PROPOSED PROBLEM (4)

Let m be a fixed positive integer. Calculate:

$$\lim_{i\to\infty} \eta(p_i^m)/p_i$$

where $\eta(n)$ is Smarandache Function defined as the smallest integer m such that m! is divisible by n, and p_i the prime series.

<u>Solution</u>:

We note by p_j a prime number greater than m. We show that

$\eta(p_i^m) = mp_i$, for any $i > j$:

if by absurd $\eta(p_i^m) = a < mp_i$ then

$a! = 1\cdot 2\cdot \ldots \cdot p_i \cdot \ldots \cdot (2p_i)\cdot \ldots \cdot ((m-k)p_i)\cdot \ldots \cdot a$, with $k > 0$, will be divisible by p_i^{m-k} but not by p_i^m.

Then this limit is equal to m.

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