PROPOSED PROBLEM

by J. Thompson

Calculate:

$$\lim_{n\to\infty}\left(1+\sum_{k=2}^{n}\frac{1}{\eta(k)}-\log\eta(n)\right)$$

where $\eta(n)$ is Smarandache Function : the smallest integer m , such that $m!$ is divisible by n .

Solution:

We know that $\left(\sum_{k=1}^{n} 1/k-\log n\right)$ converges to e for $n\to\infty$.

It's easy to show that for $k\geq 2$, $\eta(k)\leq k$. More, for k a composite number ≥ 10 , $\eta(k)\leq k/2$. Also, if $p>4$ then : $\eta(p)=p$ if and only if p is prime.

$$\sum_{k=10}^{n}\frac{1}{\eta(k)}-\log\eta(n)\geq\left(\sum_{k=10}^{n}\frac{1}{k}-\log n\right)+\sum_{\substack{k=10\\ k\neq prime}}\frac{1}{k}\xrightarrow{n\to\infty}e+\infty=\infty$$

because for any prime number p there exists a composite number p-1 such that $\frac{1}{p-1}>\frac{1}{p}$ thus :

$$\sum_{\substack{k=10\\ k\neq prime}}\frac{1}{k}=\frac{1}{10}+\frac{1}{12}+\frac{1}{14}+\frac{1}{15}+\frac{1}{16}+\frac{1}{18}+\ldots+\frac{1}{n}>\frac{1}{11}+\frac{1}{13}+\frac{1}{17}+\ldots+\frac{1}{p(n)}\xrightarrow{n\to\infty}\infty$$

where $p(n)$ is the greatest prime number less that n .

We took out the first nine terms of that series, the limit of course didn't chance.

Reference:

Smarandache F., " A function in the number theory", <Analele Univ. Timisoara>, fasc. 2, Vol. XVII,pp. 163-8, 1979;
see Mathematical Review: 82a:03012.

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