SOME PROPERTIES OF NEDIANES

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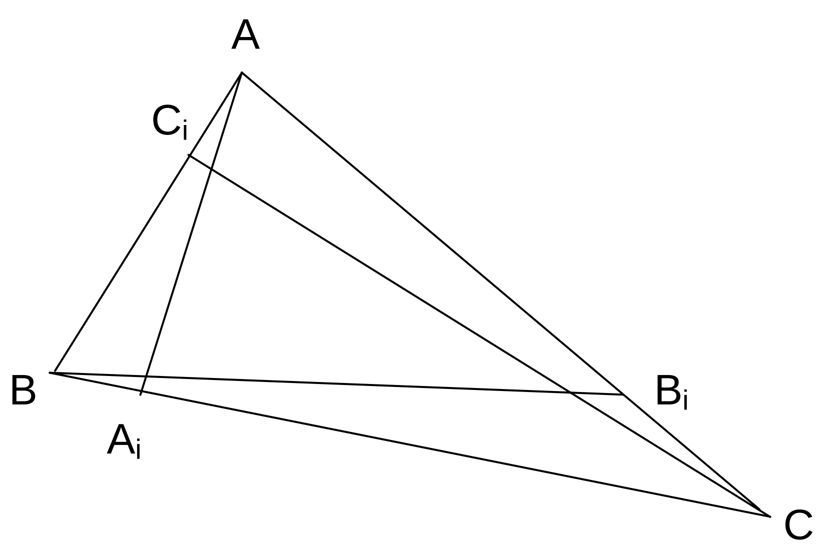
This article generalizes certain results on the nedianes (see [1], pp. 97-99). One calls *nedianes* the segments of a line that passes through a vertex of a triangle and partitions the opposite side in n equal parts. A nediane is called to be of order i if it partitions the opposite side in the rapport i/n.

For $1 \le i \le n-1$ the nedianes of order i (that is AA_i, BB_i and CC_i) have the following properties:

1) With these 3 segments one can construct a triangle.



2) $|AA_i|^2 + |BB_i|^2 + |CC_i|^2 = \dfrac{i^2 - i \cdot n + n^2}{n^2}(a^2 + b^2 + c^2)$.

Proofs:

$$\overrightarrow{AA_i} = \overrightarrow{AB} + \overrightarrow{BA_i} = \overrightarrow{AB} + \frac{i}{n}\overrightarrow{BC} \qquad (1)$$

$$\overrightarrow{BB_i} = \overrightarrow{BC} + \overrightarrow{CB_i} = \overrightarrow{BC} + \frac{i}{n}\overrightarrow{CA} \qquad (2)$$

$$\overrightarrow{CC_i} = \overrightarrow{CA} + \overrightarrow{AC_i} = \overrightarrow{CA} + \frac{i}{n}\overrightarrow{AB} \qquad (3)$$

By adding these 3 relations, we obtain:

$$\overrightarrow{AA_i} + \overrightarrow{BB_i} + \overrightarrow{CC_i} = \frac{i+n}{n}(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}) = 0$$

therefore the 3 nedianes can be the sides of a triangle.

(2) By raising to the square the relations and then adding them we obtain:

$$\left|AA_i\right|^2 + \left|BB_i\right|^2 + \left|CC_i\right|^2 = a^2 + b^2 + c^2 + \frac{i^2}{n^2}(a^2 + b^2 + c^2) +$$

$$+ \frac{i}{n}(2\overrightarrow{AB} \cdot \overrightarrow{BC} + 2\overrightarrow{BC} \cdot \overrightarrow{CA} + 2\overrightarrow{CA} \cdot \overrightarrow{AB}) \qquad (4)$$

Because $2\overrightarrow{AB} \cdot \overrightarrow{BC} = -2ca \cdot \cos B = b^2 - c^2 - a^2$ (the theorem of cosines), by substituting this in the relation (4), we obtain the requested relation.

Reference:

[1] Vodă, Dr. Viorel Gh. "Surprize în matematica elementară", Editura Albatros, Bucharest, 1981.