A Hybrid IF-FUCOM-GRA Approach and its Application to Determine Optimal Bacterial Concentrations on Mortar at Optimal Curing Day

Srila DEY[1], Florentin SMARANDACHE[2], Rama DEBBARMA[1], Priyanka MAJUMDER[3,*]

[1] *Civil Engineering Department, National Institute of Technology Agartala, 799046, Tripura, India*
[2] *University of New Mexico, Mathematics Department, 705 Gurley Ave., Gallup, NM 87301, USA*
[3] *Department of Basic Science and Humanities (Mathematics),*
  Techno College of Engineering Agartala, Tripura, India
e-mail: srilaagt10@gmail.com, smarand@unm.edu, ramadebbarma@gmail.com,
majumderpriyanka94@yahoo.com

Abstract. In this study, Intuitionistic Fuzzy Consistency Method (IF-FUCOM) and Grey Relation Analysis (GRA) were combined to assess the effects of Bacillus subtilis bacteria on concrete properties, as well as to determine the optimal bacteria concentration and curing day. Three different concentrations of bacteria were added to the mortar mixes, like 103, 105, and 107 cells/ml of water. Mortar samples were left to cure for 7 days, 14 days, and 28 days to evaluate compressive strength, water absorption, crack healing. According to the proposed algorithm, 105 bacteria are the optimal concentration, while 28 days is the ideal curing time.

Key words: IF-FUCOM, GRA, IF-FUCOM-GRA, bacterial concentrations,, curing day.

1. Introduction

Structures must become stronger, faster, and more versatile, as well as more durable, with a huge increase in the amount of cement used in the process. Most construction projects today use Portland cement concrete, which is the predominant type of concrete. Because of the low cost of construction materials and the ease of maintenance, concrete structures can be built and maintained.

Recent research found that a biomaterial can be used to treat concrete cracks (Van Tittelboom *et al.*, 2010; Ramachandran *et al.*, 2001). Scientists have discovered that inorganic substances that are deposited by microorganisms inside cement-sand mortar or the pores of concrete can be used for filling cracks (Ghosh *et al.*, 2005; Ramakrishnan *et al.*, 1999). A concrete structure's inherent weakness is its vulnerability to cracks that allow water to penetrate, causing corrosion and reducing its durability (Chahal *et al.*, 2012).

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*Corresponding author.
Priyanka Majumder

Ramachandran *et al.* (2001) pioneered microbial concrete, and since then there has been a considerable volume of research on the topic. As ureolytic bacteria are alkali-resistant and nutrition is not necessary for survival for hundreds of years, the researchers examined Bacillus sphaericus, Sporosarcina pasteurii, and Bacillus megaterium (Arunachalam *et al.*, 2010; Dhami *et al.*, 2013; Achal *et al.*, 2011). In some studies, researchers examined the effects of adding bacteria to concrete on its compressive strength and crack healing. The findings reflect that most of them considered bacterial concentrations between 103 to 107 cells/ml when considering strength enhancement. Contrary to crack healing, researchers use higher concentrations of bacterial cells (107–109 cells/ml) (Majumdar *et al.*, 2012; Mondal and Ghosh, 2018; De Muynck *et al.*, 2008).

The versatility of concrete makes it a popular choice for building materials. Locally available, strong and durable, it is versatile. Despite its capability to resist compression loads to a limit, if the load applied on the concrete exceeds their limit of load resistance, it results in cracks in the concrete, which lowers its strength. Concrete's serviceability limit is affected by cracks. Concrete may become weaker and less durable as moisture and other chemicals get into it. In addition to that, water absorption is another major issue that reduces the life of concrete. Researchers are currently using bacteria to treat concrete mortar to overcome the problems. The selection of an optimal bacteria concentration and curing day can also pose a problem. Grey Relational Analysis (GRA) can be used in this field to find an optimal solution, since various researchers use it in different fields as an optimization technique (Dagdevir and Ozceyhan, 2021; Güler *et al.*, 2021; Roy *et al.*, 2016; Si *et al.*, 2021). A major drawback of GRA is that it assigns a similar weight to all output characteristics, even though in practice not all output characteristics are equally important (Fangfang, 2021). To overcome the problem, some researchers are using Analytical Hierarchy Process (AHP) along with GRA (Erdoğan and Sayin, 2018; Erdoğan *et al.*, 2020). An advantage, as well as a reason for using the AHP method, is that results can be validated by determining the consistency of the model with actual data. A study suggests that comparing pairwise across nine criteria by the AHP method is extremely difficult since it requires a great deal of comparisons $n(n-1)/2$ (Milićević *et al.*, 2007).

The BWM has been shown to be able to resolve certain of the previously listed constraints associated with AHP models (Rezaei, 2015). Compared to AHP's many pairwise comparisons, BWM does only a small number, such as $2n-3$. There is a direct correlation between the number of pairwise criteria comparisons and the consistency of the method. Moreover, the BWM does not require a comparison of nine criteria, merely a smaller number of criteria. The AHP model is improved by forming the Best-to-Others (BO) as well as Other-to-Worst (OW) vectors, resulting in fewer pairwise comparisons, and the resulting data are more consistent. The BWM, however, has a problem in determining the optimum weight coefficients when there is a large degree of variation inconsistency. The weight coefficient can be determined by using the average of the intervals as final values in such cases, as proposed by Rezaei (2015). Despite this, the central part of the interval is not guaranteed to be representative of the optimal weight coefficient value. A better value might lie closer to the right or left end of the interval. The interval weight values do not even cover the optimum values of priority coefficients in the cases of the greater inconsistency of results (Pamučar *et al.*, 2018a).

FUCOM uses pairwise comparisons of criteria to determine criteria priority, and it validates results across a wide range of deviations from maximum consistency in order to determine criteria priority (Pamučar *et al.*, 2018b). As compared to BWM and AHP tools, FUCOM eliminates some of their weaknesses. When using FUCOM, criteria can be compared in pairs ($n − 1$ comparisons), DMC (Deviation from the Maximum Consistency) can be calculated when comparing comparisons, and transitivity can be recognized throughout pairwise comparisons. There is a subjective effect of DMs on the weighting of criteria in FUCOM, as there is in other subjective models. As such, this refers specifically to the first, as well as second steps of the FUCOM. The FUCOM, in contrast to subjective models, shows minor deviations from the optimum value in the priority value of the criteria. In addition, the FUCOM methodological procedure removes the redundancy caused by comparing criteria pairwise, which is a problem with some subjective methods for priority value determination (Božanić *et al.*, 2019; Bozanic *et al.*, 2020; Durmić *et al.*, 2020). In recent years, FOCUM is being combined with other methods by many researchers to solve problems. For the purpose of selecting the appropriate combination of construction machines to enable mobility, Darko Boana *et al.* used a hybrid model of FUCOM and fuzzified RAFSI (Božanić *et al.*, 2021). Nunić (2018) applies a hybrid model of FOCUM-MABAC for evaluating and selecting PVC carpentry manufacturers. Real-world decision-makers often use linguistic variables instead of crisp values to evaluate attributes when they have partial knowledge or little information. Decision-makers are often left with ambiguous, imprecise, or incomplete attribute information as a result of such situations. Inaccuracies such as these can be mathematically represented by fuzzy set theory, introduced by Zadeh (1975). Since they were created, fuzzy sets have been successfully used to model MCDM problems with imprecise information. A fuzzy full consistency MCDM method was presented by Pamucar and Ecer (2020). In a hybrid model used by Baig *et al.* (2022) to enhance the resilience of oil supply chains, FOCUM prioritizes vulnerabilities while Fuzzy Quality Function Deployment identifies those capabilities that can ensure their protection. As part of the sustainability plan for urban mobility, Demir *et al.* (2022) used Fuzzy-FOCUM. A hybrid fuzzy FUCOM and neutrosophic fuzzy MARCOS methodology was used to assess alternative fuel vehicles for sustainable road transportation in the United States by Pamucar *et al.* (2021). A fuzzy-focus approach was used by Tang *et al.* (2021) for prioritizing sustainability scenarios for sewage sludge. To determine the drivers for investing in cryptocurrencies, Böyükaslan and Ecer used Fuzzy FUCOM in 2021.

The drawback of fuzzy sets is that, in some circumstances, it can be quite challenging to determine a precise membership mapping for a fuzzy set (Chiao, 2016). An intuitionistic fuzzy set (IFS), which Krassimir and Parvathi proposed in 1986, is a generalized fuzzy set that considers membership and non-membership degrees, as well as hesitation degrees. IFS can handle ambiguous information in a flexible manner (Gong *et al.*, 2014). As a result, specialists have been paying more and more attention to the IFS, which is now being used in many other domains, including decision-making (Gong *et al.*, 2014). This study addresses a vacuum in the literature, since, as far as the authors are aware, the FUCOM has not yet been used to intuitionistic contexts. In fact, extending FUCOM's research to the intuitionistic fuzzy environment is motivated in part by this.

In this study, Section 2 discusses IF-FUCOM-GRA, while Section 3 discusses step-by-step methodology with experimental details and results. Results of each method listed in Section 3 are presented in Section 4 in a step-by-step manner. Sections 5 and 6 represent the discussion and conclusion sections, respectively.

1.1 Motivation of the work:

    I. However, despite the fact that many researchers have studied the effects of Bacillus subtilis bacteria on concrete properties, no studies have evaluated the optimal concentration of bacteria as per the above discussed literature. Therefore, the purpose of this study is to determine the optimal bacteria concentration as well as the effect of bacteria on concrete mortar.

    II. Intuitionistic Fuzzy FUCOM Grey Relations Analysis has never been used to determine the optimal value in such an environment.

1.2 Novelty of the work:

    I. In this paper, IF-FUCOM is developed that can be used to better define the weight coefficients of criteria.

    II. A detailed algorithm is used in this study to calculate the weights of criteria in the intuitionistic fuzzy environment.

    III. A new model for dealing with uncertainty bridges the gap between criteria weight coefficients and intuitionistic fuzzy numbers.

    IV. In order to improve the methodology, a hybrid IF-FUCOM-GRA method has been proposed. It combines novel IF-FUCOM and existing GRA techniques.

    V. In this study, the optimal bacteria concentration and curing day for concrete is determined based on its compressive strength, crack healing, and water absorption. The novel IF-FUCOM-GRA method is used to select the perfect bacteria and cure day.

2. Intuitionistic Fuzzy Full Consistency Method Grey Relational Analysis (IF-FUCOM-GRA)

IF-FUCOM-AHP has two phases, IF-FUCOM and IF-AHP, which are discussed respectively in Phases I and II. Phase I and Phase II discussed how to analyse criteria and alternatives to determine the priority value of criteria and alternatives. Figure 1 depicts the proposed method's computational procedure. Figure 1 illustrates how the method is computed.

Phase-1: Intuitionistic Fuzzy Full Consistency Method (IF-FUCOM):

In order to determine the priority value of criteria, FUCOM is used. It is proposed that a modified fuzzy FUCOM approach is used in the current study called Intuitionistic Fuzzy Full Consistency Method (IF-FUCOM) to find the priority values of each criterion. Five steps make up IF-FUCOM. Following are the steps:

Step-I: Identify the assessment criteria: This consists of n ($r = 1(1)q$) decision criteria, which are represented by $\Omega = \{\xi_r : r = 1(1)q\}$.
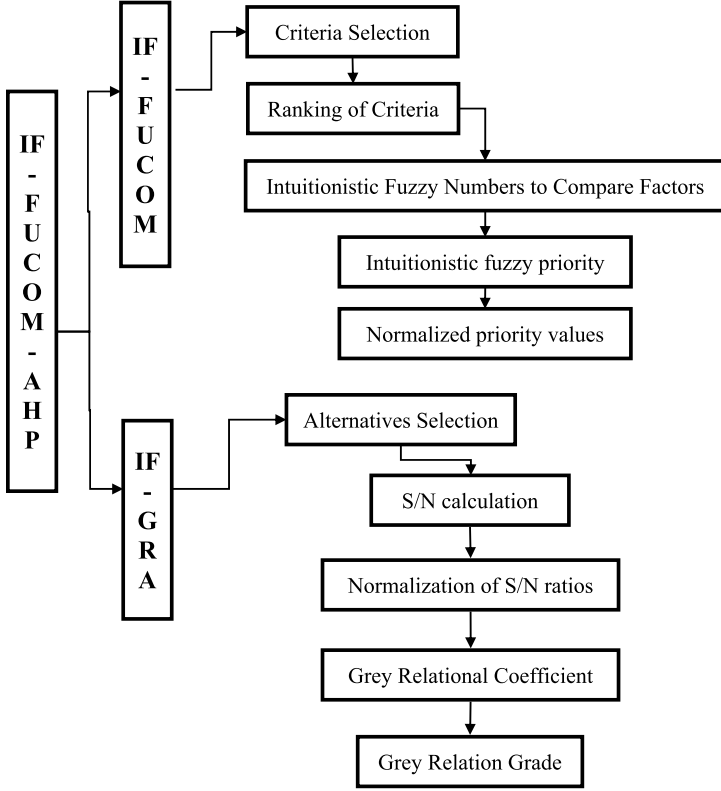
Fig. 1. Total scenario of proposed method.

Step-II: Determine the ranking of factors: The DMs determine the order of importance of factors based on their opinions. A factor is ranked in ascending order by the weight coefficient that will be assigned to it first, and so on, down to the least significant factor in the equation. The factor whose weight coefficient is expected to be the lowest is ranked last. In the resulting ranking system, $\xi_{r(1)} \succ \xi_{r(2)} \succ \cdots \succ \xi_{r(l)}$ represents the factor rank, where l represents the criterion ranking. A sign of equality replaces "$\succ$" between two or more factors that have the same ranking.

Step-III: Use intuitionistic fuzzy numbers to compare factors: Table 1 is used to compare factors. The factors are compared according to the first ranking factor. The Intuitionistic fuzzy criterion meaning ($\tilde{p}_{\xi_{r(l)}}$) is then determined for all the factors. In order to compare the remaining factors with the most important factor, a $(q-1)$ comparison is a necessity. A fuzzy Intuitionistic significance $\tilde{\wp}_{l/(l+1)}$ is derived from equation (1) by applying the defined significance of factors.

$$\tilde{\wp}_{l/(l+1)} = \frac{\tilde{p}_{\xi_{r(l+1)}}}{\tilde{p}_{\xi_{r(l)}}} = \frac{(p^l_{\xi_{r(l+1)}}, p^m_{\xi_{r(l+1)}}, p^u_{\xi_{r(l+1)}}; p'^l_{\xi_{r(l+1)}}, p'^m_{\xi_{r(l+1)}}, p'^u_{\xi_{r(l+1)}})}{(p^l_{\xi_{r(l)}}, p^m_{\xi_{r(l)}}, p^u_{\xi_{r(l)}}; p'^l_{\xi_{r(l)}}, p'^m_{\xi_{r(l)}}, p'^u_{\xi_{r(l)}})}. \tag{1}$$

Table 1
Nine-point triangular intuitionistic fuzzy scale (Otay *et al.*, 2017).

Definition	Intensity of importance $(\delta^l, \delta^m, \delta^u; \delta'^l, \delta'^m, \delta'^u)$	Reverse of intensity importance $(\frac{1}{\delta^u}, \frac{1}{\delta^m}, \frac{1}{\delta^l}; \frac{1}{\delta'^u}, \frac{1}{\delta'^m}, \frac{1}{\delta'^l})$	S.I.
EI	(1, 1, 1; 1, 1, 1)		1
AI	(8, 9, 9; 7, 9, 9)		9
MI	$(\mu - 1, \mu, \mu + 1; \mu - 2, \mu, \mu + 2)$	$(\frac{1}{\mu+1}, \frac{1}{\mu}, \frac{1}{\mu-1}; \frac{1}{\mu+2}, \frac{1}{\mu}, \frac{1}{\mu-2})$	$\mu = 3$
STI			$\mu = 5$
VSI			$\mu = 7$
Intermediate scale			$\mu = 2, 4, 6, 8$

Equation (2) provides an Intuitionistic fuzzy vector of the relative importance of the decision factors:

$$\wp = (\tilde{\wp}_{1/2}, \tilde{\wp}_{2/3}, \ldots, \tilde{\wp}_{l/(l+1)}). \tag{2}$$

Based on the factor of $\xi_{r(l+1)}$ rank, $\tilde{\wp}_{l/(l+1)}$ represents the importance that the factor of $\xi_{r(l)}$ rank possesses.

Step-IV: Calculate intuitionistic fuzzy priority: here, the Intuitionistic fuzzy priority value coefficients are calculated for factor $(\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_q)^T$. As a final priority coefficient value, the following conditions must be met:

Condition 1: The weight coefficient ratio between the observed factors ($\xi_{r(l)}$ and $\xi_{r(l+1)}$) should equal the significance ratio between them ($\tilde{\wp}_{l/(l+1)}$) defined in Step II; in other words, it should satisfy:

$$\frac{\tilde{p}_l}{\tilde{p}_{l+1}} = \tilde{\wp}_{l/(l+1)}. \tag{3}$$

Condition 2: Besides satisfying the condition in expression (3), the coefficients of weights should also qualify as transitive, i.e.

$$\tilde{\wp}_{l/(l+1)} \otimes \tilde{\wp}_{(l+1)/(l+2)} = \tilde{\wp}_{l/(l+2)}, \quad \text{i.e. } \frac{\tilde{p}_l}{\tilde{p}_{l+1}} \otimes \frac{\tilde{p}_{l+1}}{\tilde{p}_{l+2}} = \frac{\tilde{p}_l}{\tilde{p}_{l+2}}.$$

It is also necessary for the final weight coefficient values to satisfy the following condition:

$$\frac{\tilde{p}_l}{\tilde{p}_{l+2}} = \tilde{\wp}_{l/(l+1)} \otimes \tilde{\wp}_{(l+1)/(l+2)}. \tag{4}$$

DMC minimum, i.e. $\nu = 0$, can only be satisfied if there is complete transitivity among priority coefficients. Then, it can be said that $\frac{\tilde{p}_l}{\tilde{p}_{l+1}} - \tilde{\wp}_{l/(l+1)} = 0$ and $\frac{\tilde{p}_l}{\tilde{p}_{l+2}} - \tilde{\wp}_{l/(l+1)} \otimes \tilde{\wp}_{(l+1)/(l+2)} = 0$. Accordingly, DMC is $\nu = 0$, when such coefficients are obtained. To satisfy these conditions, the weight coefficients for each criterion $(\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_q)^T$ must satisfy the condition that $\left| \frac{\tilde{p}_l}{\tilde{p}_{l+1}} - \tilde{\wp}_{l/(l+1)} \right| \leqslant \nu$ and $\left| \frac{\tilde{p}_l}{\tilde{p}_{l+2}} - \tilde{\wp}_{l/(l+1)} \otimes \tilde{\wp}_{(l+1)/(l+2)} \right| \leqslant \nu$ minimize the value ν.

The final nonlinear model for computing the ideal Intuitionistic fuzzy values of the relative weights of each factor can then be set to $(\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_q)^T$.

Min v

s.t.

$$
\begin{cases}
\left| \dfrac{\tilde{p}_l}{\tilde{p}_{l+1}} - \tilde{\wp}_{l/(l+1)} \right| \leqslant v, & \text{for all } r = 1(1)q, \\[2ex]
\left| \dfrac{\tilde{p}_l}{\tilde{p}_{l+2}} - \tilde{\wp}_{l/(l+1)} \otimes \tilde{\wp}_{(l+1)/(l+2)} \right| \leqslant v, & \text{for all } r = 1(1)q, \\[2ex]
\displaystyle\sum_{r=1}^{q} \tilde{p}_r = 1, \\[2ex]
0 \leqslant p_r'^l \leqslant p_r^l \leqslant p_r^m = p_r'^m \leqslant p_r^u \leqslant p_r'^u, & \text{for all } r = 1(1)q,
\end{cases}
\tag{5}
$$

where $\tilde{p}_r = (p_r^l, p_r^m, p_r^u; p_r'^l, p_r'^m, p_r'^u)$ and $\tilde{\wp}_{l/(l+1)} = (\wp_{l/(l+1)}^l, \wp_{l/(l+1)}^m, \wp_{l/(l+1)}^u; \wp_{l/(l+1)}'^l, \wp_{l/(l+1)}'^m, \wp_{l/(l+1)}'^u)$.

The highest consistency can only be obtained by following the condition that $\frac{\tilde{p}_l}{\tilde{p}_{l+1}} - \tilde{\wp}_{l/(l+1)} = 0$ and $\frac{\tilde{p}_l}{\tilde{p}_{l+2}} - \tilde{\wp}_{l/(l+1)} \otimes \tilde{\wp}_{(l+1)/(l+2)} = 0$ are both met. In this way, the model (5) can be re-formulated into an Intuitionistic fuzzy nonlinear model (6). Intuitionistic fuzzy priority value coefficients are obtained $(\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_q)^T$, if this problem is solved.

Min v

s.t.

$$
\begin{cases}
|\tilde{p}_l - \tilde{p}_{l+1} \otimes \tilde{\wp}_{l/(l+1)}| \leqslant v, & \text{for all } r = 1(1)q, \\[1.5ex]
|\tilde{p}_l - \tilde{p}_{l+2} \otimes \tilde{\wp}_{l/(l+1)} \otimes \tilde{\wp}_{(l+1)/(l+2)}| \leqslant v, & \text{for all } r = 1(1)q, \\[1.5ex]
\displaystyle\sum_{r=1}^{q} \tilde{p}_r = 1, \\[2ex]
0 \leqslant p_r'^l \leqslant p_r^l \leqslant p_r^m = p_r'^m \leqslant p_r^u \leqslant p_r'^u, & \text{for all } r = 1(1)q,
\end{cases}
\tag{6}
$$

where $\tilde{p}_j = (p_j^l, p_j^m, p_j^u; p_j'^l, p_j'^m, p_j'^u)$ and $\tilde{\wp}_{l/(l+1)} = (\wp_{l/(l+1)}^l, \wp_{l/(l+1)}^m, \wp_{l/(l+1)}^u; \wp_{l/(l+1)}'^l, \wp_{l/(l+1)}'^m, \wp_{l/(l+1)}'^u)$.

Convert optimal Intuitionistic fuzzy priority value $(\tilde{p}_1^*, \tilde{p}_2^*, \ldots, \tilde{p}_q^*)$, where $\tilde{p}_r^* = (p_r^{l*}, p_r^{m*}, p_r^{u*}; p_r'^{l*}, p_r'^{m*}, p_r'^{u*})$, for all $r = 1(1)q$ into crisp value using the formula (7):

$$
R(\tilde{p}_r^*) = \left[\frac{(p_r^{l*} + 2p_r^{m*} + p_r^{u*}) + (p_r'^{l*} + 2p_r'^{m*} + p_r'^{u*})}{8} \right], \quad \text{for all } r = 1(1)q.
\tag{7}
$$

Step-V: Normalized priority values: equation (8) is used to calculate the normalized priority values of criteria.

$$
\tilde{w}_r^* = \frac{R(\tilde{p}_r^*)}{\sum_{r=1}^{q} R(\tilde{p}_r^*)}, \quad \forall r = 1(1)q.
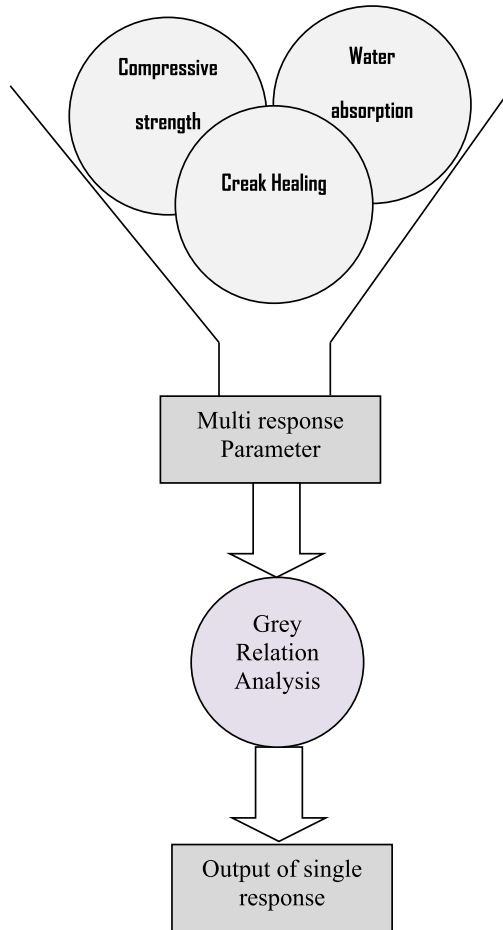\tag{8}
$$

Fig. 2. Graph of a straightforward grey relational analysis.

Phase-2: Grey Relational Analysis:

The grey theory is an immense concept used to explore uncertainty, multi-input, and discrete data. Decision analysis is used to estimate the degree of relation according to the grey relational grade. A multi-objective optimization makes it more complex to analyse the effects and relationships between design factors in experiments at their various levels that result uncertain and insignificant information. In this paper, GRA is proposed for investigating and optimizing the complexity of multi-variable problems by exploiting the concept of information. As shown in Fig. 2, GRA reduces a multi-objective question to a single objective answer (referred to as single relational grade).

The present study is conducted based upon Taguchi's orthogonal array, which corresponds to nine trails, where every trail is known as a comparison sequence. The GRA places these trails into nine subsystems. The effect of these factors on the outcome variable is assessed through regression analysis. Using GRA, the multi-objective problem is

transformed into a single-objective problem by using the parameters corresponding to the greatest weighted grey relational grade.

Step-I: S/N calculation:

Greater, nominal, and lower signal-to-noise ratio analyses are the three possible approaches. For water absorption in this study, smaller-is-better, however, higher-is-better for compressive strength and creak healing. The S/N ratios of water absorption are calculated by equation (9), and compressive strength and creak healing by equation (10).

$$(S/N)_{\text{Smller-the-better}} = -10 \times \log\left(\frac{\sum_{k=1}^{m} \lambda_k^2}{m}\right), \tag{9}$$

$$(S/N)_{\text{Higher-the-better}} = -10 \times \log\left(\frac{\sum_{k=1}^{m} \frac{1}{\lambda_k^2}}{m}\right), \tag{10}$$

where, λ_k is the kth experiment's observed data and m is representing the observations' number.

Step-II: S/N ratio normalization:

To lessen unpredictability, the S/N ratio of attribute data is modified. Data preparation is the term for this. Pre-processing of the data is needed for grey analysis (Grzenda *et al.*, 2012; Kao *et al.*, 2008). The following equation (11) normalizes the original sequence:

$$\zeta_k^*(t) = \frac{\zeta_k(t) - \min_{1 \leqslant t \leqslant n} \zeta_k(t)}{\max_{1 \leqslant t \leqslant n} \zeta_k(t) - \min_{1 \leqslant t \leqslant n} \zeta_k(t)}. \tag{11}$$

However, the data is normalized using equation (12); the smaller the characteristic, the better.

$$\zeta_k^*(t) = \frac{\max_{1 \leqslant t \leqslant n} \zeta_k(t) - \zeta_k(t)}{\max_{1 \leqslant t \leqslant n} \zeta_k(t) - \min_{1 \leqslant t \leqslant n} \zeta_k(t)}, \tag{12}$$

ζ represents desired value, $\zeta_k^*(t)$ indicates normalized value, where n stands for the number of experiments, and m for the number of answers, and $t = 1(1)n$; $k = 1(1)n$.

Step-III: Grey Relational Coefficient:

The GRC, a series of information, is used by GRA to assess the relevance of two systems. Equation (13) can be used to calculate GRC (μ_k).

$$\mu_k(t) = \frac{\delta_{\min} + \tau \times \delta_{\max}}{\delta_k(t) + \tau \times \delta_{\max}}, \tag{13}$$

where $\delta_k(t) = \|\zeta_k(0) - \zeta_k(t)\|$

$$\delta_{\min}(t) = \min\{\|\zeta_k(0) - \zeta_k(t)\| : t = 1(1)n\},$$
$$\delta_{\max}(t) = \max\{\|\zeta_k(0) - \zeta_k(t)\| : t = 1(1)n\},$$

$\zeta_k(0) =$ reference value $(= 1)$; $\zeta_k(t) =$ specific comparison value, where τ is the distinguishing coefficient $0 \leqslant \tau \leqslant 1$.

Step-IV: Grey Relation Grade:

In real engineering systems, different parts have different weights based on the circumstances. Then, equation (14) grades the grey relational coefficient (GRC) (Saaty, 1980).

$$\eta(t) = \frac{\sum_{k=1}^{m} p_k \times \mu_k(t)}{\sum_{k=1}^{m} p_k}, \tag{14}$$

where p_k stands for factor k's normalized weight. The proposed IF-FUCOM approach yields the weight of each attribute. The higher the value of grey relational grade is, the greater is the desirability.

3. Methodology

The objective of this study is to find the optimal bacterial concentrations and curing days for concrete simultaneously while considering compressive strength (CS), crack healing (CH) and water absorption (WA) as outputs using a novel MCDM technique. During the present investigation, there are six phases. A schematic representation of the detailed methodology is shown in Fig. 3.

Phase-I: The Experimental details include details on the materials, bacteria culture, mixing procedure, compression strength (ξ_1), crack healing (ξ_2) and water absorption (ξ_3) test on mortar surfaces.

Phase-II: The criteria and alternatives were discussed in this phase.

Phase-III: Determine the weights of all consideration criteria using the proposed MCDM method.

Phase-IV: Evaluate alternative weights using another existing MCDM technique.

Phase-V: Comparison of the results determined by the proposed MCDM with the existing model.
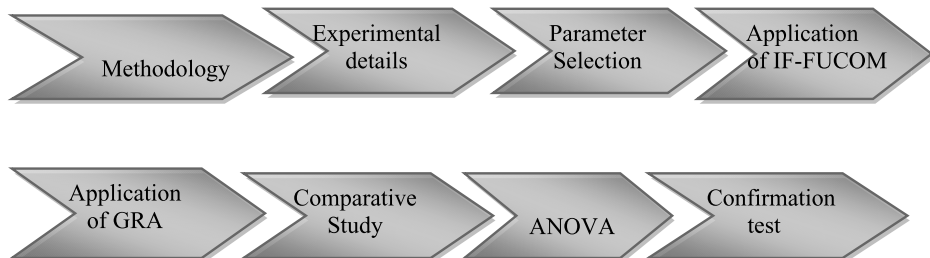

Fig. 3. A diagrammatic representation of the methodology.

Table 2

Composition and physical properties of cement.

Physical properties	
Colour	Grey
Specific gravity	3
Chemical constituents (%)	
Al_2O_3	3.78
SiO_2	21.5
MgO	1.79
Fe_2O_3	3.78
CaO	63.69
SO_3	3
Na_2O	–
K_2O	–

Phase-VI: An analysis is made of the degree to which each of the chosen parameter values contributes to the output responses.

Phase-VII: Validation tests are run to confirm forecasts and results.

Phase-VIII: Sensitivity analysis investigates how the indicators, which were calculated using the MCDM method, affect the anticipated result.

Phase-I. Experimental details:

This section explains the material choice, the bacteria mixing process, and several tests like compressive strength and water absorption.

Step-I. Materials:

Ordinary Portland Cement (OPC) 43 Grade conforms to IS 8112 : 2013, locally available Fine Aggregate, Bacillus Subtilis and potable water is used in this study. Here cement to sand ratio and water to cement ratio were 1 : 3 and 0.4 (by weight) respectively. For preparing mortar water, distilled water is used. Mortar cubes of dimension 70.6 mm^3 are prepared for both control and bacterial mortar specimens. In fresh water, curing can be conducted at room temperature 27 °C. According to information provided by the manufacturer, OPC cement's chemical composition and physical properties are presented in Table 2.

Step-II. Bacteria culture:

For this experimental work, selected bacterial sample Bacillus Subtilisis is used in this study. For bacterial culture nutrient broth was made (1.0 gm/lBeef Extract, 5.0 gm/lPeptone, 2.0 gm/lYeast Extract, 5.0 gm/NaCl). Growth conditions of Bacteria are maintained at 37 °C temperature. After 6–7 days, about 10 μl of the nutrient broth is obtained and haemocytometer counting is done. Here, the bacterial concentrations in solution used are 10^3 cells/ml, 10^5 cells/ml, 10^7 cells/ml.

Step-III. The mixing procedure:

Cement and sand is well mixed in 1:3 proportions and a mixture of water and the needed cell concentration is then prepared. After casting and compacting in a vibration

Table 3
Levels and values of the input parameters.

Parameters	Level-1	Level-2	Level-3
Concentration	0	5	7
Days	3	7	28

machine, specimens are removed and compression tests are performed after 3, 7, 14 and 28 days in air at room temperature (30 °C).

Step-IV. Compressive strength and water absorption test:

Compressive strength and water absorption of control and bacterial mortar cubes are measured in 3, 7 and 28 days after curing. The compressive strength test was done under compression testing machine.

Step-V. Crack healing on mortar surfaces:

A 28-day crack healing test is performed on microbial concrete to determine its self-healing ability at different bacteria concentrations. The crack-measuring instrument measured the crack widths. In this study, crack widths range from 0.11 mm to 1.5 mm; water is used to immerse the cracked specimens and their crack dimensions are recorded after 3, 7 and 28 days.

Phase-II. Parameter Selection:

The PV of each criterion and alternative will be calculated in the section that follows. In the present study, compressive strength (ξ_1), crack healing (ξ_2) and water absorption (ξ_3) are considered as a set of criteria. Also, nine considering trials, namely, 10^7 Concentration with 28 days, 10^5 Concentration with 28 days, 10^3 Concentration with 28 days, 10^7 Concentration with 7 days, 10^5 Concentration with 7 days, 10^3 Concentration with 7 days, 10^7 Concentration with 3 days, 10^5 Concentration with 3 days and 10^3 Concentration with 3 days as a set of alternatives.

For the present study, the design factors chosen are bacteria concentration and curing day so as to determine their influence on the outcome parameters of compressive strength, crack healing, and water absorption. Table 3 represents the levels of input parameters (bacteria concentration and curing day) that are considered as control factors for the experiment. In Table 4, based on the number of tests, a Taguchi L9 (32) orthogonal array comprising 9 rows has been calculated.

Phase-III. Application of IF-FUCOM:

Collect all factors based on the literature review, and then send them to three experts, and expert responds. Following the determination of the first-level criteria, the ranking is determined on a second level. Dimensions are ranked in this order: $\xi_1 > \xi_2 > \xi_3$. In Table 5, the linguistic variables represent the relative importance of the criteria ranked according to decision-makers preferences.

The fuzzy linguistic scale was used to transform linguistic variables into Intuitionistic fuzzy numbers (IFNs), as shown in Table 6.

Table 4

Results of an experiment using L_9 orthogonal arrays.

Trial No.	Concentration	Days	CS (Map)	CH (%)	WA (%)
1	10^3	3	30.1206	30.3167	5.5
2	10^3	7	37.6342	50.2262	4.66667
3	10^3	28	48.1967	60.6335	4.25
4	10^5	3	32.7521	70.1357	5.08333
5	10^5	7	41.2611	84.6154	4.375
6	10^5	28	52.462	90.0452	3.79167
7	10^7	3	31.4065	94.5701	4.91667
8	10^7	7	39.9128	98.5	4.04167
9	10^7	28	49.4737	99.6	3.58333

Table 5

A linguistic assessments of the main dimensions.

Dimensions	ξ_1	ξ_2	ξ_3
Linguistic variables	EI	MI	STI

Table 6

Evaluations transformed by IFNs.

Dimensions	ξ_1	ξ_2	ξ_3
IFNs	(1, 1, 1; 1, 1, 1)	(2, 3, 4; 1, 3, 5)	(4, 5, 6; 3, 5, 7)

According to expression (1), the relative importance of the criteria is as follows.

$$\tilde{\wp}_{\xi_1/\xi_2} = \frac{\tilde{p}_{\xi_1}}{\tilde{p}_{\xi_2}} = \frac{(2, 3, 4; 1, 3, 5)}{(1, 1, 1; 1, 1, 1)} = (2, 3, 4; 1, 3, 5),$$

$$\tilde{\wp}_{\xi_2/\xi_3} = \frac{\tilde{p}_{\xi_2}}{\tilde{p}_{\xi_3}} = \frac{(4, 5, 6; 3, 5, 7)}{(2, 3, 4; 1, 3, 5)} = (1, 1.667, 3; 0.6, 1.667, 7).$$

A vector comparative significance is therefore defined as follows:

$$\tilde{\wp} = \big\{(2, 3, 4; 1, 3, 5), (1, 1.667, 3; 0.6, 1.667, 7)\big\}.$$

Three constraints are imposed by equation (4) based on the conditions of relation transitivity as follows:

$$\frac{\tilde{p}_{\xi_1}}{\tilde{p}_{\xi_3}} = (2, 3, 4; 1, 3, 5).(1, 1.667, 3; 0.6, 1.667, 7) = (2, 5.001, 12; 0.6, 5.001, 35).$$

Optimization Problem:

$$\text{Min } v$$
$$\text{s.t.}$$
$$\begin{aligned}
&\left|p_1^l - 2p_2^u\right| \leqslant v,\\
&\left|p_1^u - 4p_2^l\right| \leqslant v,\\
&\left|p_1'^l - p_2'^u\right| \leqslant v,\\
&\left|p_1'^u - 5p_2'^l\right| \leqslant v,\\
&\left|p_1^m - 3p_2^m\right| \leqslant v,\\
&\left|p_2^l - p_3^u\right| \leqslant v,\\
&\left|p_2^u - 3p_3^l\right| \leqslant v,\\
&\left|p_2'^l - 0.6p_3'^u\right| \leqslant v,\\
&\left|p_2'^u - 7p_3'^l\right| \leqslant v,\\
&\left|p_2^m - 1.667p_3^m\right| \leqslant v,\\
&\left|p_1^l - 2p_3^u\right| \leqslant v,\\
&\left|p_1^u - 12p_3^l\right| \leqslant v,\\
&\left|p_1'^l - 0.6p_3'^u\right| \leqslant v,\\
&\left|p_1'^u - 35p_3'^l\right| \leqslant v,\\
&\left|p_1^m - 5.001p_3^m\right| \leqslant v,\\
&p_1^m = p_1'^m,\\
&p_2^m = p_2'^m,\\
&p_3^m = p_3'^m,\\
&\sum_{j=1}^{3} \frac{(p_j^l + 2p_j^m + p_j^u) + (p_j'^l + 2p_j'^m + p_j'^u)}{8} = 1,\\
&0 \leqslant p_j'^l \leqslant p_j^l \leqslant p_j^m = p_j'^m \leqslant p_j^u \leqslant p_j'^u \quad \text{for all } j = 1(1)3,\\
&v \geqslant 0.
\end{aligned} \tag{15}$$

Phase-IV. Application of GRA:

To calculate the score, gray relation grading is used after determining the relative weights of each criterion. GRCs are used to calculate gray reasoning grades using equation (14). GRCs are weighted from 0 to 1. IF-FUCOM decides the weight of each characteristic. A gray relational grade of the higher value indicates greater desirability.

Phase-V. Study of Comparisons:

A comparative research can identify and quantify the relationships between at least two factors by studying different groups that have been exposed to diverse treatments either by choice or circumstance. A relative study is made possible by contrasting two sets of individuals, entities, or circumstances. In the current work, the proposed technique has been contrasted with three sophisticated models.

Model-I: The local weight must be calculated using AHP (Saaty, 1980), and the global weight must be calculated using GRA (Julong, 1989).

Table 7
Best to others criteria.

Best to others	ξ_1	ξ_2	ξ_3
ξ_1	1	3	7

Table 8
Others to worst criteria.

Others to worst	ξ_3
ξ_1	7
ξ_2	5
ξ_3	1

Model-II: Identify the local weights of the BWM (Rezaei, 2015) and the GRA (Julong, 1989) alternative, as well as identifying the global weights of the BWM-GRA alternative.

Model-III: Determine the local weight of an alternative GRA (Julong, 1989), the local weight of the FUCOM (Pamučar *et al.*, 2018b), then calculate the global weight using a hybrid strategy known as FUCOM GRA.

Here are three mathematical formulations of the models discussed below.

Model-I. AHP result:

The weights of the different criteria are determined by experts within related fields who collaborate in a pairwise comparison between each criterion. This comparison matrix is shown below.

$$
\begin{array}{c}
\begin{array}{ccc} \xi_1 & \xi_2 & \xi_3 \end{array} \\
\begin{array}{c} \xi_1 \\ \xi_2 \\ \xi_3 \end{array}
\begin{pmatrix}
1 & 3 & 4 \\
 & 1 & 3 \\
 & & 1
\end{pmatrix}.
\end{array}
$$

Model-II. BWM result:

For the purpose of weighing the criteria in BWM, relevant experts are asked to identify the most and least significant factors in the case study, along with the best-to-others and other-to-worst vectors. According to expert consensus, ξ_1 and ξ_3 are the best and worst criteria, respectively. The best-to-others and worst-to-others vectors are shown in Tables 7 and 8.

The weight of each criterion can be calculated, as well as the consistency rate, using the non-linear mathematical model.

Table 9
Comparative significance levels for
the evaluation criteria.

ξ_3	ξ_3	ξ_3
1	1.08	1.25

$$
\begin{aligned}
&\max \chi \\
&\text{s.t.} \left| \frac{p_1}{p_2} - 3 \right| < \chi, \\
&\qquad \left| \frac{p_1}{p_3} - 7 \right| < \chi, \\
&\qquad \left| \frac{p_2}{p_3} - 5 \right| < \chi, \\
&\qquad \sum_{j=1}^{3} p_j = 1, \\
&\qquad p_j \geqslant 0, \quad \text{for all } j = 1(1)3.
\end{aligned}
\tag{16}
$$

Model-III. FUCOM result:

The criteria are ranked in order of importance. The ranking is determined by consensus among experts. According to experts, the relation (17) criteria are ranked. Comparisons are based on a scale of Van Tittelboom *et al.* (2010), Achal *et al.* (2011), which is shown in Table 9.

$$
\xi_1 > \xi_2 > \xi_3.
\tag{17}
$$

The relative importance of each criterion can be gauged by calculating the comparison importance values based on the obtained importance values $\theta_{C_1/C_2} = \frac{1.08}{1} = 1.08$, $\theta_{C_2/C_3} = \frac{1.25}{1.08} = 1.15740741$ and $\theta_{C_1/C_3} = 1.08 \times 1.15740741 = 1.25$.

The final weight coefficients can be determined by applying expression (18)

$$
\begin{aligned}
&\max \chi \\
&\text{s.t.} \left| \frac{p_1}{p_2} - 1.08 \right| < \chi, \\
&\qquad \left| \frac{p_2}{p_3} - 1.15740741 \right| < \chi, \\
&\qquad \left| \frac{p_1}{p_3} - 1.25 \right| < \chi, \\
&\qquad \sum_{j=1}^{3} p_j = 1, \\
&\qquad p_j \geqslant 0, \quad \text{for all } j = 1(1)3.
\end{aligned}
\tag{18}
$$

Phase-V. ANOVA:

Statistically, the difference among available scores can be evaluated through Analysis of Variance (ANOVA). In ANOVA, the level of contribution of each of the chosen parameter values over the output responses is analysed (Pattnaik *et al.*, 2013). ANOVA results can be used to determine which variables are responsible for the performance of a selected process and to control these variables to obtain a better result. ANOVA cannot provide data analysis, but this statistical method can assess variance of the data.

Phase-VI. Confirmation test:

A confirmation test is done to verify the forecast and the outcome after the S/N ratio plot is used to estimate the optimal output. The IF-FUCUM-GRG values delivered at the optimal output are predicted by equation (19).

$$\vartheta_{predicted} = \vartheta_{mean} + \sum_{i=1}^{n}(\vartheta_i - \vartheta_{mean}). \tag{19}$$

The group's reasoning grade ϑ_{mean} stands for the overall grade mean, ϑ_i for the grade at the best level, and n is the output regulating parameter.

4. Result

The six parts of the proposed model are described in the results section.

Part-I: Calculate the gray relation coefficient.

Phase-II: In this phase, IF-FUCOM is used to determine weights for the criteria. The weights and rankings of alternatives are determined using IF-FUCOM and GRC.

Phase-III: A comparison of the results offered by novel IF-FUCOM-AHP and some existing methods.

Phase-IV: Next, the ANOVA result is used to determine the influential input parameter.

Phase-V: The percentage significance of input factors can be analysed with ANOVA.

In the following, all phases are discussed in detail.

Phase-I: Result of GRC:

To assess the impact of each parameter, the SN ratio of every trail is computed based on equation (9) for compressive strength and equation (10) for crack healing and water absorption. Equations (11) and (12) are used to normalize the acquired value of the SN ratio while taking the higher-the-better and smaller-the-better qualities into consideration, respectively. Equation (13) is used to calculate the GRC after determining the normalized SN ratios for each investigation. Table 10 displays the SN ratios of the output parameters together with the corresponding GRCs.

Phase-II. Result from IF-FUCOM-GRG:

The IF-FUCOM-GRG result is divided into two parts, namely the result of IF-FUCOM and the result of GRG. All the parts are discussed below.

Step-I. Result from IF-FUCOM:

Table 10
SN ratio and GRC associated with output parameter.

SN RATIO			GRC		
CS (Mpa)	CH (%)	WA (%)	CS (Mpa)	CH (%)	WA (%)
29.57727237	29.63363852	−14.8072538	0.333333333	0.333333333	1
31.51165377	34.01860643	−13.3801418	0.455106149	0.464867652	0.565946459
33.66034607	35.65425276	−12.5677786	0.765904066	0.545102165	0.453817715
30.30478303	36.91878272	−14.1229661	0.37063052	0.629038242	0.731129591
32.31081604	38.54898824	−12.8195611	0.536003016	0.784836765	0.483508457
34.39689686	39.08921134	−11.5766106	1	0.855012953	0.365470817
29.94039081	39.51507697	−13.8334212	0.350961336	0.919850586	0.656446929
32.02224392	39.86872461	−12.131217	0.503674319	0.98166898	0.410148263
33.88748783	39.96518677	−11.0857361	0.825498313	1	0.333333333

The best values of the criteria can be found by solving the fuzzy linear model in equation (11), which is shown.

The weight coefficients for the criteria compressive strength (ξ_1), crack healing (ξ_2) and water absorption (ξ_3) are (0.443, 0.654, 0.866; 0.231, 0.654, 0.943), (0.212, 0.212, 0.212; 0.192, 0.212, 0.212) and (0.0770, 0.135, 0.231; 0.231, 0.135, 0.353), respectively, with a deviation from maximum consistency $\nu = 0.0192$. Lingo 17.0 is used to solve the model (11).

Next, use equations (7) to calculate the crisp weights for the criteria compressive strength (ξ_1), crack healing (ξ_2) and water absorption (ξ_3), which are, respectively, 0.637, 0.209, and 0.153.

Use equation (8) to calculate normalized weights for these three criteria, which are 0.499, 0.245, and 0.256.

The weights of the compressive strength (ξ_1), crack healing (ξ_2) and water absorption (ξ_3) using FUCOM-F (Pamucar and Ecer, 2020) are 0.400, 0.388, and 0.212, respectively, with $\nu = 0.001$. Despite significant discrepancies in the weights of those criteria, all FUCOM-F, as well as IF-FUCOM, algorithms rank each criterion in the same order.

Step-II: Result of IF-FUCOM-GRA:

In GRA, the relative weights of the criteria are obtained by IF-FUCOM. After determining the relative weights of the criteria, the score is calculated using grey relation grading. Using equation (16), GRCs are used to calculate the different grey reasoning grades. GRCs are weighted from 0 to 1, with p_k equal to 1. p_1, p_2, and p_3 are used as weighting factors in this study for compressive strength, crack healing, and water absorption, respectively. Gray relational grade (GRG) determined by different MCDM methods are presented in Table 11 for each trial using the L9 orthogonal array data.

Phase-III. Result of Comparative study:

This study validates the result of the proposed model by comparing it to three existing MCDM techniques. There are four steps in this phase. Determine the PV for each criterion using AHP, BWM, and FUCOM methods in the first three steps. As a last step, determine the weights of the alternatives using GRA.

Step-I: Result from AHP:

Table 11
Gray relational grade determined by IF-FUCOM.

Trail No.	IF-FUCOM-GRG	Rank
1	0.435	9
2	0.47365	8
3	0.671241	3
4	0.479423	7
5	0.579442	5
6	0.871615	1
7	0.516248	6
8	0.588762	4
9	0.785842	2

Calculate the priority value of each criterion using the AHP algorithm as described in Section. The priority value of criteria are $p_1 = 0.614$, $p_2 = 0.268$, $p_3 = 0.117$ and maximum eigen value $\lambda_{\max} = 3.074$. These values indicate that the most important criterion is CS (ξ_1), while the least important criterion is WA (ξ_3). To determine CI and CR:

$$CI = \frac{3.074 - 3}{3 - 1} = 0.037,$$

$$CR = \frac{0.037}{0.58} = 0.06379.$$

Step-II: Result from BWM:

Based on the solution to the above BWM-model (16), the following criteria weights are optimal: $p_1^* = 0.66154$, $p_2^* = 0.26154$, $p_3^* = 0.07692$ and $\chi* = 0.12308$. According to these values, the outputs CS (ξ_1) and WA (ξ_3) are the most important and the least important criteria, respectively. The degree of consistency is as follows:

$$CR = \frac{0.12308}{3.73} = 0.032996.$$

As suggested by the obtained CR value (0.032996), the obtained criteria weights have a satisfactory degree of consistency.

Step-III: Result from FUCOM:

Based on the solution to the above model (18), the following criteria weights are optimal: $p_1^* = 0.3668478$, $p_2^* = 0.3396739$, $p_3^* = 0.2934783$ and $\chi* = 0.2471142 \times E^{-08}$. According to these values, the outputs CS (ξ_1) and WA (ξ_3) are the most important and the least important criteria, respectively.

Step-IV: Different Gray relation grade:

In GRA, the relative weights of the criteria are obtained by AHP, BWM, and FUCOM. After determining the relative weights of the criteria, the score is calculated using grey relation grading. Using equation (16), GRCs are used to calculate the different grey reasoning grades. GRCs are weighted from 0 to 1, with p_k equal to 1. p_1, p_2, and p_3 are

Table 12
Gray relational grade determined by different MCDM techniques.

Trail No.	GRG	Rank	AHP-GRG	Rank	BWM-GRG	Rank	FUCOM-GRG	Rank
1	0.5555556	8	0.411	9	0.3846153	9	0.528986	8
2	0.4953068	9	0.470235	8	0.4661853	7	0.490951	9
3	0.5882746	6	0.669449	3	0.6841493	3	0.599313	6
4	0.5769328	7	0.481692	7	0.4659447	8	0.564204	7
5	0.6014494	5	0.596013	5	0.5970446	5	0.605119	5
6	0.7401613	1	0.885904	1	0.9132704	1	0.764531	1
7	0.6424196	3	0.538815	6	0.5232466	6	0.633852	4
8	0.6318305	4	0.620331	4	0.621494	4	0.638589	3
9	0.7196105	2	0.813856	2	0.8332784	2	0.740332	2

Table 13
IF-FUCOM-GRG response table.

Parameters	Level-1	Level-2	Level-3
Concentration	0.4932	0.6458	0.6391
Days	0.4632	0.5516	0.7633

used as weighting factors in this study for compressive strength, crack healing, and water absorption, respectively. Gray relational grade determined by different MCDM methods are presented in Table 12 for each trial using the L9 orthogonal array data.

Phase-IV. ANOVA result:

The S/N ratio is used to analyse the IF-FUCOM-GRG data to find the best combination of factors. According to the higher, the better criterion, the optimal combination should correspond to the highest S/N value on each factor. Results are analysed using Minitab software. Table 13 summarizes the main influences of control factors on mean grey relational grades. As a result of each level of the input control parameter, the S/N ratio plots of IF-FUCOM-GRG are shown in Fig. 4. Combining the highest factor levels calculated from bacteria concentration at 105 and 28 curing days yields the best factor level combination.

ANOVA is employed to determine whether design elements have a substantial impact on response (Haq *et al.*, 2008). ANOVA may examine the percentage importance of input factors. When $F > 4$ (Yang and Tarng, 1998) and a parameter is significant, Fisher's F-test is employed to evaluate the effect of the parameter on output quality. The ANOVA findings for the IF-FUCOM-GRG are displayed in Table 14. According to the ANOVA results, both input parameters are important for the study, but curing day is more important than bacteria concentration.

Phase-V. Confirmation test result:

In Table 15, the optimal output parameter is tested for actual and predicted IF-FUCOM-GRA values. Equation (19) predicts the IF-FUCOM-GRA values provided at the optimal output. IF-FUCOM-GRA, as predicted and experimentally determined as an optimum level, are 0.818256 and 0.88929, respectively.
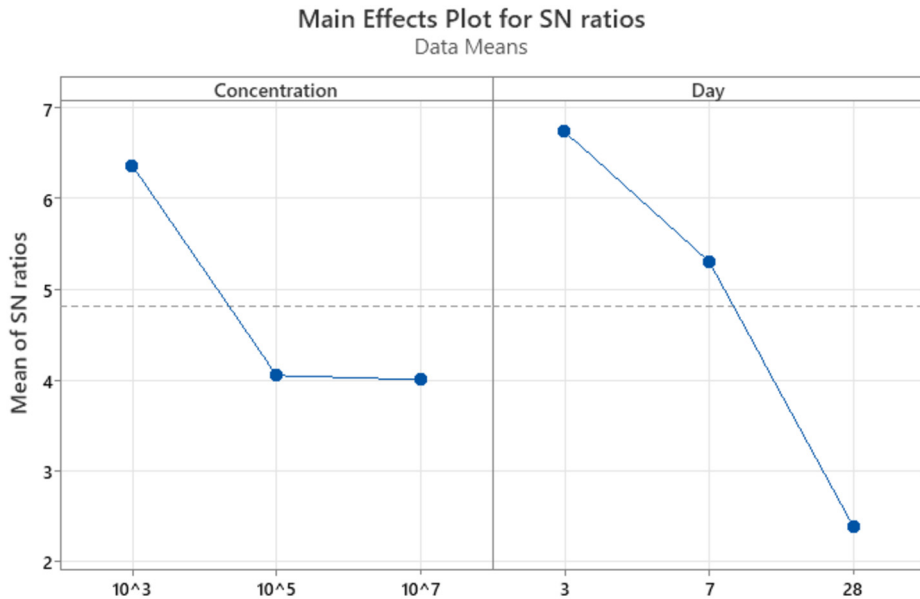
Fig. 4. Response of the IF-FUCOM-GRG SN ratio.

Table 14
Results of the ANOVA for the IF-FUCOM-GRG.

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Concentration	2	0.044646	0.022323	11.88	0.021
Days	2	0.142686	0.071343	37.96	0.003
Error	4	0.007519	0.001880		
Total	8	0.194851			

Table 15
Confirmation test table.

	Optimal input parameter	
	Predicted	Experimental
Level	10^5 bacteria concentration, 28 curing day	10^5 bacteria concentration, 28 curing day
IF-FUCOM-GRA grade	0.816385	0.88929
S/N Ratio	1.64405	1.71291

5. Discussion

In this study, three criteria, crack healing, water absorption, and compressive strength, are used to determine the optimal bacteria concentration and curing day. The values that corresponded to all three criteria are obtained through experimentation. Using the experimental data, equation (16) calculates the gray relation coefficient. A variety of MCDM

techniques are used for weighting criteria, including BWM, AHP, FUCOM, and intu-
itive fuzzy FUCOM. After that, a comparative study has been conducted among AHP-
GRG, FUCOM-GRG, BWM-GRG, and IF-FUCOM-GRG. The comparison shows that
IF-FUCOM-GRG produces similar rankings to other methods in most cases. Based on
the proposed method, the optimal bacteria concentration is 105, and the optimal curing
time is 28 days. The confirmation test result shown in Table 15 predicts IF-FUCOM-GRA
grade pretty well, and is almost in agreement with the experimental results.

6. Conclusion

MCDM problems are solved by considering different levels of importance of the crite-
ria. A number of weighting methods have been used in the literature to determine the
importance levels of expert opinions, including SAW, AHP/ANP, SWARA, BWM, and
FUCOM. The fuzzy set theory can be used to solve ambiguous and vague problems. It is
possible to improve the reliability of these weighting methods by incorporating fuzzy set
theory, which reflects the way humans think and reason. The intuitionistic fuzzy set solves
this problem by defining the non-membership degree and the two membership levels for
each element. In this study, intuitionistic fuzzy sets are combined with FUCOM to come
up with the Intuitionistic Fuzzy FUCOM (IF-FUCOM). Moreover, linguistic variables are
used instead of crisp values in pairwise comparisons for criteria in the decision-making
process.

In comparison to the IF-BWM and IF-AHP models, the IF-FUCOM model has the
advantage of offering similar results by using only $(n - 1)$ pairwise comparisons. By
eliminating the inconsistency of expert preferences with respect to the final weights of
criteria, the influence of inconsistency is reduced. The IF-FUCOM is considered the best
method for determining criteria weights because it requires a minimum number of expert
comparisons. In addition, the mathematical apparatus provides easy-to-understand weight
coefficients to facilitate rational decision-making (Fazlollahtabar *et al.*, 2019). Hence, the
IF-FUCOM tool allows decision-makers to ignore their own preferences in order to deal
with subjectivity in prioritizing criteria.

In the present study, Intuitionistic Fuzzy FUCOM Grey Relations Analysis is used to
select optimal bacteria concentrations and optimal curing time in days. The proposed al-
gorithm obtains a grey reasoning grade according to the grey relational coefficients of each
test run in order to convert multi-response optimization to single objective optimization.
The intuitionist fuzzy-FUCOM-grey reasoning grades (IF-FUCOM-GRG) are compared
with different grades like AHP-grey reasoning grade (AHP-GRG), BWM-grey reasoning
grade (BWM-GRG), FUCOM-grey reasoning grade (FUCOM-GRG). All the algorithms
have been employed to obtain the optimal input factor corresponding to the estimated val-
ues of output response. IF-FUCOM-GRG produces similar rankings with other methods
in most cases, but in some cases it produces better results. According to the proposed
method, the optimal bacteria concentration is 105, and the optimal curing time is 28 days.
Using a confirmation experiment, the computed factor combination based on the highest
ranking of IF-FUCOM-GRG is validated.

Limitations of the study:

i. There are three criteria used in this study to select the best bacteria concentration and curing time. However, the results may vary if other criteria are added.

ii. The ranking order may change as the number of alternatives increases, which is the shortcoming of this model.

Future scope:

i. In the future, the proposed method can be applied to all fields of science, engineering, and social sciences. Additionally, this method can be used in conjunction with other ranking methods (COPRAS, CODAS, ARAS, TOPSIS, EDAS, MAIRCA, etc.) to select the most appropriate alternative to solve MCDM problems.

ii. This study found optimal bacteria concentrations in concrete mortar, but optimal bacteria concentrations can also be found when cement is partially replaced by other additives like rice husk, fly ash, etc.

References

Achal, V., Mukherjee, A., Sudhakara Reddy, M. (2011). Microbial concrete way to enhance the durability of building structures. *Journal of Materials in Civil Engineering*, 23(6), 730–734.

Arunachalam, K.D., Sathyanarayanan, K.S., Darshan, B.S., Raja, R.B. (2010). Studies on the characterisation of Biosealant properties of Bacillus sphaericus. *International Journal of Engineering Science and Technology*, 2(3), 270–277.

Baig, M.M.U., Ali, Y., Rehman, O.U. (2022). Enhancing resilience of oil supply chains in context of developing countries. *Operational Research in Engineering Sciences: Theory and Applications*, 5(1), 69–89.

Böyükaslan, A., Ecer, F. (2021). Determination of drivers for investing in cryptocurrencies through a fuzzy full consistency method-Bonferroni (FUCOM-F'B) framework. *Technology in Society*, 67, 101745.

Božanić, D., Tešić, D., Kočić, J. (2019). Multi-criteria FUCOM–Fuzzy MABAC model for the selection of location for construction of single-span bailey bridge. *Decision Making: Applications in Management and Engineering*, 2(1), 132–146.

Bozanic, D., Tešić, D., Milić, A. (2020). Multicriteria decision making model with Z-numbers based on FUCOM and MABAC model. *Decision Making: Applications in Management and Engineering*, 3(2), 19–36.

Božanić, D., Milić, A., Tešić, D., Salabun, W., Pamučar, D. (2021). D numbers–FUCOM–fuzzy RAFSI model for selecting the group of construction machines for enabling mobility. *Facta Universitatis. Series: Mechanical Engineering*, 19(3), 447–471.

Chahal, N., Siddique, R., Rajor, A. (2012). Influence of bacteria on the compressive strength, water absorption and rapid chloride permeability of fly ash concrete. *Construction and Building Materials*, 28(1), 351–356.

Chiao, K.P. (2016). The multi-criteria group decision making methodology using type 2 fuzzy linguistic judgments. *Applied Soft Computing*, 49, 189–211.

Dagdevir, T., Ozceyhan, V. (2021). Optimization of process parameters in terms of stabilization and thermal conductivity on water based TiO_2 nanofluid preparation by using Taguchi method and Grey relation analysis. *International Communications in Heat and Mass Transfer*, 120, 105047.

De Muynck, W., Debrouwer, D., De Belie, N., Verstraete, W. (2008). Bacterial carbonate precipitation improves the durability of cementitious materials. *Cement and Concrete Research*, 38(7), 1005–1014.

Demir, G., Damjanović, M., Matović, B., Vujadinović, R. (2022). Toward sustainable urban mobility by using fuzzy-FUCOM and fuzzy-CoCoSo methods: the case of the SUMP podgorica. *Sustainability*, 14(9), 4972.

Dhami, N.K., Reddy, M.S., Mukherjee, A. (2013). Bacillus megaterium mediated mineralization of calcium carbonate as biogenic surface treatment of green building materials. *World Journal of Microbiology and Biotechnology*, 29(12), 2397–2406.

Durmić, E., Stević, Ž., Chatterjee, P., Vasiljević, M., Tomašević, M. (2020). Sustainable supplier selection using combined FUCOM–rough SAW model. *Reports in Mechanical Engineering*, 1(1), 34–43.

Erdoğan, S., Sayin, C. (2018). Selection of the most suitable alternative fuel depending on the fuel characteristics and price by the hybrid MCDM method. *Sustainability*, 10(5), 1583.

Erdoğan, S., Aydın, S., Balki, M.K., Sayin, C. (2020). Operational evaluation of thermal barrier coated diesel engine fueled with biodiesel/diesel blend by using MCDM method base on engine performance, emission and combustion characteristics. *Renewable Energy*, 151, 698–706.

Fangfang, W.A.N.G. (2021). Research on the model and application progress based on grey relational analysis theory. *Advances in Educational Technology and Psychology*, 5(2), 30–35.

Fazlollahtabar, H., Smailbašić, A., Stević, Ž. (2019). FUCOM method in group decision-making: selection of forklift in a warehouse. *Decision Making: Applications in Management and Engineering*, 2(1), 49–65.

Ghosh, P., Mandal, S., Chattopadhyay, B.D., Pal, S. (2005). Use of microorganism to improve the strength of cement mortar. *Cement and Concrete Research*, 35(10), 1980–1983.

Gong, Y., Feng, L., Liu, G. (2014). Fuzzy multi-attribute group decision making method with incomplete weight information under interval type-2 fuzzy environment. *Journal of Intelligent & Fuzzy Systems*, 27(1), 307–316.

Grzenda, M., Bustillo, A., Zawistowski, P. (2012). A soft computing system using intelligent imputation strategies for roughness prediction in deep drilling. *Journal of Intelligent Manufacturing*, 23(5), 1733–1743.

Güler, O., Gajević, S., Miladinović, S., Çuvalcı, H., Stojanović, B. (2021). Optimization of zinc-based hybrid nanocomposites using Taguchi grey relation analysis. In: *International Congress Motor Vehicles & Motors 2020*, Kragujevac, Serbia, October 8th–9th, 2020.

Haq, A.N., Marimuthu, P., Jeyapaul, R. (2008). Multi response optimization of machining parameters of drilling Al/SiC metal matrix composite using grey relational analysis in the Taguchi method. *The International Journal of Advanced Manufacturing Technology*, 37(3), 250–255.

Julong, D. (1989). Introduction to grey system theory. *The Journal of Grey System*, 1(1), 1–24.

Kao, M.J., Ting, C.C., Lin, B.F., Tsung, T.T. (2008). Aqueous aluminum nanofluid combustion in diesel fuel. *Journal of Testing and Evaluation*, 36(2), 503.

Krassimir, T.A., Parvathi, R. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87–96.

Majumdar, S., Sarkar, M., Chowdhury, T., Chattopadhyay, B., Mandal, S. (2012). Use of bacterial protein powder in commercial fly ash pozzolana cements for high performance construction materials. *Open Journal of Civil Engineering*, 2(4), 218–228.

Milićević, A., Pavličić, D., Kostić, A. (2007). The dynamics of change in decision making under risk. *Psihologija*, 40(1), 147–164.

Mondal, S., Ghosh, A.D. (2018). Investigation into the optimal bacterial concentration for compressive strength enhancement of microbial concrete. *Construction and Building Materials*, 183, 202–214.

Nunić, Z. (2018). Evaluation and selection of manufacturer PVC carpentry using FUCOM-MABAC model. *Operational Research in Engineering Sciences: Theory and Applications*, 1(1), 13–28.

Otay, İ., Oztaysi, B., Onar, S.C., Kahraman, C. (2017). Multi-expert performance evaluation of healthcare institutions using an integrated intuitionistic fuzzy AHP&DEA methodology. *Knowledge-Based Systems*, 133, 90–106.

Pamucar, D., Ecer, F. (2020). Prioritizing the weights of the evaluation criteria under fuzziness: the fuzzy full consistency method–FUCOM-F. *Facta Universitatis, Series: Mechanical Engineering*, 18(3), 419–437.

Pamucar, D., Ecer, F., Deveci, M. (2021). Assessment of alternative fuel vehicles for sustainable road transportation of United States using integrated fuzzy FUCOM and neutrosophic fuzzy MARCOS methodology. *Science of The Total Environment*, 788, 147763.

Pamučar, D., Lukovac, V., Božanić, D., Komazec, N. (2018a). Multi-criteria FUCOM-MAIRCA model for the evaluation of level crossings: case study in the Republic of Serbia. *Operational Research in Engineering Sciences: Theory and Applications*, 1(1), 108–129.

Pamučar, D., Stević, Ž., Sremac, S. (2018b). A new model for determining weight coefficients of criteria in MCDM models: full consistency method (FUCOM). *Symmetry*, 10(9), 393.

Pattnaik, S., Karunakar, D.B., Jha, P.K. (2013). Multi-characteristic optimization of wax patterns in the investment casting process using grey–fuzzy logic. *The International Journal of Advanced Manufacturing Technology*, 67(5–8), 1577–1587.

Ramachandran, S.K., Ramakrishnan, V., Bang, S.S. (2001). Remediation of concrete using micro-organisms. *ACI Materials Journal-American Concrete Institute*, 98(1), 3–9.

Ramakrishnan, V., Deo, K.S., Duke, E.F., Bang, S.S. (1999). SEM investigation of microbial calcite precipitation in cement. In: *Proceedings of the International Conference on Cement Microscopy*, Vol. 21, International Cement Microscopy Association, pp. 406–414.

Rezaei, J. (2015). Best-worst multi-criteria decision-making method. *Omega*, 53, 49–57.

Roy, S., Das, A.K., Banerjee, R. (2016). Grey-fuzzy Taguchi approach for multi-objective optimization of performance and emission parameters of a single cylinder crdi engine coupled with EGR. *International Journal of Automotive Technology*, 17(1), 1–12.

Saaty, T.L. (1980). *The Analytical Hierarchy Process, Planning, Priority. Resource Allocation*. RWS Publications, USA.

Si, A., Das, S., Kar, S. (2021). Picture fuzzy set-based decision-making approach using Dempster–Shafer theory of evidence and grey relation analysis and its application in COVID-19 medicine selection. *Soft Computing*, 1–15.

Tang, C., Xu, D., Chen, N. (2021). Sustainability prioritization of sewage sludge to energy scenarios with hybrid-data consideration: a fuzzy decision-making framework based on full consistency method and fusion ranking model. *Environmental Science and Pollution Research*, 28(5), 5548–5565.

Van Tittelboom, K., De Belie, N., De Muynck, W., Verstraete, W. (2010). Use of bacteria to repair cracks in concrete. *Cement and Concrete Research*, 40(1), 157–166.

Yang, W.H.P., Tarng, Y.S. (1998). Design optimization of cutting parameters for turning operations based on the Taguchi method. *Journal of Materials Processing Technology*, 84(1–3), 122–129.

Zadeh, L.A. (1975). The concept of a linguistic variable and its application to approximate reasoning—I. *Information Sciences*, 8(3), 199–249.

S. Dey holds a BE degree (2013) from Sathyabama University, an MTech degree (2015) in structural engineering from Maulana Abul Kalam Azad University of Technology and is currently pursuing a PhD degree at National Institute of Technology, Agartala.

F. Smarandache received the MSc degree in mathematics and computer science from the University of Craiova, Romania; the PhD degree in mathematics from the State University of Kishinev; and the PhD degree in applied mathematics from the Okayama University of Sciences, Japan. He has been the founder of neutrosophy (generalization of dialectics), neutrosophic set, logic, probability and statistics, since 1995. He is currently a professor of mathematics at the University of New Mexico, USA. He has published hundreds of articles and books on neutrosophic physics, superluminal and instantaneous physics, unmatter, quantum paradoxes, absolute theory of relativity, redshift and blueshift due to the medium gradient and refraction index besides the Doppler effect, paradoxism, outerart, neutrosophy as a new branch of philosophy, law of included multiple-middle, multispace and multistructure, hypersoft set, degree of dependence and independence between neutrosophic components, refined neutrosophic set, neutrosophic over-under-off-set, plithogenic set/logic/probability/statistics, neutrosophic triplet and duplet structures, quadruple neutrosophic structures, extension of algebraic structures to neutroalgebras and antialgebras, neutrogeometry & antigeometry, Dezert–Smarandache theory and so on to many peer-reviewed international journals and many books. He presented papers and plenary lectures to many international conferences around the world.

R. Debbarma received PhD Degree in civil engineering from the Indian Institute of Engineering Science and Technology, Shibpur. Her specialization is in structural engineering. Currently, she is working as an associate professor, Department of Civil Engineering, National Institute of Technology, Agartala. She has published more than 30 research papers in international journals like SCI, SCOPUS and other reputed journals. Also, she has published more than 50 papers in international conferences and 10 book chapters. She has also presented international papers in Switzerland, USA, Bangladesh and many more renowned institutions in India.

P. Majumder received the BSc degree from Tripura University, in 2010, MSc and PhD degrees in mathematics from the National Institute of Technology Agartala, India, in 2012 and 2020. Currently, he is working as an assistant professor with the Department of Basic Science and Humanities Department (Mathematics), Techno College of Engineering Agartala, Maheshkhola, Agartala, Tripura. He is the author of two books and more than 33 articles.