# Three Conjectures and Two Open Generalized Problems in Number Theory

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## 1. On a Problem with Primes.

In [1] it was asked if $\prod_{n=1}^{m} \frac{P_n+1}{P_n-1}$, where $P_n$ is the $n^{\text{th}}$ prime, is an integer for others $m \notin \{1, 2, 3, 4, 8\}$?

1) We conjecture no.
2) We also conjecture that

$$R_m = \prod_{n=1}^{m} \frac{P_n+k}{P_n-k},$$

with $k \in \mathrm{N}^* = \{1, 2, \ldots\}$, is an integer for a finite number of values of $m$.

3) Another conjecture: there is an infinite number of $k$'s for which no $R_m$ is an integer.

**Bibliography**

[1] R.K. Guy, *Unsolved Problems in Number Theory*, Problem B48, p. 57, Springer-Verlag, 1981.

## 2. On a Problem with Infinite Sequences.

Let $1 \le a_1 < a_2 < \ldots$ be an infinite sequence of integers such that any three members do not constitute an arithmetical progression. Is it true that always $\sum \frac{1}{a_i} \le 2$?

Is the function $S\left(\{a_n\}_{n\ge1}\right) = \sum_{n\ge1} \frac{1}{a_n}$ bijective (biunivocal)?

For example, $a_n = p^{n-1}$, $n \ge 1$, $p$ is an integer >1, has the property of the assumption, and $\sum_{n\ge1} \frac{1}{a_i} = 1 + \frac{1}{p-1} \le 2$,

Analogously for geometrical progressions.

4) More generally: let $f$ be a function $f : \mathrm{R}_+^m \to \mathrm{R}_+^*$. We construct a sequence $0 < a_1 < a_2 < \ldots$ such that there is no $\left(a_{i_1}, \ldots, a_{i_m}, a_{i_{m+1}}\right)$ such that $a_{i_{m+1}} = f(a_{i_1}, \ldots, a_{i_m})$.

Find $\max_{\{a_n\}_{n\geq 1}} \sum_{n\geq 1}^{m} \frac{1}{a_n}$

(It's a generalization of a question from the Problem E28, p. 127, in *Unsolved Problems in Number Theory*, by R. K. Guy, Springer-Verlag, 1981.)

5) Is the function

$$S\left(\{a_n\}_{n\geq 1}\right) = \sum_{n\geq 1} \frac{1}{a_n}$$

bijective?

*[This manuscript was confiscated by the Romanian Secret Police [Securitate] in 1980's. A copy of it was recovered by the Author after the 1989 Revolution through the CNSAS = National Council of Studying the Archives of the Secret Police.*

*Unfortunately, other confiscated manuscripts (a few hundreds of pages of mathematical proposed problems and conjectures, rebus, literary works) were never recovered, despite the fact that the Author required the CNSAS for returning his manuscripts.]*

**Addendum:**
On the next page see a copy of my holograph manuscript stamped by the C.N.S.A.S. on 23 July 2002.

## ON A PROBLEM WITH PRIMES

by FLORENTIN SMARANDACHE

David Silverman asked if $\prod_{n=1}^{m} \frac{p_n+1}{p_n-1}$, where $p_n$ is the $n$ th prime, is an integer for others $m \notin \{1,2,3,4,8\}$. We conjecture NO.

We conjecture that

$$R_m = \prod_{n=1}^{m} \frac{p_n+k}{p_n-k}, \text{ with } k \in \mathbb{N}^*,$$

is an integer for a finite number of values of $m$.

There is an infinite number of $k$ for which no $R_m$ is an integer.

Bibliography

R.K. Guy, Unsolved Problems in Number Theory, Springer-Verlag, 1981, problem B48, p. 57.

## ON A PROBLEM WITH INFINITE SEQUENCES

by FLORENTIN SMARANDACHE

Let $1 \leq a_1 < a_2 < \dots$ be an infinite sequence of integers such that any three members do not constitute an arithmetical progression. Is it true that always $\sum 1/a_i \leq 2$?

Is the function

$$S(\{a_n\}_{n\geq 1}) = \sum_{n\geq 1} 1/a_n$$

bijective? (biunivocal)

For example, $a_n = p^{n-1}$, $n \geq 1$, $p$ is an integer $> 1$, has the property of the assumption, and $\sum_{n\geq 1} 1/a_i = 1 + \frac{1}{p-1} \leq 2$.

Analogously for geometrical progressions.

More generaly: let $f$ be a function $f: \mathbb{R}_+^m \to \mathbb{R}_+^*$. We construct a sequence $0 < a_1 < a_2 < \dots$ such that there be no $(a_{i_1}, \dots, a_{i_m}, a_{i_{m+1}})$ with $f(a_{i_1}, \dots, a_{i_m}) = a_{i_{m+1}}$.

Find $\max_{\{a_n\}_{n\geq 1}} \sum_{n\geq 1} 1/a_n$.

(It's a generalization of a question from the problem E28, R.K. Guy, Unsolved Problems in Number Theory, Springer-Verlag, 1981, p. 127.)

Is the function

$$S(\{a_n\}_{n\geq 1}) = \sum_{n\geq 1} 1/a_n$$

bijective?