140GOTO 60
150END
160DEF PROCSpp(P%,N%)
170I%=1
180NPP%(0)=1
190SPP%(0)=1
200J%=1
210PJ%=0
220REPEAT
230PJ%=PJ%+P%
240X%=FNinvSpp(P%,PJ%)
250REPEAT
260SPP%(I%)=PJ%
270NPP%(I%)=P%*NPP%(I%-1)
280I%=I%+1
290UNTIL I%>X%
300J%=J%+1
310UNTIL NPP%(I%-1)>N%
320ENDPROC
330DEF FNinvSpp(P%,N%)
340LOCAL S%,T%
350S%=0
360T%=N%
370REPEAT
380T%=T% DIV P%
390S%=S%+T%
400UNTIL T%<=1
410=S%
```

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The Florentin Smarandache Function S(n)

PCW February 1993

Problem (0)

A program *SMARAND* has been designed to generate S(n) up to a preset limit N (N up to 1000000 has been used in some applications). The program requires an input of prime number up to $\sqrt{N}$. Initially the program caculates $S(p_i^k)=D(i,k)$ for all primes p_i and all exponents k needed to reach the preset limit. It then proceeds to factorize consecutive values of n. If n is prime then S(n)=n otherwise p_i^k is replaced by D(i,k) whenever $k>1$. The largest component in the resulting array is determined and is equal to S(n). Slighly different versions of the program has been used depending on the application. Up to n=32000 both S(n) and D(i,k) were registed on files. The values of S(n) for $n \leq 4800$ were listed with the help of a program *SN_TAB* and the values of $S(p_i^k)$ were listed for $p_i \leq 73$, $k \leq 75$ with the help of a program *SNP_TAB*.

Problem (i)

(a) No closed expression for S(n) has been found

(b) No asymptotic expression for S(n) has been found. The behaviour of S(n) for $n \leq 32000$ has been graphically displayed using a program *SN_DISTR*.

Problem (ii)

S(n)/(n-m), $m \neq 0$ is equivalent to

$$m=n+kS(n), \text{ k integer} \text{ .... (1)}$$

Let us assume that m is a given prime p. From the definition of S(n) it is evident that for every n there exists a prime q such that q/n (or n=lq, l integer) and S(n)=jq (i integer). We can therefore write (1) in the form

$$p=q(l+kj) \text{ ... (2)}$$

To find solutions to (1) when m is a prime p it is therefore sufficient to chose n as a multiple of p which fills the condition $l+kj=1$.

In practice (as far as I have found) this means excluding from n those multiples of m(=p) which are divisible by primes larger than p and also cases where n-m has a different parity from S(n) as for example (n,m,S(n))=(5054,19,38) is not a solution while (2527,29,38) is a solution.

When m is not a prime let p be the largest prime such that p/m, i.e. m=rp. Solutions to (1) will then be found when n is a multipe of p for which the GCD of n and S(n) is rp.

The above conditions are sufficient but may not be necessary. Lists of solutions are however easily obtained (not included) by looking for solutions to (n-m) mod S(n) = 0.

Problem (iii)
A number of solutions to $S(x^n)+S(y^n)=S(z^n)$ has been obtained and listed for n=3,5,7 and 11. The program *SMAR_iii* uses only Smarandache function values of the type $S(p_i^k)$ which had first been sorted in ascending order using a program *SNP_SORT*.

Problem (iv)
A program *SMAR_iv* has been designed to find solutions to the equation $S(k^n)^i=S(n^k)$ but no non-trivial solutions were found in the selected search area $n\leq 8000$.

Problem (v)
In a first attempt values saved on file up to 32000 were in a program *SMAR_v* to calculate the sums Z(n)=1+1/S(2)+1/S(3)+ ... +1/S(n) for n= 800, 1600, 2400, ... 32000. These sums were used to study the behaviour of Z(n)-T(n) for various functions T(n):
T(n)=log(S(n)) gave a curve "parallel" to Z(n).
T(n)=log(largest prime<n) gave a similar result.
$T(n)=1+1/2^a+1/3^a+ \dots +1/n^a$ gave interesting results. Supplemented with a linear term a "nearly straight horizontal" line was obtained.
To see if this holds for larger values the exercise was repeated for $n\leq 1000000$. Computer files to store S(n) is now out of question and the generating program *SMARAND* was revised so that the partial sums Z(n) were calculated in the same program. T(n) was calculated in a separate program for vaious values of a. For a=0.5 and it was found that

$$1+1/S(2)+1/S(3)+ \dots +1/S(n) - (1+1/\sqrt{2}+1/\sqrt{3}+ \dots +1/\sqrt{n}) - (20k-58)$$

where k= n/25000 deviates from 0 with at most 10 in the interval 1 to 1000000 (at the points of representations in the graph, 1000000 was divided in 40 interval of 25000).

Problem (vi)
Not attempted.

Equipment
Calculations were done on an dtk-computer with 486/33 Mhz processor. Programs were written in Borlands Turbo Basic. Printouts were done on an HP IIP Laser printer. Some graphs were done on an HP Paintjet. The run time to calculate S(n) up to 1000000 was 2 h 50 m. The initial calculation up 32767 took 198 s.

'SMARAND, H. Ibstedt, 930320

'The Smarandache function S(n) calculated by comparing largest prime and S(P^A). The values of S(n) are calculated and registered in a file SN.DAT up to n = 32000. The calculation goes further in other applications.

```
DEFLNG A-S
CLS :T=TIMER
DIM P(168),D(168,75),K(168),L(168)
OPEN "PA" FOR INPUT AS #1
FOR I=1 TO 168 :INPUT #1,P(I) :NEXT :CLOSE #1
```

'This part of the program calculates S(P(I)^A) and saves the result in the array D(I,A), P(I) is the Ith prime number. The routine uses the fact that D(I,A)<=P(I)*A for a downward search for the value of D(I,A). This calculation goes beyond what is required to calculate S(n) up to n = 32000.

```
FOR I=1 TO 42
A=2 :P=P(I) :D(I,1)=P
WHILE A<76
C=0 :N=0
 L:
C=C+1
N=N+P
IF C>=A THEN D(I,A)=N :GOTO LWEND
PP=P*P
 L1:
IF N-PP*INT(N/PP)=0 THEN C=C+1 :PP=PP*P :GOTO L1
IF C>=A THEN D(I,A)=N :GOTO LWEND :ELSE L
 LWEND:
INCR A
WEND
NEXT
```

'The array D(I,A) is stored in a file SNP.DAT for future use.

```
OPEN "SNP.DAT" FOR OUTPUT AS #2
FOR I=1 TO 42 :FOR J=1 TO 75
PRINT #2,P(I),J,D(I,J)
NEXT :NEXT :CLOSE #2
```

'This part of the program calculates S(N). It calls on the subroutine NFACT to express N in prime factor form. Factors P(I)^A with A>1 are replaced by D(I,A) and placed in array L() together with the factors P(I) of multiplicity 1. S(N) is then the largest component of L(). S(N) is stored in a file SN.DAT.

```
N=1
OPEN "SN.DAT" FOR APPEND AS #3
WRITE #3,1
WHILE N<32000
INCR N :print n
'Factorize N.
```

```
GOSUB NFACT
IF K(0)>0 THEN S=P(0) :GOTO LWR

'Construct L().

 FOR I=1 TO 168 :L(I)=0 :NEXT
 C=0
 FOR I=1 TO M
 INCR C
 IF K(I)=1 THEN L(C)=P(I)
 IF K(I)>1 THEN L(C)=D(I,K(I))
 NEXT

'Find the largest value of L() and hence S(N).

S=0
FOR I=1 TO C
IF L(I)>S THEN S=L(I)
NEXT
  LWR:
WRITE #3,S
WEND
CLOSE #3
T=TIMER-T :PRINT T
END

'Subroutine for factorization of N.

  NFACT:
FOR I=0 TO 168 :K(I)=0 :NEXT :P(0)=0
N1=N :M=0
FOR I=1 TO 168
 LA:
IF N1-P(I)*INT(N1/P(I))=0 THEN K(I)=K(I)+1 :M=I :N1=N1/P(I) :GOTO LA
IF N1=1 THEN I=168
NEXT
IF N1>1 THEN P(0)=N1 :K(0)=1
RETURN
```

'SN_TAB, H. Ibstedt, 930321

'This program uses the results stored in the file SN.DAT produced by the program SMARAND to tabulate the first 4800 values of the function S(N).
'Set I=21 and NB=82 on HPIIP.

```
DEFINT I-P,S :DIM S(4800)
CLS :WIDTH "LPT1:",120 :S(1)=1 :T=TIMER
OPEN "SN.DAT" FOR INPUT AS #1
FOR I=1 TO 4800
INPUT #1,S(I)
NEXT
CLOSE #1
S1$="┌──────┬──────┬" :S2$="──────┬──────┬" :S3$="──────┬──────┐"
S4$="│  n   │ S(n) │" :S5$="  n   │ S(n) │" :S6$="  n   │ S(n) │"
S7$="├──────┼──────┼" :S8$="──────┼──────┼" :S9$="──────┼──────┤"
B1$="└──────┴──────┴" :B2$="──────┴──────┴" :B3$="──────┴──────┘"
I1=1 :I2=75 :P1=1
 LW:
LPRINT TAB(8) "The Smarandache Function S(n).
LPRINT TAB(8) S1$; :FOR I=1 TO 6 :LPRINT S2$; :NEXT :LPRINT S3$;
LPRINT TAB(8) S4$; :FOR I=1 TO 6 :LPRINT S5$; :NEXT :LPRINT S6$;
LPRINT TAB(8) S7$; :FOR I=1 TO 6 :LPRINT S8$; :NEXT :LPRINT S9$;
FOR I=I1  TO I2
LPRINT  TAB(8) "¤";
FOR J=0 TO 7
LPRINT USING "#####";I+J*75; :LPRINT " °"; :LPRINT USING "#####";S(I+J*75);
LPRINT " ¤" ;
NEXT
NEXT
LPRINT TAB(8) B1$; :FOR I=1 TO 6 :LPRINT B2$; :NEXT :LPRINT B3$;
LPRINT TAB(8) "Page"P1 "of 8.
LPRINT CHR$(12)
IF P1=1 THEN P1=2 :I1=601 :I2=675 :GOTO LW
IF P1=2 THEN P1=3 :I1=1201 :I2=1275 :GOTO LW
IF P1=3 THEN P1=4 :I1=1801 :I2=1875 :GOTO LW
IF P1=4 THEN P1=5 :I1=2401 :I2=2475 :GOTO LW
IF P1=5 THEN P1=6 :I1=3001 :I2=3075 :GOTO LW
IF P1=6 THEN P1=7 :I1=3601 :I2=3675 :GOTO LW
IF P1=7 THEN P1=8 :I1=4201 :I2=4275 :GOTO LW
PRINT "END" :END
```

n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)
1	0	76	19	151	151	226	113	301	43	376	47	451	41	526	263
2	2	77	11	152	19	227	227	302	151	377	29	452	113	527	31
3	3	78	13	153	17	228	19	303	101	378	9	453	151	528	11
4	4	79	79	154	11	229	229	304	19	379	379	454	227	529	46
5	5	80	6	155	31	230	23	305	61	380	19	455	13	530	53
6	3	81	9	156	13	231	11	306	17	381	127	456	19	531	59
7	7	82	41	157	157	232	29	307	307	382	191	457	457	532	19
8	4	83	83	158	79	233	233	308	11	383	383	458	229	533	41
9	6	84	7	159	53	234	13	309	103	384	8	459	17	534	89
10	5	85	17	160	8	235	47	310	31	385	11	460	23	535	107
11	11	86	43	161	23	236	59	311	311	386	193	461	461	536	67
12	4	87	29	162	9	237	79	312	13	387	43	462	11	537	179
13	13	88	11	163	163	238	17	313	313	388	97	463	463	538	269
14	7	89	89	164	41	239	239	314	157	389	389	464	29	539	14
15	5	90	6	165	11	240	6	315	7	390	13	465	31	540	9
16	6	91	13	166	83	241	241	316	79	391	23	466	233	541	541
17	17	92	23	167	167	242	22	317	317	392	14	467	467	542	271
18	6	93	31	168	7	243	12	318	53	393	131	468	13	543	181
19	19	94	47	169	26	244	61	319	29	394	197	469	67	544	17
20	5	95	19	170	17	245	14	320	8	395	79	470	47	545	109
21	7	96	8	171	19	246	41	321	107	396	11	471	157	546	13
22	11	97	97	172	43	247	19	322	23	397	397	472	59	547	547
23	23	98	14	173	173	248	31	323	19	398	199	473	43	548	137
24	4	99	11	174	29	249	83	324	9	399	19	474	79	549	61
25	10	100	10	175	10	250	15	325	13	400	10	475	19	550	11
26	13	101	101	176	11	251	251	326	163	401	401	476	17	551	29
27	9	102	17	177	59	252	7	327	109	402	67	477	53	552	23
28	7	103	103	178	89	253	23	328	41	403	31	478	239	553	79
29	29	104	13	179	179	254	127	329	47	404	101	479	479	554	277
30	5	105	7	180	6	255	17	330	11	405	9	480	8	555	37
31	31	106	53	181	181	256	10	331	331	406	29	481	37	556	139
32	8	107	107	182	13	257	257	332	83	407	37	482	241	557	557
33	11	108	9	183	61	258	43	333	37	408	17	483	23	558	31
34	17	109	109	184	23	259	37	334	167	409	409	484	22	559	43
35	7	110	11	185	37	260	13	335	67	410	41	485	97	560	7
36	6	111	37	186	31	261	29	336	7	411	137	486	12	561	17
37	37	112	7	187	17	262	131	337	337	412	103	487	487	562	281
38	19	113	113	188	47	263	263	338	26	413	59	488	61	563	563
39	13	114	19	189	9	264	11	339	113	414	23	489	163	564	47
40	5	115	23	190	19	265	53	340	17	415	83	490	14	565	113
41	41	116	29	191	191	266	19	341	31	416	13	491	491	566	283
42	7	117	13	192	8	267	89	342	19	417	139	492	41	567	9
43	43	118	59	193	193	268	67	343	21	418	19	493	29	568	71
44	11	119	17	194	97	269	269	344	43	419	419	494	19	569	569
45	6	120	5	195	13	270	9	345	23	420	7	495	11	570	19
46	23	121	22	196	14	271	271	346	173	421	421	496	31	571	571
47	47	122	61	197	197	272	17	347	347	422	211	497	71	572	13
48	6	123	41	198	11	273	13	348	29	423	47	498	83	573	191
49	14	124	31	199	199	274	137	349	349	424	53	499	499	574	41
50	10	125	15	200	10	275	11	350	10	425	17	500	15	575	23
51	17	126	7	201	67	276	23	351	13	426	71	501	167	576	8
52	13	127	127	202	101	277	277	352	11	427	61	502	251	577	577
53	53	128	8	203	29	278	139	353	353	428	107	503	503	578	34
54	9	129	43	204	17	279	31	354	59	429	13	504	7	579	193
55	11	130	13	205	41	280	7	355	71	430	43	505	101	580	29
56	7	131	131	206	103	281	281	356	89	431	431	506	23	581	83
57	19	132	11	207	23	282	47	357	17	432	9	507	26	582	97
58	29	133	19	208	13	283	283	358	179	433	433	508	127	583	53
59	59	134	67	209	19	284	71	359	359	434	31	509	509	584	73
60	5	135	9	210	7	285	19	360	6	435	29	510	17	585	13
61	61	136	17	211	211	286	13	361	38	436	109	511	73	586	293
62	31	137	137	212	53	287	41	362	181	437	23	512	12	587	587
63	7	138	23	213	71	288	8	363	22	438	73	513	19	588	14
64	8	139	139	214	107	289	34	364	13	439	439	514	257	589	31
65	13	140	7	215	43	290	29	365	73	440	11	515	103	590	59
66	11	141	47	216	9	291	97	366	61	441	14	516	43	591	197
67	67	142	71	217	31	292	73	367	367	442	17	517	47	592	37
68	17	143	13	218	109	293	293	368	23	443	443	518	37	593	593
69	23	144	6	219	73	294	14	369	41	444	37	519	173	594	11
70	7	145	29	220	11	295	59	370	37	445	89	520	13	595	17
71	71	146	73	221	17	296	37	371	53	446	223	521	521	596	149
72	6	147	14	222	37	297	11	372	31	447	149	522	29	597	199
73	73	148	37	223	223	298	149	373	373	448	8	523	523	598	23
74	37	149	149	224	8	299	23	374	17	449	449	524	131	599	599
75	10	150	10	225	10	300	10	375	15	450	10	525	10	600	10

n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)
601	601	676	26	751	751	826	59	901	53	976	61	1051	1051	1126	563
602	43	677	677	752	47	827	827	902	41	977	977	1052	263	1127	23
603	67	678	113	753	251	828	23	903	43	978	163	1053	13	1128	47
604	151	679	97	754	29	829	829	904	113	979	89	1054	31	1129	1129
605	22	680	17	755	151	830	83	905	181	980	14	1055	211	1130	113
606	101	681	227	756	9	831	277	906	151	981	109	1056	11	1131	29
607	607	682	31	757	757	832	13	907	907	982	491	1057	151	1132	283
608	19	683	683	758	379	833	17	908	227	983	983	1058	46	1133	103
609	29	684	19	759	23	834	139	909	101	984	41	1059	353	1134	9
610	61	685	137	760	19	835	167	910	13	985	197	1060	53	1135	227
611	47	686	21	761	761	836	19	911	911	986	29	1061	1061	1136	71
612	17	687	229	762	127	837	31	912	19	987	47	1062	59	1137	379
613	613	688	43	763	109	838	419	913	83	988	19	1063	1063	1138	569
614	307	689	53	764	191	839	839	914	457	989	43	1064	19	1139	67
615	41	690	23	765	17	840	7	915	61	990	11	1065	71	1140	19
616	11	691	691	766	383	841	58	916	229	991	991	1066	41	1141	163
617	617	692	173	767	59	842	421	917	131	992	31	1067	97	1142	571
618	103	693	11	768	10	843	281	918	17	993	331	1068	89	1143	127
619	619	694	347	769	769	844	211	919	919	994	71	1069	1069	1144	13
620	31	695	139	770	11	845	26	920	23	995	199	1070	107	1145	229
621	23	696	29	771	257	846	47	921	307	996	83	1071	17	1146	191
622	311	697	41	772	193	847	22	922	461	997	997	1072	67	1147	37
623	89	698	349	773	773	848	53	923	71	998	499	1073	37	1148	41
624	13	699	233	774	43	849	283	924	11	999	37	1074	179	1149	383
625	20	700	10	775	31	850	17	925	37	1000	15	1075	43	1150	23
626	313	701	701	776	97	851	37	926	463	1001	13	1076	269	1151	1151
627	19	702	13	777	37	852	71	927	103	1002	167	1077	359	1152	8
628	157	703	37	778	389	853	853	928	29	1003	59	1078	14	1153	1153
629	37	704	11	779	41	854	61	929	929	1004	251	1079	83	1154	577
630	7	705	47	780	13	855	19	930	31	1005	67	1080	9	1155	11
631	631	706	353	781	71	856	107	931	19	1006	503	1081	47	1156	34
632	79	707	101	782	23	857	857	932	233	1007	53	1082	541	1157	89
633	211	708	59	783	29	858	13	933	311	1008	7	1083	38	1158	193
634	317	709	709	784	14	859	859	934	467	1009	1009	1084	271	1159	61
635	127	710	71	785	157	860	43	935	17	1010	101	1085	31	1160	29
636	53	711	79	786	131	861	41	936	13	1011	337	1086	181	1161	43
637	14	712	89	787	787	862	431	937	937	1012	23	1087	1087	1162	83
638	29	713	31	788	197	863	863	938	67	1013	1013	1088	17	1163	1163
639	71	714	17	789	263	864	9	939	313	1014	26	1089	22	1164	97
640	8	715	13	790	79	865	173	940	47	1015	29	1090	109	1165	233
641	641	716	179	791	113	866	433	941	941	1016	127	1091	1091	1166	53
642	107	717	239	792	11	867	34	942	157	1017	113	1092	13	1167	389
643	643	718	359	793	61	868	31	943	41	1018	509	1093	1093	1168	73
644	23	719	719	794	397	869	79	944	59	1019	1019	1094	547	1169	167
645	43	720	6	795	53	870	29	945	9	1020	17	1095	73	1170	13
646	19	721	103	796	199	871	67	946	43	1021	1021	1096	137	1171	1171
647	647	722	38	797	797	872	109	947	947	1022	73	1097	1097	1172	293
648	9	723	241	798	19	873	97	948	79	1023	31	1098	61	1173	23
649	59	724	181	799	47	874	23	949	73	1024	12	1099	157	1174	587
650	13	725	29	800	10	875	15	950	19	1025	41	1100	11	1175	47
651	31	726	22	801	89	876	73	951	317	1026	19	1101	367	1176	14
652	163	727	727	802	401	877	877	952	17	1027	79	1102	29	1177	107
653	653	728	13	803	73	878	439	953	953	1028	257	1103	1103	1178	31
654	109	729	15	804	67	879	293	954	53	1029	21	1104	23	1179	131
655	131	730	73	805	23	880	11	955	191	1030	103	1105	17	1180	59
656	41	731	43	806	31	881	881	956	239	1031	1031	1106	79	1181	1181
657	73	732	61	807	269	882	14	957	29	1032	43	1107	41	1182	197
658	47	733	733	808	101	883	883	958	479	1033	1033	1108	277	1183	26
659	659	734	367	809	809	884	17	959	137	1034	47	1109	1109	1184	37
660	11	735	14	810	9	885	59	960	8	1035	23	1110	37	1185	79
661	661	736	23	811	811	886	443	961	62	1036	37	1111	101	1186	593
662	331	737	67	812	29	887	887	962	37	1037	61	1112	139	1187	1187
663	17	738	41	813	271	888	37	963	107	1038	173	1113	53	1188	11
664	83	739	739	814	37	889	127	964	241	1039	1039	1114	557	1189	41
665	19	740	37	815	163	890	89	965	193	1040	13	1115	223	1190	17
666	37	741	19	816	17	891	11	966	23	1041	347	1116	31	1191	397
667	29	742	53	817	43	892	223	967	967	1042	521	1117	1117	1192	149
668	167	743	743	818	409	893	47	968	22	1043	149	1118	43	1193	1193
669	223	744	31	819	13	894	149	969	19	1044	29	1119	373	1194	199
670	67	745	149	820	41	895	179	970	97	1045	19	1120	8	1195	239
671	61	746	373	821	821	896	8	971	971	1046	523	1121	59	1196	23
672	8	747	83	822	137	897	23	972	12	1047	349	1122	17	1197	19
673	673	748	17	823	823	898	449	973	139	1048	131	1123	1123	1198	599
674	337	749	107	824	103	899	31	974	487	1049	1049	1124	281	1199	109
675	10	750	15	825	11	900	10	975	13	1050	10	1125	15	1200	10

n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)
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1202	601	1277	1277	1352	26	1427	1427	1502	751	1577	83	1652	59	1727	157
1203	401	1278	71	1353	41	1428	17	1503	167	1578	263	1653	29	1728	9
1204	43	1279	1279	1354	677	1429	1429	1504	47	1579	1579	1654	827	1729	19
1205	241	1280	10	1355	271	1430	13	1505	43	1580	79	1655	331	1730	173
1206	67	1281	61	1356	113	1431	53	1506	251	1581	31	1656	23	1731	577
1207	71	1282	641	1357	59	1432	179	1507	137	1582	113	1657	1657	1732	433
1208	151	1283	1283	1358	97	1433	1433	1508	29	1583	1583	1658	829	1733	1733
1209	31	1284	107	1359	151	1434	239	1509	503	1584	11	1659	79	1734	34
1210	22	1285	257	1360	17	1435	41	1510	151	1585	317	1660	83	1735	347
1211	173	1286	643	1361	1361	1436	359	1511	1511	1586	61	1661	151	1736	31
1212	101	1287	13	1362	227	1437	479	1512	9	1587	46	1662	277	1737	193
1213	1213	1288	23	1363	47	1438	719	1513	89	1588	397	1663	1663	1738	79
1214	607	1289	1289	1364	31	1439	1439	1514	757	1589	227	1664	13	1739	47
1215	12	1290	43	1365	13	1440	8	1515	101	1590	53	1665	37	1740	29
1216	19	1291	1291	1366	683	1441	131	1516	379	1591	43	1666	17	1741	1741
1217	1217	1292	19	1367	1367	1442	103	1517	41	1592	199	1667	1667	1742	67
1218	29	1293	431	1368	19	1443	37	1518	23	1593	59	1668	139	1743	83
1219	53	1294	647	1369	74	1444	38	1519	31	1594	797	1669	1669	1744	109
1220	61	1295	37	1370	137	1445	34	1520	19	1595	29	1670	167	1745	349
1221	37	1296	9	1371	457	1446	241	1521	26	1596	19	1671	557	1746	97
1222	47	1297	1297	1372	21	1447	1447	1522	761	1597	1597	1672	19	1747	1747
1223	1223	1298	59	1373	1373	1448	181	1523	1523	1598	47	1673	239	1748	23
1224	17	1299	433	1374	229	1449	23	1524	127	1599	41	1674	31	1749	53
1225	14	1300	13	1375	15	1450	29	1525	61	1600	10	1675	67	1750	15
1226	613	1301	1301	1376	43	1451	1451	1526	109	1601	1601	1676	419	1751	103
1227	409	1302	31	1377	17	1452	22	1527	509	1602	89	1677	43	1752	73
1228	307	1303	1303	1378	53	1453	1453	1528	191	1603	229	1678	839	1753	1753
1229	1229	1304	163	1379	197	1454	727	1529	139	1604	401	1679	73	1754	877
1230	41	1305	29	1380	23	1455	97	1530	17	1605	107	1680	7	1755	13
1231	1231	1306	653	1381	1381	1456	13	1531	1531	1606	73	1681	82	1756	439
1232	11	1307	1307	1382	691	1457	47	1532	383	1607	1607	1682	58	1757	251
1233	137	1308	109	1383	461	1458	15	1533	73	1608	67	1683	17	1758	293
1234	617	1309	17	1384	173	1459	1459	1534	59	1609	1609	1684	421	1759	1759
1235	19	1310	131	1385	277	1460	73	1535	307	1610	23	1685	337	1760	11
1236	103	1311	23	1386	11	1461	487	1536	12	1611	179	1686	281	1761	587
1237	1237	1312	41	1387	73	1462	43	1537	53	1612	31	1687	241	1762	881
1238	619	1313	101	1388	347	1463	19	1538	769	1613	1613	1688	211	1763	43
1239	59	1314	73	1389	463	1464	61	1539	19	1614	269	1689	563	1764	14
1240	31	1315	263	1390	139	1465	293	1540	11	1615	19	1690	26	1765	353
1241	73	1316	47	1391	107	1466	733	1541	67	1616	101	1691	89	1766	883
1242	23	1317	439	1392	29	1467	163	1542	257	1617	14	1692	47	1767	31
1243	113	1318	659	1393	199	1468	367	1543	1543	1618	809	1693	1693	1768	17
1244	311	1319	1319	1394	41	1469	113	1544	193	1619	1619	1694	22	1769	61
1245	83	1320	11	1395	31	1470	14	1545	103	1620	9	1695	113	1770	59
1246	89	1321	1321	1396	349	1471	1471	1546	773	1621	1621	1696	53	1771	23
1247	43	1322	661	1397	127	1472	23	1547	17	1622	811	1697	1697	1772	443
1248	13	1323	14	1398	233	1473	491	1548	43	1623	541	1698	283	1773	197
1249	1249	1324	331	1399	1399	1474	67	1549	1549	1624	29	1699	1699	1774	887
1250	20	1325	53	1400	10	1475	59	1550	31	1625	15	1700	17	1775	71
1251	139	1326	17	1401	467	1476	41	1551	47	1626	271	1701	12	1776	37
1252	313	1327	1327	1402	701	1477	211	1552	97	1627	1627	1702	37	1777	1777
1253	179	1328	83	1403	61	1478	739	1553	1553	1628	37	1703	131	1778	127
1254	19	1329	443	1404	13	1479	29	1554	37	1629	181	1704	71	1779	593
1255	251	1330	19	1405	281	1480	37	1555	311	1630	163	1705	31	1780	89
1256	157	1331	33	1406	37	1481	1481	1556	389	1631	233	1706	853	1781	137
1257	419	1332	37	1407	67	1482	19	1557	173	1632	17	1707	569	1782	11
1258	37	1333	43	1408	11	1483	1483	1558	41	1633	71	1708	61	1783	1783
1259	1259	1334	29	1409	1409	1484	53	1559	1559	1634	43	1709	1709	1784	223
1260	7	1335	89	1410	47	1485	11	1560	13	1635	109	1710	19	1785	17
1261	97	1336	167	1411	83	1486	743	1561	223	1636	409	1711	59	1786	47
1262	631	1337	191	1412	353	1487	1487	1562	71	1637	1637	1712	107	1787	1787
1263	421	1338	223	1413	157	1488	31	1563	521	1638	13	1713	571	1788	149
1264	79	1339	103	1414	101	1489	1489	1564	23	1639	149	1714	857	1789	1789
1265	23	1340	67	1415	283	1490	149	1565	313	1640	41	1715	21	1790	179
1266	211	1341	149	1416	59	1491	71	1566	29	1641	547	1716	13	1791	199
1267	181	1342	61	1417	109	1492	373	1567	1567	1642	821	1717	101	1792	10
1268	317	1343	79	1418	709	1493	1493	1568	14	1643	53	1718	859	1793	163
1269	47	1344	8	1419	43	1494	83	1569	523	1644	137	1719	191	1794	23
1270	127	1345	269	1420	71	1495	23	1570	157	1645	47	1720	43	1795	359
1271	41	1346	673	1421	29	1496	17	1571	1571	1646	823	1721	1721	1796	449
1272	53	1347	449	1422	79	1497	499	1572	131	1647	61	1722	41	1797	599
1273	67	1348	337	1423	1423	1498	107	1573	22	1648	103	1723	1723	1798	31
1274	14	1349	71	1424	89	1499	1499	1574	787	1649	97	1724	431	1799	257
1275	17	1350	10	1425	19	1500	15	1575	10	1650	11	1725	23	1800	10

n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)
1801	1801	1876	67	1951	1951	2026	1013	2101	191	2176	17	2251	2251	2326	1163
1802	53	1877	1877	1952	61	2027	2027	2102	1051	2177	311	2252	563	2327	179
1803	601	1878	313	1953	31	2028	26	2103	701	2178	22	2253	751	2328	97
1804	41	1879	1879	1954	977	2029	2029	2104	263	2179	2179	2254	23	2329	137
1805	38	1880	47	1955	23	2030	29	2105	421	2180	109	2255	41	2330	233
1806	43	1881	19	1956	163	2031	677	2106	13	2181	727	2256	47	2331	37
1807	139	1882	941	1957	103	2032	127	2107	43	2182	1091	2257	61	2332	53
1808	113	1883	269	1958	89	2033	107	2108	31	2183	59	2258	1129	2333	2333
1809	67	1884	157	1959	653	2034	113	2109	37	2184	13	2259	251	2334	389
1810	181	1885	29	1960	14	2035	37	2110	211	2185	23	2260	113	2335	467
1811	1811	1886	41	1961	53	2036	509	2111	2111	2186	1093	2261	19	2336	73
1812	151	1887	37	1962	109	2037	97	2112	11	2187	18	2262	29	2337	41
1813	37	1888	59	1963	151	2038	1019	2113	2113	2188	547	2263	73	2338	167
1814	907	1889	1889	1964	491	2039	2039	2114	151	2189	199	2264	283	2339	2339
1815	22	1890	9	1965	131	2040	17	2115	47	2190	73	2265	151	2340	13
1816	227	1891	61	1966	983	2041	157	2116	46	2191	313	2266	103	2341	2341
1817	79	1892	43	1967	281	2042	1021	2117	73	2192	137	2267	2267	2342	1171
1818	101	1893	631	1968	41	2043	227	2118	353	2193	43	2268	9	2343	71
1819	107	1894	947	1969	179	2044	73	2119	163	2194	1097	2269	2269	2344	293
1820	13	1895	379	1970	197	2045	409	2120	53	2195	439	2270	227	2345	67
1821	607	1896	79	1971	73	2046	31	2121	101	2196	61	2271	757	2346	23
1822	911	1897	271	1972	29	2047	89	2122	1061	2197	39	2272	71	2347	2347
1823	1823	1898	73	1973	1973	2048	14	2123	193	2198	157	2273	2273	2348	587
1824	19	1899	211	1974	47	2049	683	2124	59	2199	733	2274	379	2349	29
1825	73	1900	19	1975	79	2050	41	2125	17	2200	11	2275	13	2350	47
1826	83	1901	1901	1976	19	2051	293	2126	1063	2201	71	2276	569	2351	2351
1827	29	1902	317	1977	659	2052	19	2127	709	2202	367	2277	23	2352	14
1828	457	1903	173	1978	43	2053	2053	2128	19	2203	2203	2278	67	2353	181
1829	59	1904	17	1979	1979	2054	79	2129	2129	2204	29	2279	53	2354	107
1830	61	1905	127	1980	11	2055	137	2130	71	2205	14	2280	19	2355	157
1831	1831	1906	953	1981	283	2056	257	2131	2131	2206	1103	2281	2281	2356	31
1832	229	1907	1907	1982	991	2057	22	2132	41	2207	2207	2282	163	2357	2357
1833	47	1908	53	1983	661	2058	21	2133	79	2208	23	2283	761	2358	131
1834	131	1909	83	1984	31	2059	71	2134	97	2209	94	2284	571	2359	337
1835	367	1910	191	1985	397	2060	103	2135	61	2210	17	2285	457	2360	59
1836	17	1911	14	1986	331	2061	229	2136	89	2211	67	2286	127	2361	787
1837	167	1912	239	1987	1987	2062	1031	2137	2137	2212	79	2287	2287	2362	1181
1838	919	1913	1913	1988	71	2063	2063	2138	1069	2213	2213	2288	13	2363	139
1839	613	1914	29	1989	17	2064	43	2139	31	2214	41	2289	109	2364	197
1840	23	1915	383	1990	199	2065	59	2140	107	2215	443	2290	229	2365	43
1841	263	1916	479	1991	181	2066	1033	2141	2141	2216	277	2291	79	2366	26
1842	307	1917	71	1992	83	2067	53	2142	17	2217	739	2292	191	2367	263
1843	97	1918	137	1993	1993	2068	47	2143	2143	2218	1109	2293	2293	2368	37
1844	461	1919	101	1994	997	2069	2069	2144	67	2219	317	2294	37	2369	103
1845	41	1920	8	1995	19	2070	23	2145	13	2220	37	2295	17	2370	79
1846	71	1921	113	1996	499	2071	109	2146	37	2221	2221	2296	41	2371	2371
1847	1847	1922	62	1997	1997	2072	37	2147	113	2222	101	2297	2297	2372	593
1848	11	1923	641	1998	37	2073	691	2148	179	2223	19	2298	383	2373	113
1849	86	1924	37	1999	1999	2074	61	2149	307	2224	139	2299	22	2374	1187
1850	37	1925	11	2000	15	2075	83	2150	43	2225	89	2300	23	2375	19
1851	617	1926	107	2001	29	2076	173	2151	239	2226	53	2301	59	2376	11
1852	463	1927	47	2002	13	2077	67	2152	269	2227	131	2302	1151	2377	2377
1853	109	1928	241	2003	2003	2078	1039	2153	2153	2228	557	2303	47	2378	41
1854	103	1929	643	2004	167	2079	11	2154	359	2229	743	2304	10	2379	61
1855	53	1930	193	2005	401	2080	13	2155	431	2230	223	2305	461	2380	17
1856	29	1931	1931	2006	59	2081	2081	2156	14	2231	97	2306	1153	2381	2381
1857	619	1932	23	2007	223	2082	347	2157	719	2232	31	2307	769	2382	397
1858	929	1933	1933	2008	251	2083	2083	2158	83	2233	29	2308	577	2383	2383
1859	26	1934	967	2009	41	2084	521	2159	127	2234	1117	2309	2309	2384	149
1860	31	1935	43	2010	67	2085	139	2160	9	2235	149	2310	11	2385	53
1861	1861	1936	22	2011	2011	2086	149	2161	2161	2236	43	2311	2311	2386	1193
1862	19	1937	149	2012	503	2087	2087	2162	47	2237	2237	2312	34	2387	31
1863	23	1938	19	2013	61	2088	29	2163	103	2238	373	2313	257	2388	199
1864	233	1939	277	2014	53	2089	2089	2164	541	2239	2239	2314	89	2389	2389
1865	373	1940	97	2015	31	2090	19	2165	433	2240	8	2315	463	2390	239
1866	311	1941	647	2016	8	2091	41	2166	38	2241	83	2316	193	2391	797
1867	1867	1942	971	2017	2017	2092	523	2167	197	2242	59	2317	331	2392	23
1868	467	1943	67	2018	1009	2093	23	2168	271	2243	2243	2318	61	2393	2393
1869	89	1944	12	2019	673	2094	349	2169	241	2244	17	2319	773	2394	19
1870	17	1945	389	2020	101	2095	419	2170	31	2245	449	2320	29	2395	479
1871	1871	1946	139	2021	47	2096	131	2171	167	2246	1123	2321	211	2396	599
1872	13	1947	59	2022	337	2097	233	2172	181	2247	107	2322	43	2397	47
1873	1873	1948	487	2023	34	2098	1049	2173	53	2248	281	2323	101	2398	109
1874	937	1949	1949	2024	23	2099	2099	2174	1087	2249	173	2324	83	2399	2399
1875	20	1950	13	2025	10	2100	10	2175	29	2250	15	2325	31	2400	10

The Smarandache Function S(n).

n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)
2401	28	2476	619	2551	2551	2626	101	2701	73	2776	347	2851	2851	2926	19
2402	1201	2477	2477	2552	29	2627	71	2702	193	2777	2777	2852	31	2927	2927
2403	89	2478	59	2553	37	2628	73	2703	53	2778	463	2853	317	2928	61
2404	601	2479	67	2554	1277	2629	239	2704	26	2779	397	2854	1427	2929	101
2405	37	2480	31	2555	73	2630	263	2705	541	2780	139	2855	571	2930	293
2406	401	2481	827	2556	71	2631	877	2706	41	2781	103	2856	17	2931	977
2407	83	2482	73	2557	2557	2632	47	2707	2707	2782	107	2857	2857	2932	733
2408	43	2483	191	2558	1279	2633	2633	2708	677	2783	23	2858	1429	2933	419
2409	73	2484	23	2559	853	2634	439	2709	43	2784	29	2859	953	2934	163
2410	241	2485	71	2560	12	2635	31	2710	271	2785	557	2860	13	2935	587
2411	2411	2486	113	2561	197	2636	659	2711	2711	2786	199	2861	2861	2936	367
2412	67	2487	829	2562	61	2637	293	2712	113	2787	929	2862	53	2937	89
2413	127	2488	311	2563	233	2638	1319	2713	2713	2788	41	2863	409	2938	113
2414	71	2489	131	2564	641	2639	29	2714	59	2789	2789	2864	179	2939	2939
2415	23	2490	83	2565	19	2640	11	2715	181	2790	31	2865	191	2940	14
2416	151	2491	53	2566	1283	2641	139	2716	97	2791	2791	2866	1433	2941	173
2417	2417	2492	89	2567	151	2642	1321	2717	19	2792	349	2867	61	2942	1471
2418	31	2493	277	2568	107	2643	881	2718	151	2793	19	2868	239	2943	109
2419	59	2494	43	2569	367	2644	661	2719	2719	2794	127	2869	151	2944	23
2420	22	2495	499	2570	257	2645	46	2720	17	2795	43	2870	41	2945	31
2421	269	2496	13	2571	857	2646	14	2721	907	2796	233	2871	29	2946	491
2422	173	2497	227	2572	643	2647	2647	2722	1361	2797	2797	2872	359	2947	421
2423	2423	2498	1249	2573	83	2648	331	2723	389	2798	1399	2873	26	2948	67
2424	101	2499	17	2574	13	2649	883	2724	227	2799	311	2874	479	2949	983
2425	97	2500	20	2575	103	2650	53	2725	109	2800	10	2875	23	2950	59
2426	1213	2501	61	2576	23	2651	241	2726	47	2801	2801	2876	719	2951	227
2427	809	2502	139	2577	859	2652	17	2727	101	2802	467	2877	137	2952	41
2428	607	2503	2503	2578	1289	2653	379	2728	31	2803	2803	2878	1439	2953	2953
2429	347	2504	313	2579	2579	2654	1327	2729	2729	2804	701	2879	2879	2954	211
2430	12	2505	167	2580	43	2655	59	2730	13	2805	17	2880	8	2955	197
2431	17	2506	179	2581	89	2656	83	2731	2731	2806	61	2881	67	2956	739
2432	19	2507	109	2582	1291	2657	2657	2732	683	2807	401	2882	131	2957	2957
2433	811	2508	19	2583	41	2658	443	2733	911	2808	13	2883	62	2958	29
2434	1217	2509	193	2584	19	2659	2659	2734	1367	2809	106	2884	103	2959	269
2435	487	2510	251	2585	47	2660	19	2735	547	2810	281	2885	577	2960	37
2436	29	2511	31	2586	431	2661	887	2736	19	2811	937	2886	37	2961	47
2437	2437	2512	157	2587	199	2662	33	2737	23	2812	37	2887	2887	2962	1481
2438	53	2513	359	2588	647	2663	2663	2738	74	2813	97	2888	38	2963	2963
2439	271	2514	419	2589	863	2664	37	2739	83	2814	67	2889	107	2964	19
2440	61	2515	503	2590	37	2665	41	2740	137	2815	563	2890	34	2965	593
2441	2441	2516	37	2591	2591	2666	43	2741	2741	2816	11	2891	59	2966	1483
2442	37	2517	839	2592	9	2667	127	2742	457	2817	313	2892	241	2967	43
2443	349	2518	1259	2593	2593	2668	29	2743	211	2818	1409	2893	263	2968	53
2444	47	2519	229	2594	1297	2669	157	2744	21	2819	2819	2894	1447	2969	2969
2445	163	2520	7	2595	173	2670	89	2745	61	2820	47	2895	193	2970	11
2446	1223	2521	2521	2596	59	2671	2671	2746	1373	2821	31	2896	181	2971	2971
2447	2447	2522	97	2597	53	2672	167	2747	67	2822	83	2897	2897	2972	743
2448	17	2523	58	2598	433	2673	12	2748	229	2823	941	2898	23	2973	991
2449	79	2524	631	2599	113	2674	191	2749	2749	2824	353	2899	223	2974	1487
2450	14	2525	101	2600	13	2675	107	2750	15	2825	113	2900	29	2975	17
2451	43	2526	421	2601	34	2676	223	2751	131	2826	157	2901	967	2976	31
2452	613	2527	38	2602	1301	2677	2677	2752	43	2827	257	2902	1451	2977	229
2453	223	2528	79	2603	137	2678	103	2753	2753	2828	101	2903	2903	2978	1489
2454	409	2529	281	2604	31	2679	47	2754	17	2829	41	2904	22	2979	331
2455	491	2530	23	2605	521	2680	67	2755	29	2830	283	2905	83	2980	149
2456	307	2531	2531	2606	1303	2681	383	2756	53	2831	149	2906	1453	2981	271
2457	13	2532	211	2607	79	2682	149	2757	919	2832	59	2907	19	2982	71
2458	1229	2533	149	2608	163	2683	2683	2758	197	2833	2833	2908	727	2983	157
2459	2459	2534	181	2609	2609	2684	61	2759	89	2834	109	2909	2909	2984	373
2460	41	2535	26	2610	29	2685	179	2760	23	2835	9	2910	97	2985	199
2461	107	2536	317	2611	373	2686	79	2761	251	2836	709	2911	71	2986	1493
2462	1231	2537	59	2612	653	2687	2687	2762	1381	2837	2837	2912	13	2987	103
2463	821	2538	47	2613	67	2688	8	2763	307	2838	43	2913	971	2988	83
2464	11	2539	2539	2614	1307	2689	2689	2764	691	2839	167	2914	47	2989	61
2465	29	2540	127	2615	523	2690	269	2765	79	2840	71	2915	53	2990	23
2466	137	2541	22	2616	109	2691	23	2766	461	2841	947	2916	15	2991	997
2467	2467	2542	41	2617	2617	2692	673	2767	2767	2842	29	2917	2917	2992	17
2468	617	2543	2543	2618	17	2693	2693	2768	173	2843	2843	2918	1459	2993	73
2469	823	2544	53	2619	97	2694	449	2769	71	2844	79	2919	139	2994	499
2470	19	2545	509	2620	131	2695	14	2770	277	2845	569	2920	73	2995	599
2471	353	2546	67	2621	2621	2696	337	2771	163	2846	1423	2921	127	2996	107
2472	103	2547	283	2622	23	2697	31	2772	11	2847	73	2922	487	2997	37
2473	2473	2548	14	2623	61	2698	71	2773	59	2848	89	2923	79	2998	1499
2474	1237	2549	2549	2624	41	2699	2699	2774	73	2849	37	2924	43	2999	2999
2475	11	2550	17	2625	15	2700	10	2775	37	2850	19	2925	13	3000	15

The Smarandache Function S(n).

n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)
3001	3001	3076	769	3151	137	3226	1613	3301	3301	3376	211	3451	29	3526	43
3002	79	3077	181	3152	197	3227	461	3302	127	3377	307	3452	863	3527	3527
3003	13	3078	19	3153	1051	3228	269	3303	367	3378	563	3453	1151	3528	14
3004	751	3079	3079	3154	83	3229	3229	3304	59	3379	109	3454	157	3529	3529
3005	601	3080	11	3155	631	3230	19	3305	661	3380	26	3455	691	3530	353
3006	167	3081	79	3156	263	3231	359	3306	29	3381	23	3456	9	3531	107
3007	97	3082	67	3157	41	3232	101	3307	3307	3382	89	3457	3457	3532	883
3008	47	3083	3083	3158	1579	3233	61	3308	827	3383	199	3458	19	3533	3533
3009	59	3084	257	3159	13	3234	14	3309	1103	3384	47	3459	1153	3534	31
3010	43	3085	617	3160	79	3235	647	3310	331	3385	677	3460	173	3535	101
3011	3011	3086	1543	3161	109	3236	809	3311	43	3386	1693	3461	3461	3536	17
3012	251	3087	21	3162	31	3237	83	3312	23	3387	1129	3462	577	3537	131
3013	131	3088	193	3163	3163	3238	1619	3313	3313	3388	22	3463	3463	3538	61
3014	137	3089	3089	3164	113	3239	79	3314	1657	3389	3389	3464	433	3539	3539
3015	67	3090	103	3165	211	3240	9	3315	17	3390	113	3465	11	3540	59
3016	29	3091	281	3166	1583	3241	463	3316	829	3391	3391	3466	1733	3541	3541
3017	431	3092	773	3167	3167	3242	1621	3317	107	3392	53	3467	3467	3542	23
3018	503	3093	1031	3168	11	3243	47	3318	79	3393	29	3468	34	3543	1181
3019	3019	3094	17	3169	3169	3244	811	3319	3319	3394	1697	3469	3469	3544	443
3020	151	3095	619	3170	317	3245	59	3320	83	3395	97	3470	347	3545	709
3021	53	3096	43	3171	151	3246	541	3321	41	3396	283	3471	89	3546	197
3022	1511	3097	163	3172	61	3247	191	3322	151	3397	79	3472	31	3547	3547
3023	3023	3098	1549	3173	167	3248	29	3323	3323	3398	1699	3473	151	3548	887
3024	9	3099	1033	3174	46	3249	38	3324	277	3399	103	3474	193	3549	26
3025	22	3100	31	3175	127	3250	15	3325	19	3400	17	3475	139	3550	71
3026	89	3101	443	3176	397	3251	3251	3326	1663	3401	179	3476	79	3551	67
3027	1009	3102	47	3177	353	3252	271	3327	1109	3402	12	3477	61	3552	37
3028	757	3103	107	3178	227	3253	3253	3328	13	3403	83	3478	47	3553	19
3029	233	3104	97	3179	34	3254	1627	3329	3329	3404	37	3479	71	3554	1777
3030	101	3105	23	3180	53	3255	31	3330	37	3405	227	3480	29	3555	79
3031	433	3106	1553	3181	3181	3256	37	3331	3331	3406	131	3481	118	3556	127
3032	379	3107	239	3182	43	3257	3257	3332	17	3407	3407	3482	1741	3557	3557
3033	337	3108	37	3183	1061	3258	181	3333	101	3408	71	3483	43	3558	593
3034	41	3109	3109	3184	199	3259	3259	3334	1667	3409	487	3484	67	3559	3559
3035	607	3110	311	3185	14	3260	163	3335	29	3410	31	3485	41	3560	89
3036	23	3111	61	3186	59	3261	1087	3336	139	3411	379	3486	83	3561	1187
3037	3037	3112	389	3187	3187	3262	233	3337	71	3412	853	3487	317	3562	137
3038	31	3113	283	3188	797	3263	251	3338	1669	3413	3413	3488	109	3563	509
3039	1013	3114	173	3189	1063	3264	17	3339	53	3414	569	3489	1163	3564	11
3040	19	3115	89	3190	29	3265	653	3340	167	3415	683	3490	349	3565	31
3041	3041	3116	41	3191	3191	3266	71	3341	257	3416	61	3491	3491	3566	1783
3042	26	3117	1039	3192	19	3267	22	3342	557	3417	67	3492	97	3567	41
3043	179	3118	1559	3193	103	3268	43	3343	3343	3418	1709	3493	499	3568	223
3044	761	3119	3119	3194	1597	3269	467	3344	19	3419	263	3494	1747	3569	83
3045	29	3120	13	3195	71	3270	109	3345	223	3420	19	3495	233	3570	17
3046	1523	3121	3121	3196	47	3271	3271	3346	239	3421	311	3496	23	3571	3571
3047	277	3122	223	3197	139	3272	409	3347	3347	3422	59	3497	269	3572	47
3048	127	3123	347	3198	41	3273	1091	3348	31	3423	163	3498	53	3573	397
3049	3049	3124	71	3199	457	3274	1637	3349	197	3424	107	3499	3499	3574	1787
3050	61	3125	25	3200	10	3275	131	3350	67	3425	137	3500	15	3575	13
3051	113	3126	521	3201	97	3276	13	3351	1117	3426	571	3501	389	3576	149
3052	109	3127	59	3202	1601	3277	113	3352	419	3427	149	3502	103	3577	73
3053	71	3128	23	3203	3203	3278	149	3353	479	3428	857	3503	113	3578	1789
3054	509	3129	149	3204	89	3279	1093	3354	43	3429	127	3504	73	3579	1193
3055	47	3130	313	3205	641	3280	41	3355	61	3430	21	3505	701	3580	179
3056	191	3131	101	3206	229	3281	193	3356	839	3431	73	3506	1753	3581	3581
3057	1019	3132	29	3207	1069	3282	547	3357	373	3432	13	3507	167	3582	199
3058	139	3133	241	3208	401	3283	67	3358	73	3433	3433	3508	877	3583	3583
3059	23	3134	1567	3209	3209	3284	821	3359	3359	3434	101	3509	29	3584	12
3060	17	3135	19	3210	107	3285	73	3360	8	3435	229	3510	13	3585	239
3061	3061	3136	14	3211	26	3286	53	3361	3361	3436	859	3511	3511	3586	163
3062	1531	3137	3137	3212	73	3287	173	3362	82	3437	491	3512	439	3587	211
3063	1021	3138	523	3213	17	3288	137	3363	59	3438	191	3513	1171	3588	23
3064	383	3139	73	3214	1607	3289	23	3364	58	3439	181	3514	251	3589	97
3065	613	3140	157	3215	643	3290	47	3365	673	3440	43	3515	37	3590	359
3066	73	3141	349	3216	67	3291	1097	3366	17	3441	37	3516	293	3591	19
3067	3067	3142	1571	3217	3217	3292	823	3367	37	3442	1721	3517	3517	3592	449
3068	59	3143	449	3218	1609	3293	89	3368	421	3443	313	3518	1759	3593	3593
3069	31	3144	131	3219	37	3294	61	3369	1123	3444	41	3519	23	3594	599
3070	307	3145	37	3220	23	3295	659	3370	337	3445	53	3520	11	3595	719
3071	83	3146	22	3221	3221	3296	103	3371	3371	3446	1723	3521	503	3596	31
3072	12	3147	1049	3222	179	3297	157	3372	281	3447	383	3522	587	3597	109
3073	439	3148	787	3223	293	3298	97	3373	3373	3448	431	3523	271	3598	257
3074	53	3149	67	3224	31	3299	3299	3374	241	3449	3449	3524	881	3599	61
3075	41	3150	10	3225	43	3300	11	3375	15	3450	23	3525	47	3600	10

n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)
3601	277	3676	919	3751	31	3826	1913	3901	83	3976	71	4051	4051	4126	2063
3602	1801	3677	3677	3752	67	3827	89	3902	1951	3977	97	4052	1013	4127	4127
3603	1201	3678	613	3753	139	3828	29	3903	1301	3978	17	4053	193	4128	43
3604	53	3679	283	3754	1877	3829	547	3904	61	3979	173	4054	2027	4129	4129
3605	103	3680	23	3755	751	3830	383	3905	71	3980	199	4055	811	4130	59
3606	601	3681	409	3756	313	3831	1277	3906	31	3981	1327	4056	26	4131	17
3607	3607	3682	263	3757	34	3832	479	3907	3907	3982	181	4057	4057	4132	1033
3608	41	3683	127	3758	1879	3833	3833	3908	977	3983	569	4058	2029	4133	4133
3609	401	3684	307	3759	179	3834	71	3909	1303	3984	83	4059	41	4134	53
3610	38	3685	67	3760	47	3835	59	3910	23	3985	797	4060	29	4135	827
3611	157	3686	97	3761	3761	3836	137	3911	3911	3986	1993	4061	131	4136	47
3612	43	3687	1229	3762	19	3837	1279	3912	163	3987	443	4062	677	4137	197
3613	3613	3688	461	3763	71	3838	101	3913	43	3988	997	4063	239	4138	2069
3614	139	3689	31	3764	941	3839	349	3914	103	3989	3989	4064	127	4139	4139
3615	241	3690	41	3765	251	3840	10	3915	29	3990	19	4065	271	4140	23
3616	113	3691	3691	3766	269	3841	167	3916	89	3991	307	4066	107	4141	101
3617	3617	3692	71	3767	3767	3842	113	3917	3917	3992	499	4067	83	4142	109
3618	67	3693	1231	3768	157	3843	61	3918	653	3993	33	4068	113	4143	1381
3619	47	3694	1847	3769	3769	3844	62	3919	3919	3994	1997	4069	313	4144	37
3620	181	3695	739	3770	29	3845	769	3920	14	3995	47	4070	37	4145	829
3621	71	3696	11	3771	419	3846	641	3921	1307	3996	37	4071	59	4146	691
3622	1811	3697	3697	3772	41	3847	3847	3922	53	3997	571	4072	509	4147	29
3623	3623	3698	86	3773	21	3848	37	3923	3923	3998	1999	4073	4073	4148	61
3624	151	3699	137	3774	37	3849	1283	3924	109	3999	43	4074	97	4149	461
3625	29	3700	37	3775	151	3850	11	3925	157	4000	15	4075	163	4150	83
3626	37	3701	3701	3776	59	3851	3851	3926	151	4001	4001	4076	1019	4151	593
3627	31	3702	617	3777	1259	3852	107	3927	17	4002	29	4077	151	4152	173
3628	907	3703	46	3778	1889	3853	3853	3928	491	4003	4003	4078	2039	4153	4153
3629	191	3704	463	3779	3779	3854	47	3929	3929	4004	13	4079	4079	4154	67
3630	22	3705	19	3780	9	3855	257	3930	131	4005	89	4080	17	4155	277
3631	3631	3706	109	3781	199	3856	241	3931	3931	4006	2003	4081	53	4156	1039
3632	227	3707	337	3782	61	3857	29	3932	983	4007	4007	4082	157	4157	4157
3633	173	3708	103	3783	97	3858	643	3933	23	4008	167	4083	1361	4158	11
3634	79	3709	3709	3784	43	3859	227	3934	281	4009	211	4084	1021	4159	4159
3635	727	3710	53	3785	757	3860	193	3935	787	4010	401	4085	43	4160	13
3636	101	3711	1237	3786	631	3861	13	3936	41	4011	191	4086	227	4161	73
3637	3637	3712	29	3787	541	3862	1931	3937	127	4012	59	4087	67	4162	2081
3638	107	3713	79	3788	947	3863	3863	3938	179	4013	4013	4088	73	4163	181
3639	1213	3714	619	3789	421	3864	23	3939	101	4014	223	4089	47	4164	347
3640	13	3715	743	3790	379	3865	773	3940	197	4015	73	4090	409	4165	17
3641	331	3716	929	3791	223	3866	1933	3941	563	4016	251	4091	4091	4166	2083
3642	607	3717	59	3792	79	3867	1289	3942	73	4017	103	4092	31	4167	463
3643	3643	3718	26	3793	3793	3868	967	3943	3943	4018	41	4093	4093	4168	521
3644	911	3719	3719	3794	271	3869	73	3944	29	4019	4019	4094	89	4169	379
3645	15	3720	31	3795	23	3870	43	3945	263	4020	67	4095	13	4170	139
3646	1823	3721	122	3796	73	3871	79	3946	1973	4021	4021	4096	16	4171	97
3647	521	3722	1861	3797	3797	3872	22	3947	3947	4022	2011	4097	241	4172	149
3648	19	3723	73	3798	211	3873	1291	3948	47	4023	149	4098	683	4173	107
3649	89	3724	19	3799	131	3874	149	3949	359	4024	503	4099	4099	4174	2087
3650	73	3725	149	3800	19	3875	31	3950	79	4025	23	4100	41	4175	167
3651	1217	3726	23	3801	181	3876	19	3951	439	4026	61	4101	1367	4176	29
3652	83	3727	3727	3802	1901	3877	3877	3952	19	4027	4027	4102	293	4177	4177
3653	281	3728	233	3803	3803	3878	277	3953	67	4028	53	4103	373	4178	2089
3654	29	3729	113	3804	317	3879	431	3954	659	4029	79	4104	19	4179	199
3655	43	3730	373	3805	761	3880	97	3955	113	4030	31	4105	821	4180	19
3656	457	3731	41	3806	173	3881	3881	3956	43	4031	139	4106	2053	4181	113
3657	53	3732	311	3807	47	3882	647	3957	1319	4032	8	4107	74	4182	41
3658	59	3733	3733	3808	17	3883	353	3958	1979	4033	109	4108	79	4183	89
3659	3659	3734	1867	3809	293	3884	971	3959	107	4034	2017	4109	587	4184	523
3660	61	3735	83	3810	127	3885	37	3960	11	4035	269	4110	137	4185	31
3661	523	3736	467	3811	103	3886	67	3961	233	4036	1009	4111	4111	4186	23
3662	1831	3737	101	3812	953	3887	26	3962	283	4037	367	4112	257	4187	79
3663	37	3738	89	3813	41	3888	12	3963	1321	4038	673	4113	457	4188	349
3664	229	3739	3739	3814	1907	3889	3889	3964	991	4039	577	4114	22	4189	71
3665	733	3740	17	3815	109	3890	389	3965	61	4040	101	4115	823	4190	419
3666	47	3741	43	3816	53	3891	1297	3966	661	4041	449	4116	21	4191	127
3667	193	3742	1871	3817	347	3892	139	3967	3967	4042	47	4117	179	4192	131
3668	131	3743	197	3818	83	3893	229	3968	31	4043	311	4118	71	4193	599
3669	1223	3744	13	3819	67	3894	59	3969	14	4044	337	4119	1373	4194	233
3670	367	3745	107	3820	191	3895	41	3970	397	4045	809	4120	103	4195	839
3671	3671	3746	1873	3821	3821	3896	487	3971	38	4046	34	4121	317	4196	1049
3672	17	3747	1249	3822	14	3897	433	3972	331	4047	71	4122	229	4197	1399
3673	3673	3748	937	3823	3823	3898	1949	3973	137	4048	23	4123	31	4198	2099
3674	167	3749	163	3824	239	3899	557	3974	1987	4049	4049	4124	1031	4199	19
3675	14	3750	20	3825	17	3900	13	3975	53	4050	10	4125	15	4200	10

The Smarandache Function S(n).

n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)	n	S(n)
4201	4201	4276	1069	4351	229	4426	2213	4501	643	4576	13	4651	4651	4726	139
4202	191	4277	47	4352	17	4427	233	4502	2251	4577	199	4652	1163	4727	163
4203	467	4278	31	4353	1451	4428	41	4503	79	4578	109	4653	47	4728	197
4204	1051	4279	389	4354	311	4429	103	4504	563	4579	241	4654	179	4729	4729
4205	58	4280	107	4355	67	4430	443	4505	53	4580	229	4655	19	4730	43
4206	701	4281	1427	4356	22	4431	211	4506	751	4581	509	4656	97	4731	83
4207	601	4282	2141	4357	4357	4432	277	4507	4507	4582	79	4657	4657	4732	26
4208	263	4283	4283	4358	2179	4433	31	4508	23	4583	4583	4658	137	4733	4733
4209	61	4284	17	4359	1453	4434	739	4509	167	4584	191	4659	1553	4734	263
4210	421	4285	857	4360	109	4435	887	4510	41	4585	131	4660	233	4735	947
4211	4211	4286	2143	4361	89	4436	1109	4511	347	4586	2293	4661	79	4736	37
4212	13	4287	1429	4362	727	4437	29	4512	47	4587	139	4662	37	4737	1579
4213	383	4288	67	4363	4363	4438	317	4513	4513	4588	37	4663	4663	4738	103
4214	43	4289	4289	4364	1091	4439	193	4514	61	4589	353	4664	53	4739	677
4215	281	4290	13	4365	97	4440	37	4515	43	4590	17	4665	311	4740	79
4216	31	4291	613	4366	59	4441	4441	4516	1129	4591	4591	4666	2333	4741	431
4217	4217	4292	37	4367	397	4442	2221	4517	4517	4592	41	4667	359	4742	2371
4218	37	4293	53	4368	13	4443	1481	4518	251	4593	1531	4668	389	4743	31
4219	4219	4294	113	4369	257	4444	101	4519	4519	4594	2297	4669	29	4744	593
4220	211	4295	859	4370	23	4445	127	4520	113	4595	919	4670	467	4745	73
4221	67	4296	179	4371	47	4446	19	4521	137	4596	383	4671	173	4746	113
4222	2111	4297	4297	4372	1093	4447	4447	4522	19	4597	4597	4672	73	4747	101
4223	103	4298	307	4373	4373	4448	139	4523	4523	4598	22	4673	4673	4748	1187
4224	11	4299	1433	4374	18	4449	1483	4524	29	4599	73	4674	41	4749	1583
4225	26	4300	43	4375	20	4450	89	4525	181	4600	23	4675	17	4750	19
4226	2113	4301	23	4376	547	4451	4451	4526	73	4601	107	4676	167	4751	4751
4227	1409	4302	239	4377	1459	4452	53	4527	503	4602	59	4677	1559	4752	11
4228	151	4303	331	4378	199	4453	73	4528	283	4603	4603	4678	2339	4753	97
4229	4229	4304	269	4379	151	4454	131	4529	647	4604	1151	4679	4679	4754	2377
4230	47	4305	41	4380	73	4455	11	4530	151	4605	307	4680	13	4755	317
4231	4231	4306	2153	4381	337	4456	557	4531	197	4606	47	4681	151	4756	41
4232	46	4307	73	4382	313	4457	4457	4532	103	4607	271	4682	2341	4757	71
4233	83	4308	359	4383	487	4458	743	4533	1511	4608	12	4683	223	4758	61
4234	73	4309	139	4384	137	4459	21	4534	2267	4609	419	4684	1171	4759	4759
4235	22	4310	431	4385	877	4460	223	4535	907	4610	461	4685	937	4760	17
4236	353	4311	479	4386	43	4461	1487	4536	9	4611	53	4686	71	4761	46
4237	223	4312	14	4387	107	4462	97	4537	349	4612	1153	4687	109	4762	2381
4238	163	4313	227	4388	1097	4463	4463	4538	2269	4613	659	4688	293	4763	433
4239	157	4314	719	4389	19	4464	31	4539	89	4614	769	4689	521	4764	397
4240	53	4315	863	4390	439	4465	47	4540	227	4615	71	4690	67	4765	953
4241	4241	4316	83	4391	4391	4466	29	4541	239	4616	577	4691	4691	4766	2383
4242	101	4317	1439	4392	61	4467	1489	4542	757	4617	19	4692	23	4767	227
4243	4243	4318	127	4393	191	4468	1117	4543	59	4618	2309	4693	38	4768	149
4244	1061	4319	617	4394	39	4469	109	4544	71	4619	149	4694	2347	4769	251
4245	283	4320	9	4395	293	4470	149	4545	101	4620	11	4695	313	4770	53
4246	193	4321	149	4396	157	4471	263	4546	2273	4621	4621	4696	587	4771	367
4247	137	4322	2161	4397	4397	4472	43	4547	4547	4622	2311	4697	61	4772	1193
4248	59	4323	131	4398	733	4473	71	4548	379	4623	67	4698	29	4773	43
4249	607	4324	47	4399	83	4474	2237	4549	4549	4624	34	4699	127	4774	31
4250	17	4325	173	4400	11	4475	179	4550	13	4625	37	4700	47	4775	191
4251	109	4326	103	4401	163	4476	373	4551	41	4626	257	4701	1567	4776	199
4252	1063	4327	4327	4402	71	4477	37	4552	569	4627	661	4702	2351	4777	281
4253	4253	4328	541	4403	37	4478	2239	4553	157	4628	89	4703	4703	4778	2389
4254	709	4329	37	4404	367	4479	1493	4554	23	4629	1543	4704	14	4779	59
4255	37	4330	433	4405	881	4480	8	4555	911	4630	463	4705	941	4780	239
4256	19	4331	71	4406	2203	4481	4481	4556	67	4631	421	4706	181	4781	683
4257	43	4332	38	4407	113	4482	83	4557	31	4632	193	4707	523	4782	797
4258	2129	4333	619	4408	29	4483	4483	4558	53	4633	113	4708	107	4783	4783
4259	4259	4334	197	4409	4409	4484	59	4559	97	4634	331	4709	277	4784	23
4260	71	4335	34	4410	14	4485	23	4560	19	4635	103	4710	157	4785	29
4261	4261	4336	271	4411	401	4486	2243	4561	4561	4636	61	4711	673	4786	2393
4262	2131	4337	4337	4412	1103	4487	641	4562	2281	4637	4637	4712	31	4787	4787
4263	29	4338	241	4413	1471	4488	17	4563	26	4638	773	4713	1571	4788	19
4264	41	4339	4339	4414	2207	4489	134	4564	163	4639	4639	4714	2357	4789	4789
4265	853	4340	31	4415	883	4490	449	4565	83	4640	29	4715	41	4790	479
4266	79	4341	1447	4416	23	4491	499	4566	761	4641	17	4716	131	4791	1597
4267	251	4342	167	4417	631	4492	1123	4567	4567	4642	211	4717	89	4792	599
4268	97	4343	101	4418	94	4493	4493	4568	571	4643	4643	4718	337	4793	4793
4269	1423	4344	181	4419	491	4494	107	4569	1523	4644	43	4719	22	4794	47
4270	61	4345	79	4420	17	4495	31	4570	457	4645	929	4720	59	4795	137
4271	4271	4346	53	4421	4421	4496	281	4571	653	4646	101	4721	4721	4796	109
4272	89	4347	23	4422	67	4497	1499	4572	127	4647	1549	4722	787	4797	41
4273	4273	4348	1087	4423	4423	4498	173	4573	269	4648	83	4723	4723	4798	2399
4274	2137	4349	4349	4424	79	4499	409	4574	2287	4649	4649	4724	1181	4799	4799
4275	19	4350	29	4425	59	4500	15	4575	61	4650	31	4725	10	4800	10

'SNP_TAB, H. Ibstedt, 930322

'This program tabulates S(n) for n=P^J, 1<P<74, 1<J,76, using data form the file SNP_ASC.

```
DEFLNG I-S :DIM KP(1575),KJ(1575),SP(1575)
CLS:
WIDTH "LPT1:",120
OPEN "SNP.DAT" FOR INPUT AS #1
FOR I=1 TO 1575
INPUT #1,KP(I),KJ(I),SP(I)
NEXT
CLOSE #1
S1$="┏━━━━━━━┯━━━━━━━┯━━━━━━━┳" :S2$="━━━━━━━┯━━━━━━━┯━━━━━━━┳"
S4$="┃ P(I)  │   J   │D(I,J)┃" :S5$=" P(I)  │   J   │D(I,J)┃"
S7$="┣━━━━━━━┿━━━━━━━┿━━━━━━━╋" :S8$="━━━━━━━┿━━━━━━━┿━━━━━━━╋"
B1$="┗━━━━━━━┷━━━━━━━┷━━━━━━━┻" :B2$="━━━━━━━┷━━━━━━━┷━━━━━━━┻"
S3$="━━━━━━━┯━━━━━━━┯━━━━━━━┓"
S6$=" P(I)  │   J   │D(I,J)┃"
S9$="━━━━━━━┿━━━━━━━┿━━━━━━━┫"
B3$="━━━━━━━┷━━━━━━━┷━━━━━━━┛"
I1=1 :I2=75 :P1=1
 LW:
LPRINT TAB(12) "The Smarandache Function S(n)=D(I,J) for powers of primes P(I)^J."
LPRINT TAB(12) S1$; :FOR I=1 TO 3 :LPRINT S2$; :NEXT :LPRINT S3$;
LPRINT TAB(12) S4$; :FOR I=1 TO 3 :LPRINT S5$; :NEXT :LPRINT S6$;
LPRINT TAB(12) S7$; :FOR I=1 TO 3 :LPRINT S8$; :NEXT :LPRINT S9$;
FOR I=I1  TO I2
LPRINT  TAB(12) "┃";
FOR J=0 TO 4
LPRINT USING "#####";KP(I+J*75);
LPRINT "  │"; :LPRINT USING "#####";KJ(I+J*75); :LPRINT "  │"; :LPRINT USING
"#####";SP(I+J*75);
LPRINT "  ┃" ;
NEXT
NEXT
LPRINT TAB(12) B1$; :FOR I=1 TO 3 :LPRINT B2$; :NEXT :LPRINT B3$;
LPRINT TAB(12) "Page"P1 "of 3.
LPRINT CHR$(12)
IF P1=1 THEN P1=2 :I1=601 :I2=675 :GOTO LW
IF P1=2 THEN P1=3 :I1=1201 :I2=1275 :GOTO LW
PRINT "END" :END
```

The Smarandache Function S(n)=D(I,J) for powers of primes P(I)^J.

P(I)	J	D(I,J)	P(I)	J	D(I,J)	P(I)	J	D(I,J)	P(I)	J	D(I,J)	P(I)	J	D(I,J)
2	1	2	3	1	3	5	1	5	7	1	7	11	1	11
2	2	4	3	2	6	5	2	10	7	2	14	11	2	22
2	3	4	3	3	9	5	3	15	7	3	21	11	3	33
2	4	6	3	4	9	5	4	20	7	4	28	11	4	44
2	5	8	3	5	12	5	5	25	7	5	35	11	5	55
2	6	8	3	6	15	5	6	25	7	6	42	11	6	66
2	7	8	3	7	18	5	7	30	7	7	49	11	7	77
2	8	10	3	8	18	5	8	35	7	8	49	11	8	88
2	9	12	3	9	21	5	9	40	7	9	56	11	9	99
2	10	12	3	10	24	5	10	45	7	10	63	11	10	110
2	11	14	3	11	27	5	11	50	7	11	70	11	11	121
2	12	16	3	12	27	5	12	50	7	12	77	11	12	121
2	13	16	3	13	27	5	13	55	7	13	84	11	13	132
2	14	16	3	14	30	5	14	60	7	14	91	11	14	143
2	15	16	3	15	33	5	15	65	7	15	98	11	15	154
2	16	18	3	16	36	5	16	70	7	16	98	11	16	165
2	17	20	3	17	36	5	17	75	7	17	105	11	17	176
2	18	20	3	18	39	5	18	75	7	18	112	11	18	187
2	19	22	3	19	42	5	19	80	7	19	119	11	19	198
2	20	24	3	20	45	5	20	85	7	20	126	11	20	209
2	21	24	3	21	45	5	21	90	7	21	133	11	21	220
2	22	24	3	22	48	5	22	95	7	22	140	11	22	231
2	23	26	3	23	51	5	23	100	7	23	147	11	23	242
2	24	28	3	24	54	5	24	100	7	24	147	11	24	242
2	25	28	3	25	54	5	25	105	7	25	154	11	25	253
2	26	30	3	26	54	5	26	110	7	26	161	11	26	264
2	27	32	3	27	57	5	27	115	7	27	168	11	27	275
2	28	32	3	28	60	5	28	120	7	28	175	11	28	286
2	29	32	3	29	63	5	29	125	7	29	182	11	29	297
2	30	32	3	30	63	5	30	125	7	30	189	11	30	308
2	31	32	3	31	66	5	31	125	7	31	196	11	31	319
2	32	34	3	32	69	5	32	130	7	32	196	11	32	330
2	33	36	3	33	72	5	33	135	7	33	203	11	33	341
2	34	36	3	34	72	5	34	140	7	34	210	11	34	352
2	35	38	3	35	75	5	35	145	7	35	217	11	35	363
2	36	40	3	36	78	5	36	150	7	36	224	11	36	363
2	37	40	3	37	81	5	37	150	7	37	231	11	37	374
2	38	40	3	38	81	5	38	155	7	38	238	11	38	385
2	39	42	3	39	81	5	39	160	7	39	245	11	39	396
2	40	44	3	40	81	5	40	165	7	40	245	11	40	407
2	41	44	3	41	84	5	41	170	7	41	252	11	41	418
2	42	46	3	42	87	5	42	175	7	42	259	11	42	429
2	43	48	3	43	90	5	43	175	7	43	266	11	43	440
2	44	48	3	44	90	5	44	180	7	44	273	11	44	451
2	45	48	3	45	93	5	45	185	7	45	280	11	45	462
2	46	48	3	46	96	5	46	190	7	46	287	11	46	473
2	47	50	3	47	99	5	47	195	7	47	294	11	47	484
2	48	52	3	48	99	5	48	200	7	48	294	11	48	484
2	49	52	3	49	102	5	49	200	7	49	301	11	49	495
2	50	54	3	50	105	5	50	205	7	50	308	11	50	506
2	51	56	3	51	108	5	51	210	7	51	315	11	51	517
2	52	56	3	52	108	5	52	215	7	52	322	11	52	528
2	53	56	3	53	108	5	53	220	7	53	329	11	53	539
2	54	58	3	54	111	5	54	225	7	54	336	11	54	550
2	55	60	3	55	114	5	55	225	7	55	343	11	55	561
2	56	60	3	56	117	5	56	230	7	56	343	11	56	572
2	57	62	3	57	117	5	57	235	7	57	343	11	57	583
2	58	64	3	58	120	5	58	240	7	58	350	11	58	594
2	59	64	3	59	123	5	59	245	7	59	357	11	59	605
2	60	64	3	60	126	5	60	250	7	60	364	11	60	605
2	61	64	3	61	126	5	61	250	7	61	371	11	61	616
2	62	64	3	62	129	5	62	250	7	62	378	11	62	627
2	63	64	3	63	132	5	63	255	7	63	385	11	63	638
2	64	66	3	64	135	5	64	260	7	64	392	11	64	649
2	65	68	3	65	135	5	65	265	7	65	392	11	65	660
2	66	68	3	66	135	5	66	270	7	66	399	11	66	671
2	67	70	3	67	138	5	67	275	7	67	406	11	67	682
2	68	72	3	68	141	5	68	275	7	68	413	11	68	693
2	69	72	3	69	144	5	69	280	7	69	420	11	69	704
2	70	72	3	70	144	5	70	285	7	70	427	11	70	715
2	71	74	3	71	147	5	71	290	7	71	434	11	71	726
2	72	76	3	72	150	5	72	295	7	72	441	11	72	726
2	73	76	3	73	153	5	73	300	7	73	441	11	73	737
2	74	78	3	74	153	5	74	300	7	74	448	11	74	748
2	75	80	3	75	156	5	75	305	7	75	455	11	75	759

The Smarandache Function S(n)=D(I,J) for powers of primes P(I)^J.

P(I)	J	D(I,J)	P(I)	J	D(I,J)	P(I)	J	D(I,J)	P(I)	J	D(I,J)	P(I)	J	D(I,J)
23	1	23	29	1	29	31	1	31	37	1	37	41	1	41
23	2	46	29	2	58	31	2	62	37	2	74	41	2	82
23	3	69	29	3	87	31	3	93	37	3	111	41	3	123
23	4	92	29	4	116	31	4	124	37	4	148	41	4	164
23	5	115	29	5	145	31	5	155	37	5	185	41	5	205
23	6	138	29	6	174	31	6	186	37	6	222	41	6	246
23	7	161	29	7	203	31	7	217	37	7	259	41	7	287
23	8	184	29	8	232	31	8	248	37	8	296	41	8	328
23	9	207	29	9	261	31	9	279	37	9	333	41	9	369
23	10	230	29	10	290	31	10	310	37	10	370	41	10	410
23	11	253	29	11	319	31	11	341	37	11	407	41	11	451
23	12	276	29	12	348	31	12	372	37	12	444	41	12	492
23	13	299	29	13	377	31	13	403	37	13	481	41	13	533
23	14	322	29	14	406	31	14	434	37	14	518	41	14	574
23	15	345	29	15	435	31	15	465	37	15	555	41	15	615
23	16	368	29	16	464	31	16	496	37	16	592	41	16	656
23	17	391	29	17	493	31	17	527	37	17	629	41	17	697
23	18	414	29	18	522	31	18	558	37	18	666	41	18	738
23	19	437	29	19	551	31	19	589	37	19	703	41	19	779
23	20	460	29	20	580	31	20	620	37	20	740	41	20	820
23	21	483	29	21	609	31	21	651	37	21	777	41	21	861
23	22	506	29	22	638	31	22	682	37	22	814	41	22	902
23	23	529	29	23	667	31	23	713	37	23	851	41	23	943
23	24	529	29	24	696	31	24	744	37	24	888	41	24	984
23	25	552	29	25	725	31	25	775	37	25	925	41	25	1025
23	26	575	29	26	754	31	26	806	37	26	962	41	26	1066
23	27	598	29	27	783	31	27	837	37	27	999	41	27	1107
23	28	621	29	28	812	31	28	868	37	28	1036	41	28	1148
23	29	644	29	29	841	31	29	899	37	29	1073	41	29	1189
23	30	667	29	30	841	31	30	930	37	30	1110	41	30	1230
23	31	690	29	31	870	31	31	961	37	31	1147	41	31	1271
23	32	713	29	32	899	31	32	961	37	32	1184	41	32	1312
23	33	736	29	33	928	31	33	992	37	33	1221	41	33	1353
23	34	759	29	34	957	31	34	1023	37	34	1258	41	34	1394
23	35	782	29	35	986	31	35	1054	37	35	1295	41	35	1435
23	36	805	29	36	1015	31	36	1085	37	36	1332	41	36	1476
23	37	828	29	37	1044	31	37	1116	37	37	1369	41	37	1517
23	38	851	29	38	1073	31	38	1147	37	38	1369	41	38	1558
23	39	874	29	39	1102	31	39	1178	37	39	1406	41	39	1599
23	40	897	29	40	1131	31	40	1209	37	40	1443	41	40	1640
23	41	920	29	41	1160	31	41	1240	37	41	1480	41	41	1681
23	42	943	29	42	1189	31	42	1271	37	42	1517	41	42	1681
23	43	966	29	43	1218	31	43	1302	37	43	1554	41	43	1722
23	44	989	29	44	1247	31	44	1333	37	44	1591	41	44	1763
23	45	1012	29	45	1276	31	45	1364	37	45	1628	41	45	1804
23	46	1035	29	46	1305	31	46	1395	37	46	1665	41	46	1845
23	47	1058	29	47	1334	31	47	1426	37	47	1702	41	47	1886
23	48	1058	29	48	1363	31	48	1457	37	48	1739	41	48	1927
23	49	1081	29	49	1392	31	49	1488	37	49	1776	41	49	1968
23	50	1104	29	50	1421	31	50	1519	37	50	1813	41	50	2009
23	51	1127	29	51	1450	31	51	1550	37	51	1850	41	51	2050
23	52	1150	29	52	1479	31	52	1581	37	52	1887	41	52	2091
23	53	1173	29	53	1508	31	53	1612	37	53	1924	41	53	2132
23	54	1196	29	54	1537	31	54	1643	37	54	1961	41	54	2173
23	55	1219	29	55	1566	31	55	1674	37	55	1998	41	55	2214
23	56	1242	29	56	1595	31	56	1705	37	56	2035	41	56	2255
23	57	1265	29	57	1624	31	57	1736	37	57	2072	41	57	2296
23	58	1288	29	58	1653	31	58	1767	37	58	2109	41	58	2337
23	59	1311	29	59	1682	31	59	1798	37	59	2146	41	59	2378
23	60	1334	29	60	1682	31	60	1829	37	60	2183	41	60	2419
23	61	1357	29	61	1711	31	61	1860	37	61	2220	41	61	2460
23	62	1380	29	62	1740	31	62	1891	37	62	2257	41	62	2501
23	63	1403	29	63	1769	31	63	1922	37	63	2294	41	63	2542
23	64	1426	29	64	1798	31	64	1922	37	64	2331	41	64	2583
23	65	1449	29	65	1827	31	65	1953	37	65	2368	41	65	2624
23	66	1472	29	66	1856	31	66	1984	37	66	2405	41	66	2665
23	67	1495	29	67	1885	31	67	2015	37	67	2442	41	67	2706
23	68	1518	29	68	1914	31	68	2046	37	68	2479	41	68	2747
23	69	1541	29	69	1943	31	69	2077	37	69	2516	41	69	2788
23	70	1564	29	70	1972	31	70	2108	37	70	2553	41	70	2829
23	71	1587	29	71	2001	31	71	2139	37	71	2590	41	71	2870
23	72	1587	29	72	2030	31	72	2170	37	72	2627	41	72	2911
23	73	1610	29	73	2059	31	73	2201	37	73	2664	41	73	2952
23	74	1633	29	74	2088	31	74	2232	37	74	2701	41	74	2993
23	75	1656	29	75	2117	31	75	2263	37	75	2738	41	75	3034

The Smarandache Function S(n)=D(I,J) for powers of primes P(I)^J.

P(I)	J	D(I,J)	P(I)	J	D(I,J)	P(I)	J	D(I,J)	P(I)	J	D(I,J)	P(I)	J	D(I,J)
59	1	59	61	1	61	67	1	67	71	1	71	73	1	73
59	2	118	61	2	122	67	2	134	71	2	142	73	2	146
59	3	177	61	3	183	67	3	201	71	3	213	73	3	219
59	4	236	61	4	244	67	4	268	71	4	284	73	4	292
59	5	295	61	5	305	67	5	335	71	5	355	73	5	365
59	6	354	61	6	366	67	6	402	71	6	426	73	6	438
59	7	413	61	7	427	67	7	469	71	7	497	73	7	511
59	8	472	61	8	488	67	8	536	71	8	568	73	8	584
59	9	531	61	9	549	67	9	603	71	9	639	73	9	657
59	10	590	61	10	610	67	10	670	71	10	710	73	10	730
59	11	649	61	11	671	67	11	737	71	11	781	73	11	803
59	12	708	61	12	732	67	12	804	71	12	852	73	12	876
59	13	767	61	13	793	67	13	871	71	13	923	73	13	949
59	14	826	61	14	854	67	14	938	71	14	994	73	14	1022
59	15	885	61	15	915	67	15	1005	71	15	1065	73	15	1095
59	16	944	61	16	976	67	16	1072	71	16	1136	73	16	1168
59	17	1003	61	17	1037	67	17	1139	71	17	1207	73	17	1241
59	18	1062	61	18	1098	67	18	1206	71	18	1278	73	18	1314
59	19	1121	61	19	1159	67	19	1273	71	19	1349	73	19	1387
59	20	1180	61	20	1220	67	20	1340	71	20	1420	73	20	1460
59	21	1239	61	21	1281	67	21	1407	71	21	1491	73	21	1533
59	22	1298	61	22	1342	67	22	1474	71	22	1562	73	22	1606
59	23	1357	61	23	1403	67	23	1541	71	23	1633	73	23	1679
59	24	1416	61	24	1464	67	24	1608	71	24	1704	73	24	1752
59	25	1475	61	25	1525	67	25	1675	71	25	1775	73	25	1825
59	26	1534	61	26	1586	67	26	1742	71	26	1846	73	26	1898
59	27	1593	61	27	1647	67	27	1809	71	27	1917	73	27	1971
59	28	1652	61	28	1708	67	28	1876	71	28	1988	73	28	2044
59	29	1711	61	29	1769	67	29	1943	71	29	2059	73	29	2117
59	30	1770	61	30	1830	67	30	2010	71	30	2130	73	30	2190
59	31	1829	61	31	1891	67	31	2077	71	31	2201	73	31	2263
59	32	1888	61	32	1952	67	32	2144	71	32	2272	73	32	2336
59	33	1947	61	33	2013	67	33	2211	71	33	2343	73	33	2409
59	34	2006	61	34	2074	67	34	2278	71	34	2414	73	34	2482
59	35	2065	61	35	2135	67	35	2345	71	35	2485	73	35	2555
59	36	2124	61	36	2196	67	36	2412	71	36	2556	73	36	2628
59	37	2183	61	37	2257	67	37	2479	71	37	2627	73	37	2701
59	38	2242	61	38	2318	67	38	2546	71	38	2698	73	38	2774
59	39	2301	61	39	2379	67	39	2613	71	39	2769	73	39	2847
59	40	2360	61	40	2440	67	40	2680	71	40	2840	73	40	2920
59	41	2419	61	41	2501	67	41	2747	71	41	2911	73	41	2993
59	42	2478	61	42	2562	67	42	2814	71	42	2982	73	42	3066
59	43	2537	61	43	2623	67	43	2881	71	43	3053	73	43	3139
59	44	2596	61	44	2684	67	44	2948	71	44	3124	73	44	3212
59	45	2655	61	45	2745	67	45	3015	71	45	3195	73	45	3285
59	46	2714	61	46	2806	67	46	3082	71	46	3266	73	46	3358
59	47	2773	61	47	2867	67	47	3149	71	47	3337	73	47	3431
59	48	2832	61	48	2928	67	48	3216	71	48	3408	73	48	3504
59	49	2891	61	49	2989	67	49	3283	71	49	3479	73	49	3577
59	50	2950	61	50	3050	67	50	3350	71	50	3550	73	50	3650
59	51	3009	61	51	3111	67	51	3417	71	51	3621	73	51	3723
59	52	3068	61	52	3172	67	52	3484	71	52	3692	73	52	3796
59	53	3127	61	53	3233	67	53	3551	71	53	3763	73	53	3869
59	54	3186	61	54	3294	67	54	3618	71	54	3834	73	54	3942
59	55	3245	61	55	3355	67	55	3685	71	55	3905	73	55	4015
59	56	3304	61	56	3416	67	56	3752	71	56	3976	73	56	4088
59	57	3363	61	57	3477	67	57	3819	71	57	4047	73	57	4161
59	58	3422	61	58	3538	67	58	3886	71	58	4118	73	58	4234
59	59	3481	61	59	3599	67	59	3953	71	59	4189	73	59	4307
59	60	3481	61	60	3660	67	60	4020	71	60	4260	73	60	4380
59	61	3540	61	61	3721	67	61	4087	71	61	4331	73	61	4453
59	62	3599	61	62	3721	67	62	4154	71	62	4402	73	62	4526
59	63	3658	61	63	3782	67	63	4221	71	63	4473	73	63	4599
59	64	3717	61	64	3843	67	64	4288	71	64	4544	73	64	4672
59	65	3776	61	65	3904	67	65	4355	71	65	4615	73	65	4745
59	66	3835	61	66	3965	67	66	4422	71	66	4686	73	66	4818
59	67	3894	61	67	4026	67	67	4489	71	67	4757	73	67	4891
59	68	3953	61	68	4087	67	68	4489	71	68	4828	73	68	4964
59	69	4012	61	69	4148	67	69	4556	71	69	4899	73	69	5037
59	70	4071	61	70	4209	67	70	4623	71	70	4970	73	70	5110
59	71	4130	61	71	4270	67	71	4690	71	71	5041	73	71	5183
59	72	4189	61	72	4331	67	72	4757	71	72	5041	73	72	5256
59	73	4248	61	73	4392	67	73	4824	71	73	5112	73	73	5329
59	74	4307	61	74	4453	67	74	4891	71	74	5183	73	74	5329
59	75	4366	61	75	4514	67	75	4958	71	75	5254	73	75	5402

'SN_DISTR, H. Ibstedt, 930322

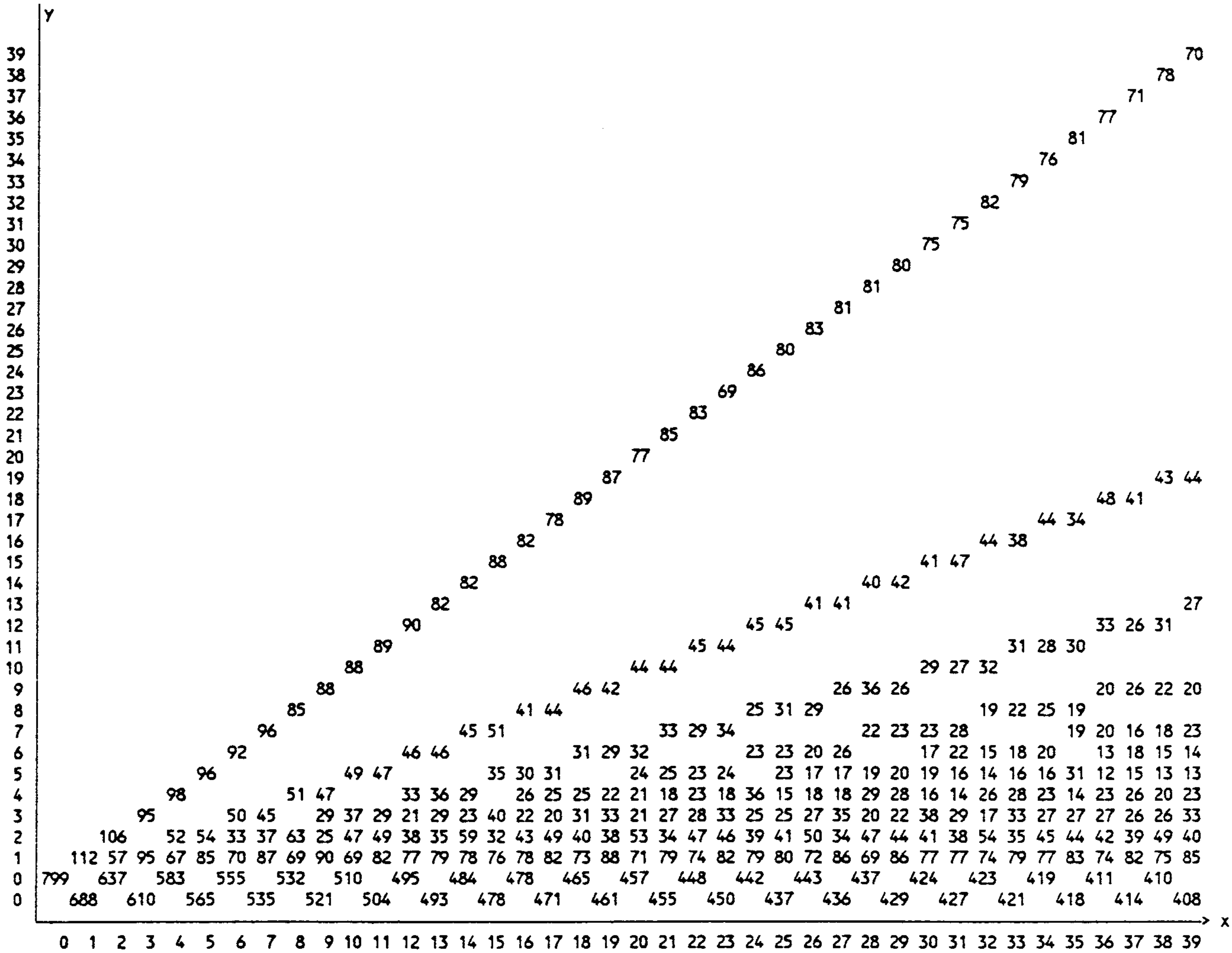
'The values of S(n) for n<32000 are input from the file SN.DAT and the number of values falling into each square of a 40 x 40 matrix are counted and displayed in a graph. An interresting pattern is formed by large primes while the bottom layer mainly resulting form composite numbers requires two lines in the graph.

'Set I=1021 and NB=82 on HPIIP.

```
DEFLNG A-S
DIM C(51,51)
CLS :WIDTH "LPT1:",130
LPRINT TAB(50) "The Smarandache function S(n)"
LPRINT TAB(50) "-----------------------------" :LPRINT :LPRINT
LPRINT TAB(35) "Number of values of S(n) in the interval 800y µ S(n) < 800(y+1).
LPRINT
LPRINT TAB(4)" | y"
LPRINT TAB(4)" | "
K=0
OPEN "SN.DAT" FOR INPUT AS #1
WHILE K<32000
INCR K
INPUT #1,S
I=K\800+1 :J=S\800+1
C(I,J)=C(I,J)+1
WEND
CLOSE #1
FOR J=40 TO 2 STEP -1 :LPRINT USING "###";J-1; :LPRINT " | ";
FOR I=1 TO 40
IF C(I,J)=0 THEN LPRINT SPC(3); ELSE LPRINT USING "###";C(I,J);
NEXT :LPRINT
NEXT
LPRINT USING "###";J-1; :LPRINT " | ";
J=1 :FOR I=1 TO 39 STEP 2
LPRINT USING "###";C(I,J); :LPRINT SPC(3);
NEXT :LPRINT
LPRINT USING "###";J-1; :LPRINT " | ";
FOR I=2 TO 40 STEP 2
LPRINT SPC(3); :LPRINT USING "###";C(I,J);
NEXT :LPRINT
LPRINT TAB(5) "└"; :FOR I=1 TO 120 :LPRINT "-"; :NEXT :LPRINT "> x"
LPRINT TAB(6); :FOR I=0 TO 39:LPRINT USING "###";I; :NEXT :LPRINT
LPRINT :LPRINT TAB(5) "Intervals: 800x≤ n < 800(x+1).
LPRINT :LPRINT TAB(5) "SN_DISTR "DATE$
LPRINT CHR$(12)
END
```

The Smarandache function S(n)

Number of values of S(n) in the interval 800y ≤ S(n) < 800(y+1).



Intervals: 800x ≤ n < 800(x+1).

SN_DISTR 03-24-1993

```
'SNP_SORT, H. Ibstedt, 930322

'This program inputs the Smarandache function S(n) for powers of primes from the file
SNP.DAT , sorts S(n) in ascending order and writes the result to a file SNP_ASC

DEFLNG A-S
CLS
DIM D(42,75),KP(3150),KJ(3150),SP(3150)
OPEN "SNP.DAT" FOR INPUT AS #1
FOR I=1 TO 42 :FOR J=1 TO 75
INCR L
INPUT #1,K1,K2,K3
KP(L)=K1 :KJ(L)=K2 :SP(L)=K3
NEXT :NEXT
FOR I=1 TO L :FOR J=I+1 TO L-1
IF SP(I)>SP(J) THEN SWAP SP(I),SP(J) :SWAP KJ(I),KJ(J) :SWAP KP(I),KP(J)
NEXT :NEXT
OPEN "SNP_ASC" FOR OUTPUT AS #2
FOR I=1 TO L
PRINT #2,KP(I),KJ(I),SP(I)
NEXT
CLOSE #2
PRINT "END" :END
```

'SMAR_iii, H. Ibstedt, 930322

'This program searches for solutions to S(x^n)+s(y^n)=S(z^n). Two parameters are set in the program: n (=NM) and the largest value of S(z^n) (=NS). x, y and z are restricted to powers of prime numbers. The input to the program is provided by the file SNP_ASC, which contains S(p^n) sorted in ascending order.

```
DEFLNG A-P :DEFDBL X,Y,Z
CLS :WIDTH "LPT1:",120
DIM D(42,75),K1(3150),K2(3150),K3(3150)
NM=3 :NS=120 :L=0
OPEN "SNP_ASC" FOR INPUT AS #1
WHILE NOT EOF(1)
INCR L :INPUT #1,K1(L),K2(L),K3(L)
WEND :CLOSE #1 :COUNT=0
LPRINT TAB(16) "Solutions to S(x^n) + S(y^n) = S(z^n) for n ="NM
LPRINT TAB(16) :LPRINT
"┌──────┬──────┬──────┬────────────┬────────────┬────────────┬──────┬──────┬──────
──┐"
LPRINT TAB(16) :LPRINT "│  P1  │  P2  │  P3  │      x      │      y      │      z
│ S(x^n) │ S(y^n) │ S(z^n) │"
LPRINT TAB(16) :LPRINT
"├──────┼──────┼──────┼────────────┼────────────┼────────────┼──────┼──────┼──────
──┤"
FOR I=NS TO 4 STEP -1 :PRINT I
IF K2(I) MOD NM <> 0 THEN III
FOR J=I-1 TO 3 STEP -1
IF K2(J) MOD NM <> 0 THEN JJJ
FOR K=J-1 TO 2 STEP -1
IF COUNT=75 THEN GOTO KKK
IF K2(K) MOD NM <> 0 THEN KKK
IF K1(I)=K1(J) OR K1(I)=K1(K) OR K1(J)=K1(K) THEN KKK
IF K3(I)=K3(J)+K3(K) THEN
INCR COUNT
X=K1(K)^(K2(K)/NM) :Y=K1(J)^(K2(J)/NM) :Z=K1(I)^(K2(I)/NM)
LPRINT TAB(16) "│"; :LPRINT USING "######";K1(K);
LPRINT "│"; :LPRINT USING "######";K1(J);
LPRINT "│"; :LPRINT USING "######";K1(I);
LPRINT "│"; :LPRINT USING "##############";X;
LPRINT "│"; :LPRINT USING "##############";Y;
LPRINT "│"; :LPRINT USING "##############";Z;
LPRINT "│"; :LPRINT USING "#######";K3(K);
LPRINT "│"; :LPRINT USING "#######";K3(J);
LPRINT "│"; :LPRINT USING "#######";K3(I);
LPRINT "│"
END IF
 KKK:
NEXT
 JJJ:
NEXT
 III:
NEXT
LPRINT TAB(16) :LPRINT
"└──────┴──────┴──────┴────────────┴────────────┴────────────┴──────┴──────┴──────
──┘"
LPRINT CHR$(12)
PRINT "COUNT ="COUNT
PRINT "END" :END
```

Solutions to $S(x^n) + S(y^n) = S(z^n)$ for $n = 3$

P1	P2	P3	x	y	z	$S(x^n)$	$S(y^n)$	$S(z^n)$
2	17	3	8	17	59049	12	51	63
5	2	3	5	32768	59049	15	48	63
7	2	3	7	8192	59049	21	42	63
2	13	3	128	13	59049	24	39	63
5	3	2	25	243	262144	25	33	58
5	11	2	25	11	262144	25	33	58
5	7	3	5	49	19683	15	42	57
5	2	3	5	8192	19683	15	42	57
7	2	3	7	2048	19683	21	36	57
2	11	3	128	11	19683	24	33	57
5	2	3	25	1024	19683	25	32	57
5	2	3	25	512	19683	25	32	57
3	2	19	3	32768	19	9	48	57
2	3	19	8	2187	19	12	45	57
3	7	19	9	49	19	15	42	57
5	7	19	5	49	19	15	42	57
3	2	19	9	8192	19	15	42	57
5	2	19	5	8192	19	15	42	57
7	2	19	7	2048	19	21	36	57
3	2	19	27	2048	19	21	36	57
2	3	19	128	243	19	24	33	57
2	11	19	128	11	19	24	33	57
5	2	19	25	1024	19	25	32	57
5	2	19	25	512	19	25	32	57
2	5	7	32	125	343	16	40	56
2	5	7	16	125	343	16	40	56
2	5	3	2	625	6561	4	50	54
2	7	3	8	49	6561	12	42	54
5	13	3	5	13	6561	15	39	54
7	11	3	7	11	6561	21	33	54
5	3	2	25	81	65536	25	27	52
3	7	17	3	49	17	9	42	51
3	2	17	3	8192	17	9	42	51
2	13	17	8	13	17	12	39	51
2	3	17	8	729	17	12	39	51
3	2	17	9	2048	17	15	36	51
5	2	17	5	2048	17	15	36	51
2	3	17	128	81	17	24	27	51
2	7	5	4	49	625	8	42	50
3	13	2	3	13	32768	9	39	48
5	3	2	5	243	32768	15	33	48
3	11	2	9	11	32768	15	33	48
5	11	2	5	11	32768	15	33	48
7	3	2	7	81	32768	21	27	48
7	5	2	7	25	16384	21	25	46
3	5	2	27	25	16384	21	25	46
2	11	3	8	11	2187	12	33	45
2	5	3	64	25	2187	20	25	45
7	2	3	7	128	2187	21	24	45
3	11	7	3	11	49	9	33	42
5	3	7	5	81	49	15	27	42
3	11	2	3	11	8192	9	33	42
5	3	2	5	81	8192	15	27	42
3	7	2	27	7	8192	21	21	42
3	5	2	9	25	4096	15	25	40
2	3	13	8	81	13	12	27	39
3	2	13	9	128	13	15	24	39
5	2	13	5	128	13	15	24	39
5	2	3	5	128	729	15	24	39
3	7	2	9	7	2048	15	21	36
5	7	2	5	7	2048	15	21	36
5	3	2	5	27	2048	15	21	36
2	5	3	4	25	243	8	25	33
2	7	3	8	7	243	12	21	33
2	5	11	4	25	11	8	25	33
3	2	11	3	128	11	9	24	33
2	7	11	8	7	11	12	21	33
2	3	11	8	27	11	12	21	33
2	5	3	8	5	81	12	15	27
2	7	5	2	7	25	4	21	25
2	3	5	2	27	25	4	21	25
3	2	5	3	32	25	9	16	25
3	2	5	3	16	25	9	16	25
3	5	2	3	5	128	9	15	24
3	2	7	3	8	7	9	12	21

Solutions to S(x^n) + S(y^n) = S(z^n) for n = 5

P1	P2	P3	x	y	z	S(x^n)	S(y^n)	S(z^n)
7	11	5	7	121	78125	35	110	145
2	17	5	2048	17	78125	60	85	145
2	3	5	4096	6561	78125	64	81	145
13	2	5	13	32768	78125	65	80	145
5	11	3	5	121	1594323	25	110	135
11	2	3	11	32768	1594323	55	80	135
7	2	3	49	16384	1594323	63	72	135
2	3	13	8	177147	169	16	114	130
5	3	13	5	59049	169	25	105	130
2	7	13	64	343	169	32	98	130
7	19	13	7	19	169	35	95	130
3	5	13	81	625	169	45	85	130
5	17	13	25	17	169	45	85	130
3	17	13	81	17	169	45	85	130
11	3	13	11	2187	169	55	75	130
2	11	3	8	121	531441	16	110	126
2	7	3	32	343	531441	28	98	126
2	3	7	4	177147	2401	12	114	126
2	11	7	8	121	2401	16	110	126
5	3	7	25	6561	2401	45	81	126
3	2	7	243	16384	2401	54	72	126
2	3	5	64	19683	15625	32	93	125
2	3	5	256	6561	15625	44	81	125
3	2	5	81	32768	15625	45	80	125
2	13	5	2048	13	15625	60	65	125
7	2	23	7	32768	23	35	80	115
11	2	23	11	2048	23	55	60	115
2	7	3	8	343	177147	16	98	114
2	7	11	4	343	121	12	98	110
3	7	11	3	343	121	12	98	110
5	17	11	5	17	121	25	85	110
7	3	11	7	2187	121	35	75	110
3	5	11	81	125	121	45	65	110
5	13	11	25	13	121	45	65	110
3	13	11	81	13	121	45	65	110
5	2	3	5	32768	59049	25	80	105
5	2	3	25	2048	59049	45	60	105
2	3	5	4	19683	3125	12	93	105
2	3	5	16	6561	3125	24	81	105
3	2	5	27	16384	3125	33	72	105
3	2	5	81	2048	3125	45	60	105
3	5	7	27	125	343	33	65	98
3	13	7	27	13	343	33	65	98
2	3	7	256	243	343	44	54	98
2	3	19	64	729	19	32	63	95
2	7	19	64	49	19	32	63	95
7	2	19	7	2048	19	35	60	95
2	5	3	2	625	19683	8	85	93
2	17	3	2	17	19683	8	85	93
5	2	3	5	8192	19683	25	68	93
2	5	3	32	125	19683	28	65	93
2	13	3	32	13	19683	28	65	93
2	11	3	128	11	19683	38	55	93
5	2	3	25	512	19683	45	48	93
5	2	17	5	2048	17	25	60	85
2	5	3	8	125	6561	16	65	81
2	13	3	8	13	6561	16	65	81
5	11	2	5	11	32768	25	55	80
7	5	2	7	25	32768	35	45	80
7	3	2	7	81	32768	35	45	80
2	7	3	4	49	2187	12	63	75
3	7	2	27	7	8192	33	35	68
2	3	5	64	27	125	32	33	65
2	3	13	64	27	13	32	33	65
2	11	3	2	11	729	8	55	63
5	2	3	5	128	729	25	38	63
2	7	3	32	7	729	28	35	63
2	11	7	2	11	49	8	55	63
5	2	7	5	128	49	25	38	63
5	7	2	5	7	2048	25	35	60
2	3	5	4	27	25	12	33	45
2	5	3	2	5	27	8	25	33

Solutions to S(x^n) + S(y^n) = S(z^n) for n = 7

P1	P2	P3	x	y	z	S(x^n)	S(y^n)	S(z^n)
3	11	23	2187	1331	529	102	220	322
17	29	23	17	29	529	119	203	322
2	5	7	4	9765625	823543	16	285	301
2	5	7	64	1953125	823543	46	255	301
3	13	7	19683	169	823543	132	169	301
2	5	43	4	9765625	43	16	285	301
2	5	43	64	1953125	43	46	255	301
3	13	43	19683	169	43	132	169	301
2	5	41	16	1953125	41	32	255	287
7	17	41	7	289	41	49	238	287
3	5	41	729	78125	41	87	200	287
11	3	41	121	59049	41	143	144	287
3	13	11	6561	169	14641	117	169	286
2	11	19	64	1331	361	46	220	266
7	31	19	7	31	361	49	217	266
13	7	19	13	2401	361	91	175	266
7	5	19	49	15625	361	91	175	266
13	5	19	13	15625	361	91	175	266
3	5	13	9	390625	2197	30	230	260
3	5	13	81	78125	2197	60	200	260
2	5	13	256	78125	2197	60	200	260
3	11	13	6561	121	2197	117	143	260
5	13	7	125	169	117649	90	169	259
5	13	37	125	169	37	90	169	259
2	7	5	32	16807	1953125	38	217	255
2	31	5	32	31	1953125	38	217	255
2	29	5	128	29	1953125	52	203	255
2	5	17	2	390625	289	8	230	238
3	11	17	3	1331	289	18	220	238
2	5	17	32	78125	289	38	200	238
11	23	17	11	23	289	77	161	238
3	11	5	729	121	390625	87	143	230
3	7	11	27	2401	1331	45	175	220
3	5	11	27	15625	1331	45	175	220
3	5	11	243	3125	1331	75	145	220
3	19	11	729	19	1331	87	133	220
3	7	11	729	343	1331	87	133	220
2	5	7	1024	3125	16807	72	145	217
2	5	31	1024	3125	31	72	145	217
5	11	29	25	121	29	60	143	203
3	11	29	81	121	29	60	143	203
2	11	29	256	121	29	60	143	203
3	5	7	9	3125	2401	30	145	175
2	11	7	16	121	2401	32	143	175
2	11	5	16	121	15625	32	143	175
2	23	13	2	23	169	8	161	169
2	5	13	8	3125	169	24	145	169
7	5	13	7	625	169	49	120	169
2	3	13	128	6561	169	52	117	169
2	5	23	4	3125	23	16	145	161
3	11	23	3	121	23	18	143	161
2	5	3	8	625	59049	24	120	144
2	17	11	8	17	121	24	119	143
2	7	11	128	49	121	52	91	143
2	13	11	128	13	121	52	91	143
2	3	19	4	6561	19	16	117	133
2	3	19	64	729	19	46	87	133
2	3	7	4	6561	343	16	117	133
2	3	7	64	729	343	46	87	133
5	2	3	25	1024	19683	60	72	132
2	3	5	256	81	625	60	60	120
2	3	17	16	729	17	32	87	119
5	2	3	5	1024	2187	30	72	102
2	3	7	4	243	49	16	75	91
3	2	7	27	64	49	45	46	91
2	3	13	4	243	13	16	75	91
3	2	13	27	64	13	45	46	91
3	2	5	3	1024	125	18	72	90
3	2	5	9	256	125	30	60	90
2	7	3	32	7	729	38	49	87
2	3	11	16	27	11	32	45	77
5	3	2	5	9	256	30	30	60

Solutions to S(x^n) + S(y^n) = S(z^n) for n = 11

P1	P2	P3	x	y	z	S(x^n)	S(y^n)	S(z^n)
17	53	41	17	2809	68921	187	1166	1353
29	47	41	29	2209	68921	319	1034	1353
37	43	41	37	1849	68921	407	946	1353
11	31	41	1771561	961	68921	671	682	1353
61	31	41	61	961	68921	671	682	1353
5	29	61	25	707281	3721	95	1247	1342
11	37	61	11	50653	3721	121	1221	1342
17	19	61	4913	130321	3721	544	798	1342
11	71	61	161051	71	3721	561	781	1342
3	23	31	243	6436343	923521	114	1219	1333
47	71	59	47	71	3481	517	781	1298
11	67	59	161051	67	3481	561	737	1298
5	19	29	5	47045881	707281	50	1197	1247
13	17	29	28561	83521	707281	533	714	1247
2	19	37	4	47045881	50653	24	1197	1221
17	47	37	17	2209	50653	187	1034	1221
29	41	37	29	1681	50653	319	902	1221
53	29	37	53	841	50653	583	638	1221
13	43	23	169	1849	6436343	273	946	1219
7	43	23	2401	1849	6436343	273	946	1219
5	17	53	25	24137569	2809	95	1071	1166
47	59	53	47	59	2809	517	649	1166
13	29	17	13	24389	24137569	143	928	1071
13	19	17	169	130321	24137569	273	798	1071
7	19	17	2401	130321	24137569	273	798	1071
3	19	47	3	2476099	2209	27	1007	1034
11	73	47	121	73	2209	231	803	1034
23	71	47	23	71	2209	253	781	1034
11	53	47	14641	53	2209	451	583	1034
41	53	47	41	53	2209	451	583	1034
43	11	47	43	161051	2209	473	561	1034
5	67	19	15625	67	2476099	270	737	1007
3	41	31	81	1681	29791	90	902	992
7	59	31	16807	59	29791	343	649	992
7	13	43	49	4826809	1849	140	806	946
13	73	43	13	73	1849	143	803	946
19	67	43	19	67	1849	209	737	946
5	13	43	15625	371293	1849	270	676	946
3	37	29	243	1369	24389	114	814	928
11	71	41	11	71	1681	121	781	902
11	61	41	121	61	1681	231	671	902
23	59	41	23	59	1681	253	649	902
29	53	41	29	53	1681	319	583	902
31	11	41	31	161051	1681	341	561	902
7	37	17	7	1369	1419857	70	814	884
13	11	37	13	1771561	1369	143	671	814
13	61	37	13	61	1369	143	671	814
11	53	37	121	53	1369	231	583	814
23	11	37	23	161051	1369	253	561	814
5	17	37	15625	4913	1369	270	544	814
31	43	37	31	43	1369	341	473	814
11	43	37	1331	43	1369	341	473	814
7	23	13	7	12167	4826809	70	736	806
3	11	13	729	1771561	4826809	135	671	806
5	11	13	125	1771561	4826809	135	671	806
3	61	13	729	61	4826809	135	671	806
5	61	13	125	61	4826809	135	671	806
7	37	13	117649	37	4826809	399	407	806
19	37	13	361	37	4826809	399	407	806
11	31	73	11	961	73	121	682	803
5	13	73	15625	28561	73	270	533	803
13	29	71	13	841	71	143	638	781
11	23	67	121	529	67	231	506	737
2	13	23	32	371293	12167	60	676	736
7	13	23	343	28561	12167	203	533	736
19	43	31	19	43	961	209	473	682
11	41	31	121	41	961	231	451	682
3	59	13	3	59	371293	27	649	676
2	19	13	64	6859	371293	68	608	676
7	43	13	343	43	371293	203	473	676
5	11	13	3125	14641	371293	225	451	676
5	41	13	3125	41	371293	225	451	676
29	17	13	29	289	371293	319	357	676
13	23	59	13	529	59	143	506	649
11	47	29	11	47	841	121	517	638

'SMAR_iv, H. Ibstedt, 930323

'This program searches for solutions to the equation S(k^n)^i = S(n^k). The search is limited to the first 8000 values of S(n) loaded from the file SN.DAT. No non-trivial solutions were found.

```
DEFLNG I-S :DEFDBL X,Y,Z
CLS :WIDTH "LPT1:",120
DIM S(8000)
L=0 :NS=90 :KS=90
OPEN "SN.DAT" FOR INPUT AS #1
FOR L=1 TO 8000
INPUT #1,S(L)
NEXT
CLOSE #1
PRINT S(8000)
LPRINT TAB(12) "Search for solutions to S(k^n)^i = S(n^k). "
LPRINT TAB(12) :LPRINT "┌────────┬────────┬────────┬────────┬────────┬────────┬─────────┐"
LPRINT TAB(12) :LPRINT "│   k    │   n    │ S(k^n) │   n    │   k    │ S(n^k) │    i    │"
LPRINT TAB(12) :LPRINT "├────────┼────────┼────────┼────────┼────────┼────────┼─────────┤"
FOR K=2 TO KS :PRINT K
FOR N=2 TO NS
IF K^N>8000 OR N^K>8000 THEN N=NS :GOTO L2
C=S(K^N) :D=S(N^K)
IF C>D THEN SWAP C,D
E=1 :I=0
 L1:
E=E*C :I=I+1
IF E>D THEN L2
IF E=D AND K<>N THEN GOSUB LW
IF E<D THEN L1
 L2:
NEXT
NEXT
LPRINT TAB(12) :LPRINT "└────────┴────────┴────────┴────────┴────────┴────────┴─────────┘"
LPRINT CHR$(12)
PRINT "END" :END
 LW:
LPRINT TAB(12) "│"; :LPRINT USING "######";K;
LPRINT "│"; :LPRINT USING "######";N;
LPRINT "│"; :LPRINT USING "######";S(K^N);
LPRINT "│"; :LPRINT USING "######";N;
LPRINT "│"; :LPRINT USING "######";K;
LPRINT "│"; :LPRINT USING "######";S(N^K);
LPRINT "│"; :LPRINT USING "#######";I;
LPRINT "│"
RETURN
```

Search for solutions to S(k^n)^i = S(n^k).

k	n	S(k^n)	n	k	S(n^k)	i
2	4	6	4	2	6	1
4	2	6	2	4	6	1

'SMAR_v, H. Ibstedt, 930323

'This program uses the first 32000 values of S(n) input from the file SN.DAT to calculate 1+1/S(2)+1/S(3)+ ... +1/S(n) for n=800, 1600, 2400, ... 32000. The sums are used to study the behaviour of 1+1/S(2)+1/S(3)+ ... 1/S(n)-T(n). The following cases are examined T(n)=0, T(n)=Log(S(n)), T(n) equal to the logarithm of the largest prime less than n and finally T(n)=(rK-V)+1/2^a+1/3^a+ ... +1/n^a, where a=.5164, r=0.96 and V=27. K=n/800.

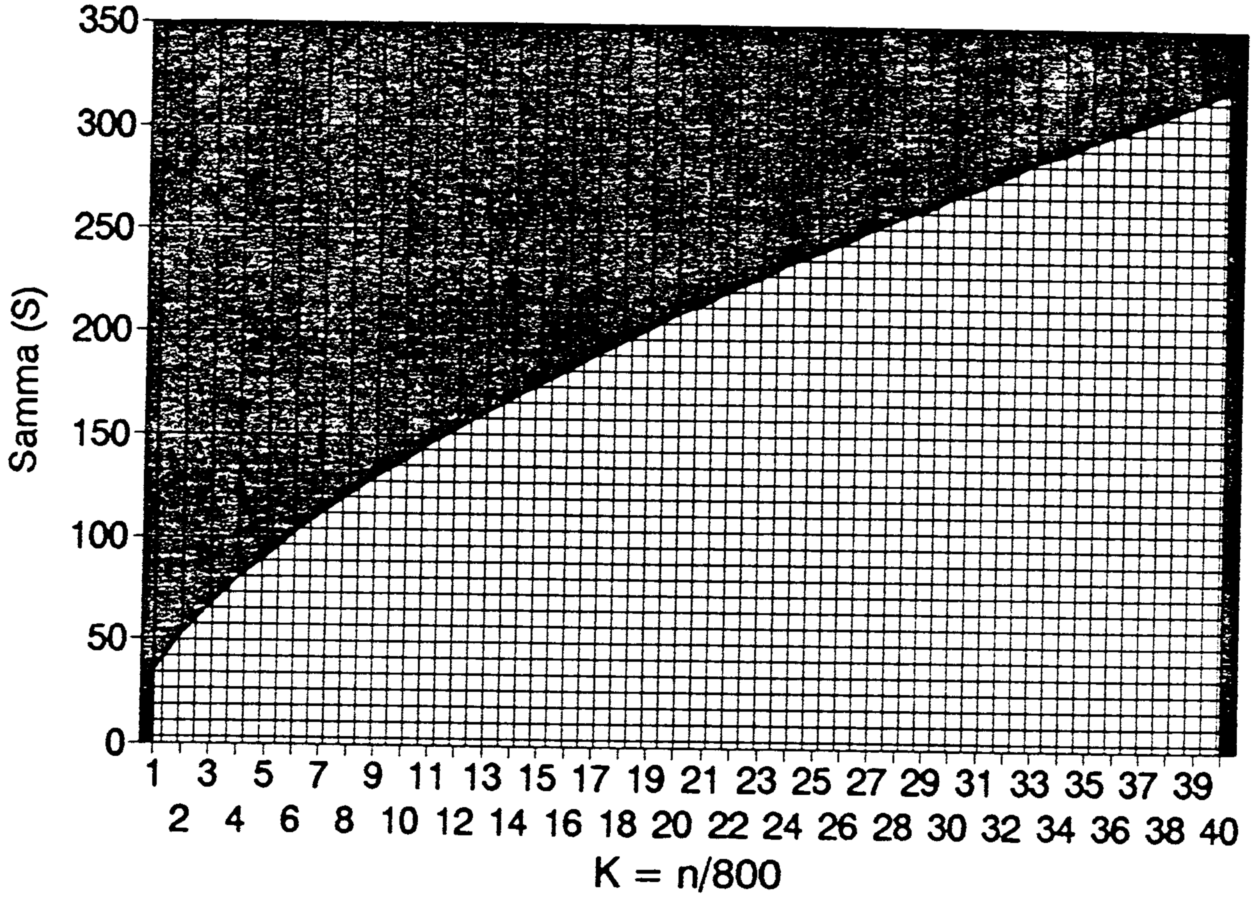
'Set I=22 and NB=62 on HPIIP
'Number of intervals = NN (40), Size of interval NI (800).

```
DEFDBL A-Z :DIM LAM(40),SAM(40),ZAM(40),HAM(40)
NN=40 :NI=800
CLS :WIDTH "LPT1:",120
Z=1 :K=1 :Z1=0
LPRINT :LPRINT
LPRINT TAB(12); "Behaviour of 1+1/S(2)+1/S(3)+ ... 1/S(n)-T(n). K=n\800."
LPRINT TAB(12); "In the column LAM(K) T(n)=0, in SAM(K) T(n)=Log(S(n)),in"
LPRINT TAB(12); "ZAM(K) T(n) is the logarithm of the largest prime less"
LPRINT TAB(12); "than n and in HAM(K) T(n)=1+1/2^.52+1/3^.52+ ... 1/n^.52." :LPRINT
LPRINT TAB(12); "┌──────────┬──────────┬──────────┬──────────┬──────────┐"
LPRINT TAB(12); "│    K     │  LAM(K)  │  SAM(K)  │  ZAM(K)  │  HAM(K)  │"
LPRINT TAB(12); "├──────────┼──────────┼──────────┼──────────┼──────────┤"
OPEN "SN.DAT" FOR INPUT AS #1
WHILE K<41
G=1
WHILE I<K*NI
INCR I
INPUT #1,S
IF S>G THEN G=S
Z=Z+1/S
Z1=Z1+1/I^.5164
WEND
LAM(K)=Z
HAM(K)=Z-Z1-.96*K+V
SAM(K)=Z-LOG(S)
ZAM(K)=Z-LOG(G)
LPRINT TAB(12); :LPRINT "│"; :LPRINT USING "##########";K;
LPRINT "│"; :LPRINT USING "##########";LAM(K);
LPRINT "│"; :LPRINT USING "##########";SAM(K);
LPRINT "│"; :LPRINT USING "##########";ZAM(K);
LPRINT "│"; :LPRINT USING "##########";HAM(K);
LPRINT "│"
INCR K
WEND
CLOSE #1
LPRINT TAB(12); "└──────────┴──────────┴──────────┴──────────┴──────────┘"
LPRINT :LPRINT TAB(12) "SMAR_v"
LPRINT CHR$(12)
PRINT "END" :END
```

PCW, Numbers Count: Problem (v), February 1993

K	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
S	35	53	67	80	91	102	112	121	130	138	146	154	162	169	176	183	190	197	203	210
K	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
S	216	223	228	235	240	246	252	258	262	268	273	279	285	289	295	300	304	310	315	320

1+1/S(2)+ ... +1/S(n)-log(S(n))
n = 800, 1600, 2400, ... 32000



Behaviour of 1+1/S(2)+1/S(3)+ ... 1/S(n)-T(n). K=n\800.
In the column LAM(K) T(n)=0, in SAM(K) T(n)=Log(S(n)),in ZAM(K) T(n) is the logarithm of the largest prime less than n and in HAM(K) T(n)=(r*K-V)+1/2^a+1/3^a+ ... 1/n^a, where r=0.96, V=27 and a=0.5164 have been chosen to fit HAM(K) as closely as possible to 0.

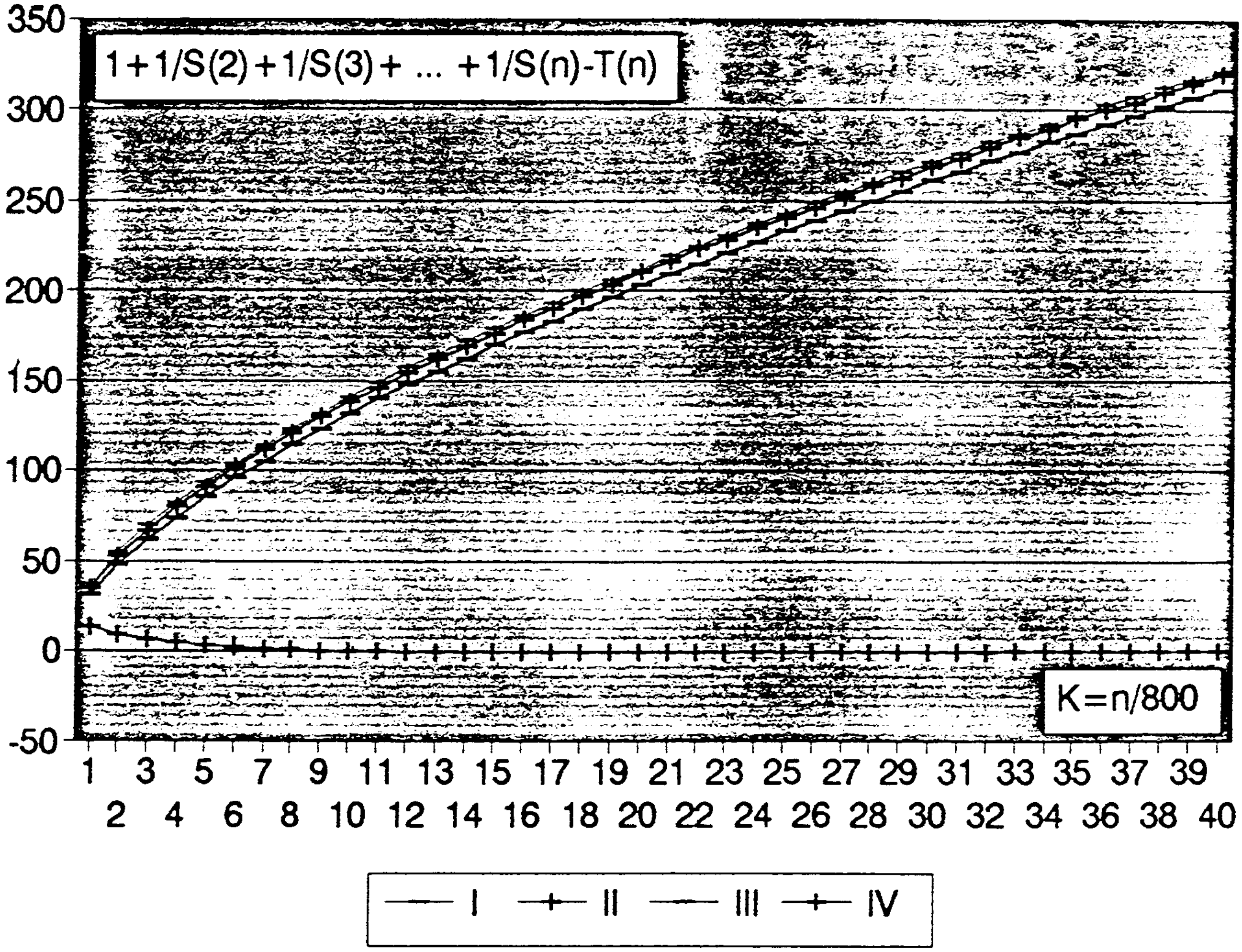
K	LAM(K)	SAM(K)	ZAM(K)	HAM(K)
1	37	35	31	13
2	55	53	48	9
3	70	67	62	6
4	82	80	74	5
5	94	91	85	3
6	104	102	96	2
7	114	112	105	1
8	123	121	115	1
9	132	130	123	0
10	141	138	132	-0
11	149	146	140	-0
12	157	154	148	-1
13	164	162	155	-1
14	172	169	162	-1
15	179	176	170	-1
16	186	183	176	-1
17	193	190	183	-1
18	200	197	190	-1
19	206	203	196	-1
20	212	210	203	-1
21	219	216	209	-1
22	225	223	215	-1
23	231	228	221	-1
24	237	235	227	-1
25	243	240	233	-1
26	249	246	239	-1
27	254	252	244	-1
28	260	258	250	-1
29	265	262	255	-1
30	271	268	261	-1
31	276	273	266	-1
32	282	279	272	-1
33	287	285	277	-0
34	292	289	282	-0
35	297	295	287	-0
36	303	300	292	-0
37	307	304	297	-0
38	313	310	302	0
39	317	315	307	0
40	322	320	312	0

SMAR_v

The Smarandache Function S(n)

Graphical representation of $1 + 1/S(2) + 1/S(3) + \ldots + 1/S(n) - T(n)$ for $n < 32000$.

Graph I:	$T(n) = 0$.
Graph II:	$T(n) = \log(S(n))$.
Graph III:	$T(n) = \log(\text{largest prime} < n)$.
Graph IV:	$T(n) = (rK-V) + 1/2^a + 1/3^a + \ldots + 1/n^a$, where the parameters $r = 0.96$, $V = 27$ and $a = 0.5164$ were chosen to fit the graph as closely as possible to a horizontal line.



'SMARAND1, H. Ibstedt, 930327

'The Smarandache function S(n) calculated by comparing largest prime and S(P^A). The upper limit for the calculation is n = 1000000. The results are used to calculate 1+1/S(2)+1/S(3)+ ... +1/S(n) registering partial sums for n = 25000, 50000, 75000, ... 1000000.

```
DEFLNG A-S
CLS :T=TIMER
DIM P(168),D(168,20),K(168),L(168)
OPEN "PA" FOR INPUT AS #1
FOR I=1 TO 168 :INPUT #1,P(I) :NEXT :CLOSE #1
```

*'This part of the program calculates S(P(I)^A) and saves the result in the array D(I,A), P(I) is the Ith prime number. The routine uses the fact that D(I,A)<=P(I)*A in the search for the value of D(I,A). This calculation goes as far as is required to calculate S(n) up to n = 1000000.*

```
FOR I=1 TO 168
A=2 :P=P(I) :D(I,1)=P
WHILE A<21
C=0 :N=0
 L:
INCR C
INCR N,P
IF C>=A THEN D(I,A)=N :GOTO LWEND
PP=P*P
 L1:
IF N-PP*INT(N/PP)=0 THEN INCR C :PP=PP*P :GOTO L1
IF C>=A THEN D(I,A)=N :GOTO LWEND :ELSE L
 LWEND:
INCR A
WEND
NEXT
```

'This part of the program calculates S(N) and the sum of reciprocals. It calls on the subroutine NFACT to express N in prime factor form. Factors P(I)^A with A>1 are replaced by D(I,A) and placed in array L() together with the factors P(I) of multiplicity 1. S(N) is then the largest component of L(). S(N) is stored in a file SN.DAT.

```
N=1 :Z=1 :D=0 :ZC=0
OPEN "SAM.DAT" FOR APPEND AS #3
WHILE N<1000001
if inkey$<>"" then print "end" :end
INCR N :INCR D :PRINT N
'Factorize N.
GOSUB NFACT
IF K(0)>0 THEN S=P(0) :GOTO LWR
'Construct L().
FOR I=1 TO 168 :L(I)=0 :NEXT
C=0
FOR I=1 TO M
INCR C
```

```
IF K(I)=1 THEN L(C)=P(I)
IF K(I)>1 THEN L(C)=D(I,K(I))
NEXT
```

'Find the largest value of L() and hence S(N).

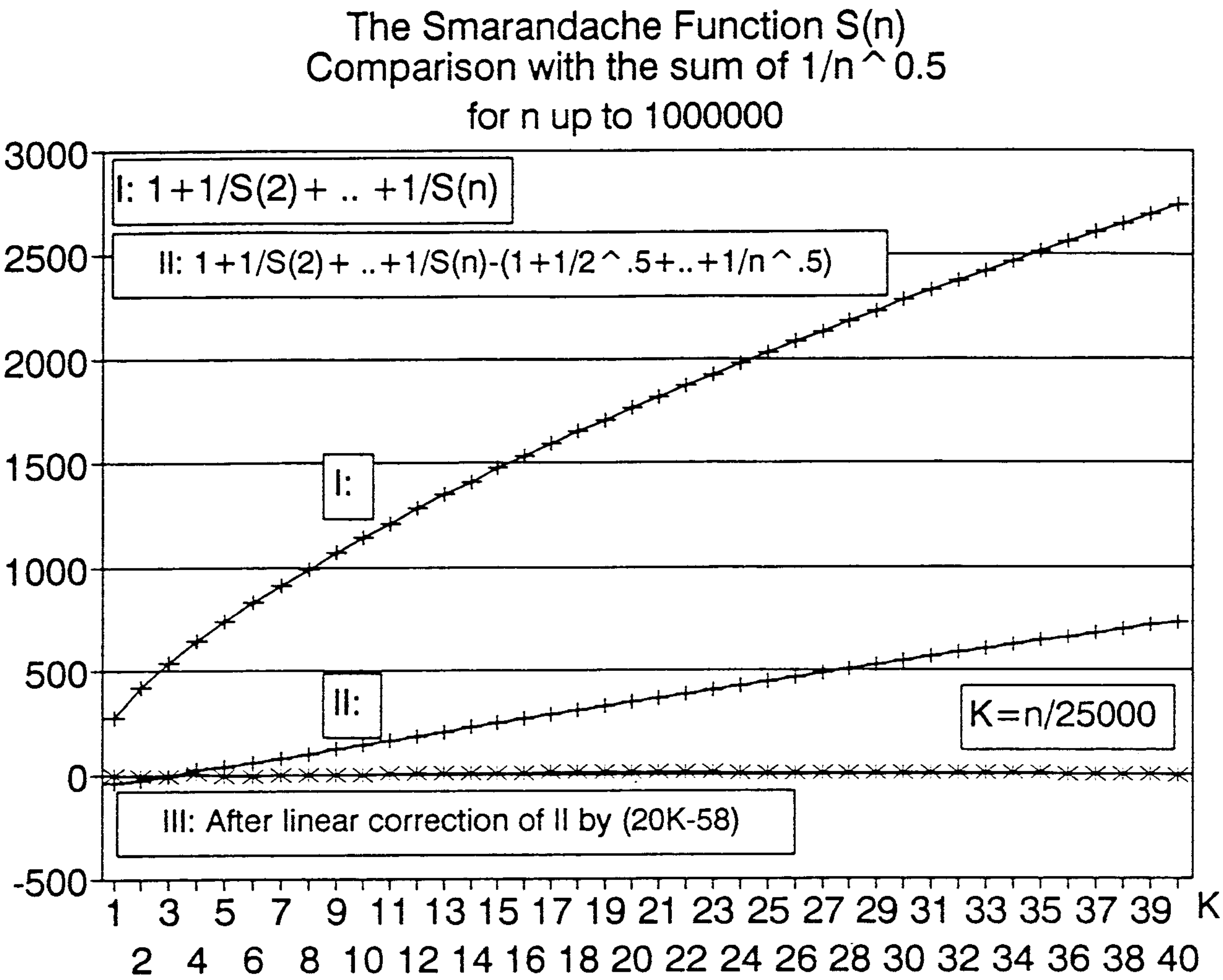
```
 S=0
 FOR I=1 TO C
 IF L(I)>S THEN S=L(I)
 NEXT
  LWR:
 Z=Z+1/S
 IF D=25000 THEN ZC=ZC+Z :WRITE #3,Z,ZC :Z=0 :D=0
 WEND
 CLOSE #3
 T=TIMER-T :PRINT T
 END
```

'Subroutine for factorization of N. (Improved from SMARAND to avoid large primes)

```
  NFACT:
FOR I=0 TO 168 :K(I)=0 :NEXT :P(0)=0
N1=N :I=0 :M=0
FOR I=1 TO 168
 LA:
IF N1-P(I)*INT(N1/P(I))=0 THEN K(I)=K(I)+1 :M=I :N1=N1/P(I) :GOTO LA
IF N1=1 THEN I=168
NEXT
IF N1>1 THEN P(0)=N1 :K(0)=1
RETURN
```

Problem v: Summary of obtained data.

K	Sum interval	Graph I Sum 1/S(n)	Sum 1/n^2	Graph II Diff.	Lin. Corr.	Graph III after lin. corr.
1	277	277	315	-38	-38	0
2	145	422	446	-24	-18	-6
3	119	541	546	-5	2	-7
4	105	646	621	25	22	3
5	96	742	706	36	42	-6
6	89	831	773	58	62	-4
7	83	914	835	79	82	-3
8	79	993	893	100	102	-2
9	76	1069	947	122	122	0
10	73	1142	999	143	142	1
11	70	1212	1047	165	162	3
12	68	1280	1094	186	182	4
13	66	1346	1139	207	202	5
14	64	1410	1182	228	222	6
15	63	1473	1223	250	242	8
16	61	1534	1263	271	262	9
17	60	1594	1302	292	282	10
18	58	1652	1340	312	302	10
19	57	1709	1377	332	322	10
20	56	1765	1413	352	342	10
21	55	1820	1448	372	362	10
22	54	1874	1482	392	382	10
23	53	1927	1515	412	402	10
24	52	1979	1548	431	422	9
25	52	2031	1580	451	442	9
26	51	2082	1611	471	462	9
27	50	2132	1642	490	482	8
28	50	2182	1672	510	502	8
29	49	2231	1701	530	522	8
30	49	2280	1731	549	542	7
31	48	2328	1759	569	562	7
32	47	2375	1787	588	582	6
33	47	2422	1815	607	602	5
34	46	2468	1842	626	622	4
35	46	2514	1869	645	642	3
36	45	2559	1896	663	662	1
37	45	2604	1922	682	682	0
38	44	2648	1948	700	702	-2
39	44	2692	1973	719	722	-3
40	44	2736	1999	737	742	-5


The Smarandache Function S(n)
Comparison with the sum of 1/n^0.5
for n up to 1000000
I: 1+1/S(2)+ .. +1/S(n)
II: 1+1/S(2)+ ..+1/S(n)-(1+1/2^.5+..+1/n^.5)
I:
II:
K=n/25000
III: After linear correction of II by (20K-58)
3000
2500
2000
1500
1000
500
0
-500
K

C O N T E N T S

$ 9.95

A collection of papers concerning Smarandache type functions, numbers, sequences, integer algorithms, paradoxes, experimental geometries, algebraic structures, neutrosophic probability, set, and logic, etc.

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