Open Questions about Concatenated Primes and Metasequences

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Abstract.
We define a metasequence as a sequence constructed with the terms of other given sequence(s). In this short note we present some open questions on concatenated primes involved in metasequences.

First Class of Concatenated Sequences.

1) Let $a_1, a_2, \ldots, a_{k-1}, a_k$ be given $k \geq 1$ digits in the numeration base b.
a) There exists a prime number P of the concatenated form:

$$P = \overline{*\ldots*a_1*\ldots*a_2*\ldots*\ldots*\ldots*a_{k-1}*\ldots*a_k*\ldots*}$$

where the stars "$*\ldots*$" represent various (from none to any finite positive integer) numbers of digits in base b.

Of course, if a_k is the last digit then a_k should belong to the set {1, 3, 7, 9} in base *10*. Similar restriction for the last number's digit a_k in other base b.

b) Are there infinitely many such primes?
c) What about considering fixed positions for the given digits: i.e. each given a_i on a given position n_i ?
d) As a consequence, for any group of given digits $a_1, a_2, \ldots, a_{k-1}, a_k$ do we have finitely or infinitely many primes starting with this group of digits (i.e. in the following concatenated form):

$$\overline{a_1a_2\ldots a_{k-1}a_k*\ldots*}$$

?
e) As a consequence, for any group of given digits $a_1, a_2, \ldots, a_{k-1}, a_k$ do we have finitely or infinitely many primes ending with this group of digits (i.e. in the following concatenated form):

$$\overline{*\ldots*a_1a_2\ldots a_{k-1}a_k}$$

(of course considering the primality restriction on the last digit a_k) ?

f) As a consequence, for any group of given digits $a_1, a_2, \ldots, a_{k-1}, a_k$ and any given digits $b_1, b_2, \ldots, b_{j-1}, b_j$ do we have finitely or infinitely many primes beginning with the group of digits $a_1, a_2, \ldots, a_{k-1}, a_k$ and ending with the group of digits $b_1, b_2, \ldots, b_{j-1}, b_j$ (i.e. in the following concatenated form):

$$\overline{a_1 a_2 \ldots a_{k-1} a_k * \ldots * b_1 b_2 \ldots b_{j-1} b_j}$$

(of course considering the primality restriction on the last digit bj) ?

g) As a consequence, for any group of given digits $a_1, a_2, \ldots, a_{k-1}, a_k$ do we have finitely or infinitely many primes having inside of their concatenated form this group of digits (i.e. in the following concatenated form):

$$\overline{* \ldots * a_1 a_2 \ldots a_{k-1} a_k * \ldots *}$$

?

h) As a consequence, for any groups of given digits $a_1, a_2, \ldots, a_{k-1}, a_k$ and $b_1, b_2, \ldots, b_{j-1}, b_j$ and $c_1, c_2, \ldots, c_{i-1}, c_i$ do we have finitely or infinitely many primes beginning with the group of digits $a_1, a_2, \ldots, a_{k-1}, a_k$, ending with the group of digits $b_1, b_2, \ldots, b_{j-1}, b_j$, and having inside the group of digits $c_1, c_2, \ldots, c_{i-1}, c_i$ (i.e. in the following concatenated form):

$$\overline{a_1 a_2 \ldots a_{k-1} a_k * \ldots * c_1 c_2 \ldots c_{i-1} c_i * \ldots * b_1 b_2 \ldots b_{j-1} b_j}$$

(of course considering the primality restriction on the last digit bj) ?

i) What general condition has a sequence $s_1, s_2, \ldots, s_n, \ldots$ to satisfy in order for the concatenated metasequence

$$\overline{s_1 s_2 \ldots s_n}$$

for $n = 1, 2, \ldots$ to contain infinitely many primes?

Second Class of Metasequences.

2) Let's note the sequence of prime numbers by $p_1 = 2, p_2 = 3, p_3 = 5, \ldots, p_n$ the n^{st} prime number, for any natural number n.

a) Does the metasequence

$$p_1 \cdot p_2 \cdot \ldots \cdot p_n + 1$$

for $n = 1, 2, \ldots$ contains finitely or infinitely many primes?

b) What about the metasequence:

$$p_1 \cdot p_2 \cdot \ldots \cdot p_n - 1$$

?

c) What general condition has a sequence $s_1, s_2, \ldots, s_n, \ldots$ to satisfy in order for the metasequence

$$s_1 \cdot s_2 \cdot \ldots \cdot s_n \pm 1$$

for $n = 1, 2, \ldots$ to contain infinitely many primes?

Reference:

F. Smarandache, Sequences of Numbers Involved in Unsolved Problems, 139 p., HeXis, 2006.