

## A PROPERTY OF THE CIRCUMSCRIBED OCTAGON

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### Abstract

In this article we'll obtain through the duality method a property in relation to the contact cords of the opposite sides of a circumscribable octagon.

In an inscribed hexagon the following theorem proved by Blaise Pascal in 1640 is true.

### Theorem 1 (Blaise Pascal)

The opposite sides of a hexagon inscribed in a circle intersect in collinear points.

To prove the Pascal theorem one may use [1].

In [2] there is a discussion that the Pascal's theorem will be also true if two or more pairs of vertexes of the hexagon coincide. In this case, for example the side  $AB$  for  $B \rightarrow A$  must be substituted with the tangent in  $A$ . For example we suppose that two pairs of vertexes coincide. The hexagon  $AA'BCC'D$  for  $A' \rightarrow A, C' \rightarrow C$  becomes the inscribed quadrilateral  $ABCD$ . This quadrilateral viewed as a degenerated hexagon of sides  $AB, BC, CC' \rightarrow$  the tangent in  $C, C'D \rightarrow CD, D'A \rightarrow DA, AA' \rightarrow$  the tangent in  $A$  and the Pascal theorem leads to:

### Theorem 2

In an inscribed quadrilateral the opposite sides and the tangents in the opposite vertexes intersect in four collinear points.

### Remark 1

In figure 1 is presented the corresponding configuration of theorem 2.

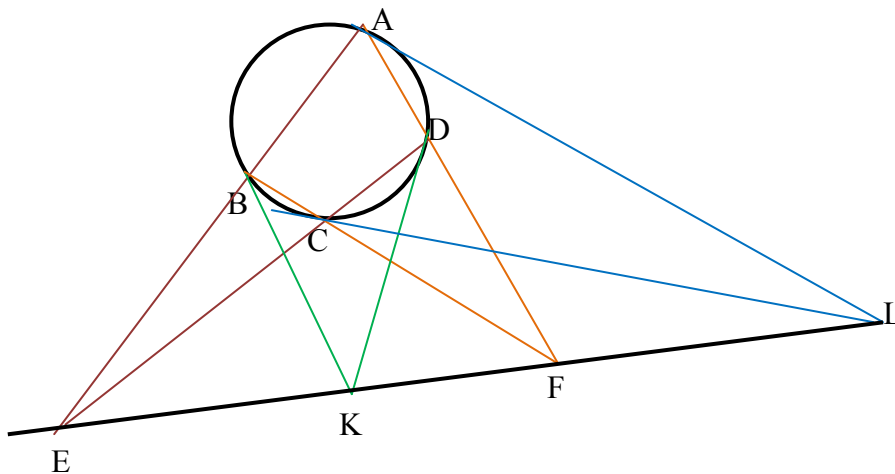


Fig. 1

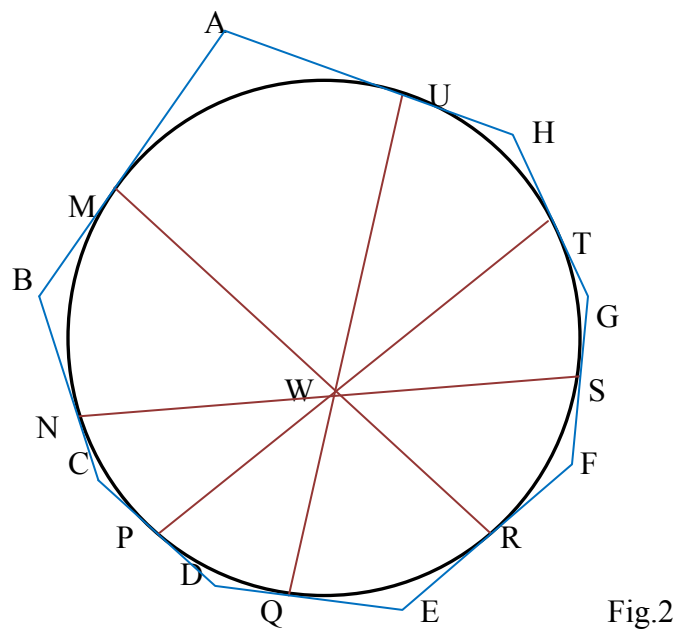
For the tangents constructed in  $B$  and  $D$  the property is also true if we consider the  $ABCD$  as a degenerated hexagon  $ABB'CDD'A$ .

**Theorem 3**

In an inscribed octagon the four cords determined by the contact points with the circle of the opposite sides are concurrent.

**Proof**

We'll transform through reciprocal polar the configuration from figure 1. To point  $E$  will correspond, through this transformation the line determined by the tangent points with the circle of the tangents constructed from  $E$  (its polar). To point  $K$  corresponds the side  $BD$ .



To point  $F$  corresponds the line determined by the contact points of the tangents constructed from  $F$  to the circle. To point  $L$  corresponds its polar  $AC$ . To point  $A$  corresponds, by duality, the tangent  $AL$ , also to points  $B, C, D$  correspond the tangents  $BK, CL, DK$ . These four tangents together with the tangents constructed from  $E$  and  $F$  (also four) will contain the sides of an octagon circumscribed to the given circle.

In this octagon  $(AC)$  and  $(BD)$  will connect the contact points of two pairs of opposite sides with the circle; the other two lines determined by the contact points of the opposite sides of the octagon with the circle will be the polar of the points  $E$  and  $F$ . Because the polar transformation through reciprocal polar leads to the fact that to collinear points correspond concurrent lines; the points' polar  $E, K, F, L$  are concurrent; these lines are the cords to which the theorem refers to.

**Remark 2**

In figure 2 we represented an octagon circumscribed  $ABCDEFGH$  . As it can be seen the cords  $MR$ ,  $NS$ ,  $PT$ ,  $QU$  are concurrent in the point  $W$  .

### **References**

- [1] Roger A Johnson – *Advanced Euclidean Geometry*, Dover Publications, Inc. Mineola, New-York, 2007
- [2] N. Mihăileanu – *Leții complementare de geometrie*, Editura Didactică și Pedagogică, București, 1976
- [3] Florentin Smarandache – *Multispace & Multistructure, Neutrosophic Trandisciplinarity (100 Collected Papers of Sciences)*, Vol IV, 800 p., North-European Scientific Publishers, Hanko, Finland, 2010