Real Examples of NeutroGeometry & AntiGeometry

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Abstract: For the classical *Geometry*, in a geometrical space, all items (concepts, axioms, theorems, etc.) are totally (100%) true. But, in the real world, many items are not totally true. The *NeutroGeometry* is a geometrical space that has some items that are only partially true (and partially indeterminate, and partially false), and no item that is totally false. The *AntiGeometry* is a geometrical space that has some item that are totally (100%) false. While the Non-Euclidean Geometries [hyperbolic and elliptic geometries] resulted from the total negation of only one specific axiom (Euclid's Fifth Postulate), the AntiGeometry results from the total negation of any axiom [and in general: theorem, concept, idea etc.] and even of more axioms [theorem, concept, idea, etc.] and in general from any geometric axiomatic system (Euclid's five postulates, Hilbert's 20 axioms, etc.), and the NeutroAxiom results from the partial negation of any axiom (or concept, theorem, idea, etc.). Clearly, the AntiGeometry is a generalization of Non-Euclidean Geometries. [5]

Keywords: Non-Euclidean Geometries, Euclidean Geometry, Lobachevski-Bolyai-Gauss Geometry, Riemannian Geometry, NeutroManifold, AntiManifold, NeutroAlgebra, AntiAlgebra, NeutroGeometry, AntiGeometry, Hybrid (Smarandache) Geometry, NeutroAxiom, AntiAxiom, NeutroTheorem, AntiTheorem, Partial Function, NeutroFunction, AntiFunction, NeutroOperation, AntiOperation, NeutroAttribute, AntiAttribute, NeutroRelation, AntiRelation, NeutroStructure, AntiStructure.

1. Introduction

This is a review paper on the newly emerging field of NeutroStructures and AntiStructures, introduced by Smarandache [1] since 2019 and developed [2, 3, 4] in 2020-2021, inspired from our real world since the laws and regulations do not equally apply to all citizens, but in different degrees.

Let T = true, I = indeterminacy, F = false,

where $T, I, F \in [0, 1]$ and $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$.

The following neutrosophic triplets occur in our real world:

2. <Structure(1, 0, 0), NeutroStructure(T, I, F), AntiStructure(0, 0, 1)>

In any theoretical field of knowledge, the classical **Structures** have all items (concepts, axioms, theorems, properties, ideas, relationships, etc.) totally (100%) true.

But in the real world, most structures have items that are only partially true (and partially indeterminate, or partially false) and no item that is totally false (as in NeutroStructure), we call them **NeutroStructures**.

And structures that have some items that are totally (100%) false, we call them **AntiStructures**.

3. <Algebra(1, 0, 0), NeutroAlgebra(T, I, F), AntiAlgebra(0, 0, 1)>

As particular cases, when the structures are algebras or geometries, one gets the above neutrosophic triplets.

The Classical Algebraic Structures [Algebra] have all operations totally (100%) well-defined, and all axioms [theorems, concepts, ideas, etc.] totally (100%) true.

The NeutroAlgebraic Structures have operations or axioms (and in general: theorems, concepts, ideas, etc.) that are not totally (100%) well-defined or respectively totally (100%) true, but only partially well-defined or partially true [and none of them is 0% well-defined or respectively 0% true as in AntiAlgebraic Structures]. The NeutroAlgebraic Structures are in between Classical Algebraic Structures and AntiAlgebraic Structures.

And the AntiAlgebraic Structures have at least one operation or one axiom that is 0% well-defined or respectively 0% true.

4. <Geometry(1, 0, 0), NeutroGeometry(T, I, F), AntiGeometry(0, 0, 1)>

1) A geometric structure whose all axioms (and theorems, propositions, etc.) are totally true is called a classical Geometric Structure (or **Geometry**).

2) A geometric structure that has at least one NeutroOperation or one NeutroAxiom (and no AntiOperation and no AntiAxiom) is called a NeutroAlgebraic Structure (or **NeutroGeometry**).

3) A geometric structure that has at least one AntiOperation or one Anti Axiom is called an AntiAlgebraic Structure (or **AntiGeometry**).

Therefore, a neutrosophic triplet is formed: *<Geometry, NeutroGeometry, AntiGeometry>*, where "Geometry" can be any classical Euclidean, Projective, Affine, Discrete, Differential, etc. geometric structure.

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Similarly, for any field of knowledge, the axioms (and theorems, propositions, concepts, ideas etc.) are categorized in three groups [1 – 4]:

5. <Axiom(1, 0, 0), NeutroAxiom(T, I, F), AntiAxiom(0, 0, 1)>

An axiom, defined on a given set, endowed with some operation(s). When we define an axiom on a given set, it does not automatically mean that the axiom is true for all set's elements. We have three possibilities again:

i) The axiom is true for all set's elements (totally true) [degree of truth T = 1] (as in classical algebraic structures; this is a classical **Axiom**). Neutrosophically we write: Axiom(1,0,0).

ii) The axiom if true for some elements [degree of truth T], indeterminate for other elements [degree of indeterminacy I], and false for other elements [degree of falsehood F], where (T,I,F) is different from (1,0,0) and from (0,0,1) (this is **NeutroAxiom**). Neutrosophically we write NeutroAxiom(T,I,F).

iii) The axiom is false for all set's elements [degree of falsehood F = 1](this is **AntiAxiom**). Neutrosophically we write AntiAxiom(0,0,1).

And, of course, the Axiom may be replaced by Theorem, Property, Concept, etc.

6. Examples of AntiGeometry

6.1. The Hyperbolic (Non-Euclidean) Geometry [or Lobachevski-Bolyai-Gauss Geometry] resulted from the total negation of the axiom called Euclid's Fifth Postulate [through a point exterior to a line only one parallel can be drawn to that line] by the AntiAxiom: through a point exterior to a line many parallels can be drawn to that line.

6.2. The Elliptic (Non-Euclidean) Geometry [or Riemannian Geometry] resulted from the total negation of the axiom called Euclid's Fifth Postulate [through a point exterior to a line only one parallel can be drawn to that line] by another AntiAxiom: through a point exterior to a line no parallel can be drawn to that line.

6.3. The second class of the Hybrid (Smarandache) Geometry (or SG)[1] where an axiom is totally denied but in multiple different ways in the same geometric space, which combined the Hyperbolic and Elliptic Geometries into the same geometric space, by totally denying Euclid's Fifth Postulate in two different ways:

a) there are lines and points exteriors to them such that through a point exterior to a line many parallels can be drawn to that line; and

b) there are other lines and points exteriors to them such that through a point exterior to a line no parallel can be drawn to that line.

6.4. New example of AntiGeometry that is not a Non-Euclidean Geometry

Let us have on a plane (π) all circles of radius r > 0 and centered into the origin (0, 0).

For example, the below drawn circles (Fig. 1).

By "point" we understand any classical point, and by "line" we understand the circumference of a circle.

Let's take any three distinct points on the circumference of the small circle (similarly it will be for all other circles).

Clearly,   the points A, B, C lie on the same line (circumference), and:

the point B lies between the point A and point C;

the point C lies between the point B and point A;

and the point A lies between the point C and point B.

Therefore, Hilbert's Postulate B.3 of the Axioms of Betweenness, stated as follows:

"If A, B and C are three distinct points lying on the same line, then one and only one of the points lies between the other two."

is totally denied, because for any three distinct points lying on a line one has any point lies between the other two.
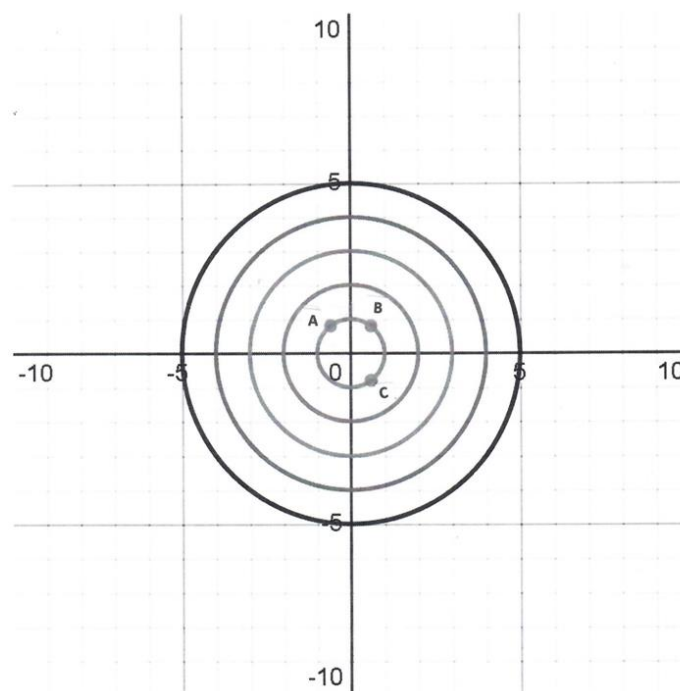

Figure 1. New Example of AntiGeometry that is not a Non-Euclidean Geometry.

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[1] Linfan Mao, *Pseudo-Manifold Geometries with Applications*, Cornell University, New York City, USA, 2006. Abstract: https://arxiv.org/abs/math/0610307, Full paper: https://arxiv.org/pdf/math/0610307. "A Smarandache geometry is a geometry which has at least one Smarandachely denied axiom (1969), i.e., an axiom behaves in at least two different ways within the same space, i.e., validated and invalided, or only invalided but in multiple distinct ways."

This geometrical model does not represent a Non-Euclidean Geometry because the Euclid's Fifth Postulate is 100% true. Two lines are considered parallel if they do not intersect.

See the proof below:

Giving a line (circumference C_1 centered in the origin) and a point P_1 that does not lie on it, there exists a unique line (circumference C_2 centered in the origin) that passes though the point P_1 and does not intersect C_1.
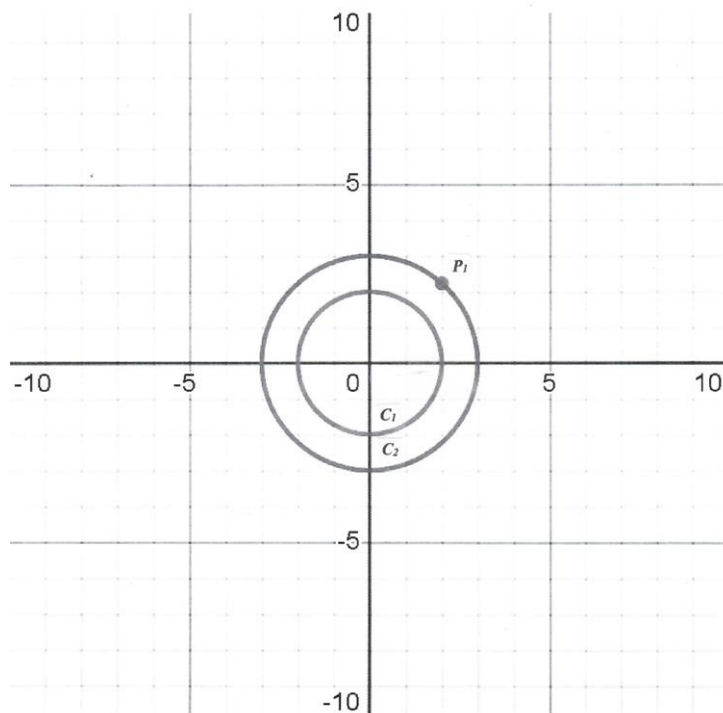

Figure 2. Euclid's Fifth Postulate is totally true.

7. Examples of NeutroGeometry

7.1. The first class of the Hybrid (or Smarandache) Geometry[1] where an axiom is partially true and partially false in the same geometric space.

For example, there are two distinct points that determine a single line, and other two distinct points that determine no line in the same geometric space.

Thus, *Hilbert's Postulate I.1.* of the Axioms of Incidence, announced as follows:

"For every point P and every point Q not equal to P, there exists a unique line incident with the points P and Q"

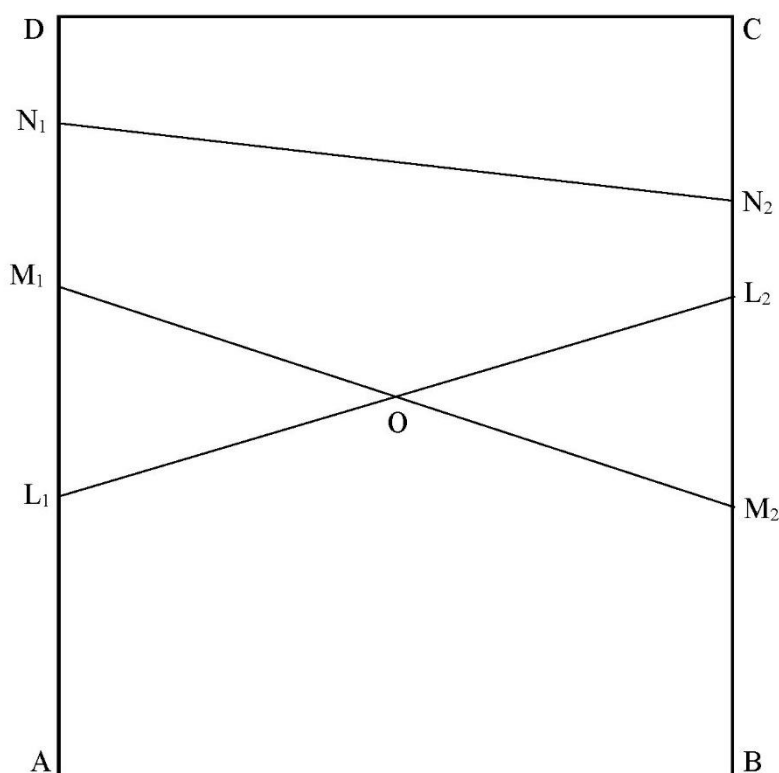becomes partially true and partially false.

Figure 3*: Example of a NeutroGeometry that is an SG*

Assume the rectangle ABCD is a geometric space, where "point" means any classical point on the sides AB and CD or interior to this rectangle, and "line" is any segment of line connecting a point from AB with a point of CD and passing through the center O of the rectangle.

For example, L_1L_2 is a line since it connects the point L_1 lying on AB, and point L_2 lying on CD, and passes through the center O. Similarly for the line M_1M_2.

But N_1N_2 is not a line, since it does not pass through the center O.

7.2. Example of NeutroGeometry that is not an SG

We consider the previous model of the rectangular geometric space ABCD, but adding some indeterminacy (I), as in our everyday life, i.e. the dark spot below, which represents some marsh area, so M_1M_2 although it is a line since it passes through the origin O, but it has also some degree of indeterminacy when crossing through the indeterminate zone (I).

While L_1L_2 is a totally determinate line, M_1M_2 is partially determinate and partially indeterminate (as in neutrosophy).
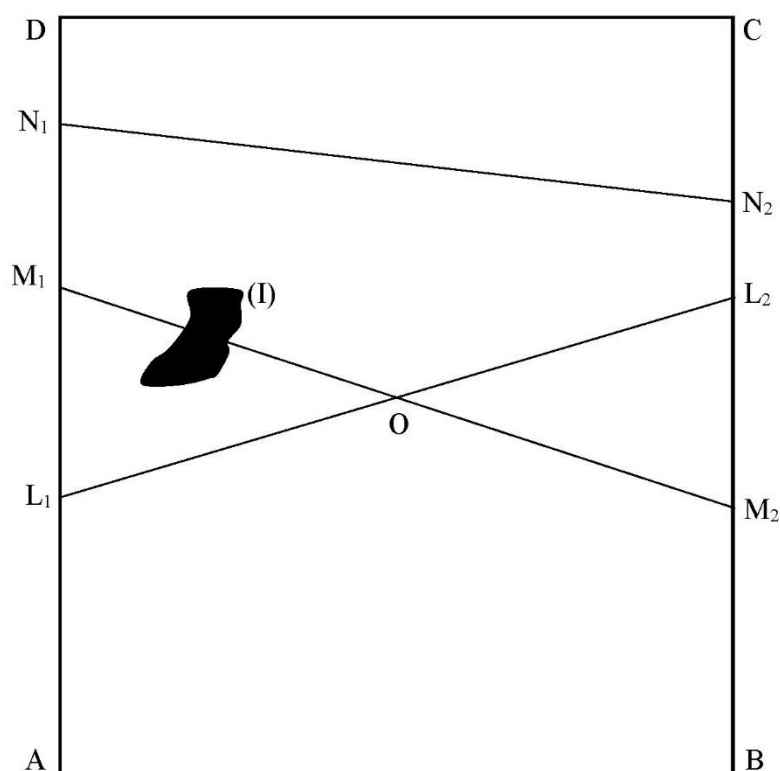
Figure 34*: Example of a NeutroGeometry that is not an SG*

5. Conclusions

In this paper we presented simple examples of NeutroGeometry, AntiGeometry, SG, and Non-Euclidean Geometries..

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