The Characteristic Function of a Neutrosophic Set

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Abstract. The purpose of this paper is to introduce and study the characteristic function of a neutrosophic set. After given the fundamental definitions of neutrosophic set operations generated by the characteristic function of a neutrosophic set (Ng for short), we obtain several properties, and discussed the relationship between neutrosophic sets generated by Ng and others. Finally, we introduce the neutrosophic topological spaces generated by . Possible application to GIS topology rules are touched upon.

Keywords: Neutrosophic Set; Neutrosophic Topology; Characteristic Function.

1 Introduction

Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts, such as a neutrosophic set theory. After the introduction of the neutrosophic set concepts in [2-13]. In this paper we introduce definitions of neutrosophic sets by characteristic function. After given the fundamental definitions of neutrosophic set operations by , we obtain several properties, and discussed the relationship between neutrosophic sets and others. Added to, we introduce the neutrosophic topological spaces generated by Ng.

2 Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [7- 9], Hanafy, Salama et al. [2- 13] and Demirci in [1].

3 Neutrosophic Sets generated by Ng

We shall now consider some possible definitions for basic concepts of the neutrosophic sets generated by Ng and its operations.

3.1 Definition

Let X is a non-empty fixed set. A neutrosophic set (NS for short) A is an object having the form

\[ A = \{ x, \mu_A(x), \sigma_A(x), \nu_A(x) \} \]

where \( \mu_A(x) \) and \( \gamma_A(x) \) represent the degree of membership function (namely \( \mu_A(x) \)), the degree of indeterminacy (namely \( \sigma_A(x) \)), and the degree of non-membership (namely \( \gamma_A(x) \)) respectively of each element \( x \in X \) to the set A

and let \( g_A : X \times [0,1] \rightarrow [0,1] \) be reality function, then \( N_{g_A}(\lambda) = N_{g_A}(\{ x, \lambda_1, \lambda_2, \lambda_3 \}) \) is said to be the characteristic function of a neutrosophic set on X if

\[ N_{g_A}(\lambda) = \begin{cases} 1 & \text{if } \mu_A(x) = \lambda_1, \sigma_A(x) = \lambda_2, \nu_A(x) = \lambda_3 \\ 0 & \text{otherwise} \end{cases} \]

where \( \lambda = (x, \lambda_1, \lambda_2, \lambda_3) \). Then the object

\[ G(A) = \{ x, \mu(A)(x), \sigma(A)(x), \nu(A)(x) \} \]

is a neutrosophic set generated by where

\[ \mu_{G(A)} = \sup \lambda_1 \{ N_{g_A}(\lambda \land \lambda) \} \]

\[ \sigma_{G(A)} = \sup \lambda_2 \{ N_{g_A}(\lambda \land \lambda) \} \]

\[ \nu_{G(A)} = \sup \lambda_3 \{ N_{g_A}(\lambda \land \lambda) \} \]

3.1 Proposition

1) \( A \subseteq N_{g_A} B \Leftrightarrow G(A) \subseteq G(B) \).
2) \( A = \neg G \) \( B \Rightarrow G(A) = G(B) \)

3.2 Definition

Let \( A \) be neutrosophic set of \( X \). Then the neutrosophic complement of \( A \) generated by \( \neg G(\{A\}) \) may be defined as the following:

\[
\begin{align*}
(\neg G)^1 &= \{ x, \mu_x^c, \nu_x^c \} \\
(\neg G)^2 &= \{ x, \nu_x, \sigma_x \} \\
(\neg G)^3 &= \{ x, \sigma_x, \mu_x \}
\end{align*}
\]

3.1 Example. Let \( X = \{ x \} \), \( A = \{ (0.5, 0.2, 0.06) \} \), \( \neg G_A = 1 \), \( \neg G_A = 0 \). Then \( G(A) = \{ (0.5, 0.2, 0.06) \} \)
Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the \( G(\{A\}) \) and \( G(\{A\}) \) as follows \( G(\{N\}) \) may be defined as:

- i) \( G(\{N\}) = \{ (0.0, 0.1) \} \)
- ii) \( G(\{N\}) = \{ (0.0, 0.1) \} \)
- iii) \( G(\{N\}) = \{ (0.0, 0.0) \} \)

\( G(\{N\}) \) may be defined as:

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We will define the following operations intersection and union for neutrosophic sets generated by \( \neg G \) by \( \cap \neg G \) and \( \cup \neg G \) respectively.

3.3 Definition. Let two neutrosophic sets \( A = \{ (x, \mu_A(x), \sigma_A(x), \nu_A(x) \} \) and \( B = \{ (x, \mu_B(x), \sigma_B(x), \nu_B(x) \} \) on \( X \), and

\[ G(A) = \{ x, \mu_G(A)(x), \sigma_G(A)(x), \nu_G(A)(x) \} \],
\[ G(B) = \{ x, \mu_G(B)(x), \sigma_G(B)(x), \nu_G(B)(x) \} \]. Then \( A \cap \neg G B \) may be defined as three types:

- Type I:

\[ \mu_G(A)(x) \vee \mu_G(B)(x) \wedge \sigma_G(A)(x) \wedge \sigma_G(B)(x) \wedge \nu_G(A)(x) \wedge \nu_G(B)(x) \]

- Type II:

\[ \mu_G(A)(x) \vee \mu_G(B)(x) \wedge \sigma_G(A)(x) \wedge \sigma_G(B)(x) \wedge \nu_G(A)(x) \wedge \nu_G(B)(x) \]

- Type III:

\[ \mu_G(A)(x) \vee \mu_G(B)(x) \wedge \sigma_G(A)(x) \wedge \sigma_G(B)(x) \wedge \nu_G(A)(x) \wedge \nu_G(B)(x) \]

We can easily generalize the operations of intersection and union in definition 3.2 to arbitrary family of neutrosophic subsets generated by \( \neg G \) as follows:

3.3 Proposition.
Let \( \{ A_j : j \in J \} \) be arbitrary family of neutrosophic subsets in \( X \) generated by \( \neg G \), then the following are true

- 1) \( (A \cap B)^\neg G = A^\neg G \cup B^\neg G \).
- 2) \( (A \cup B)^\neg G = A^\neg G \cap B^\neg G \).

\[ G(A \cap B) = \{ \mu_G(A)(x) \wedge \mu_G(B)(x), \sigma_G(A)(x) \wedge \sigma_G(B)(x), \nu_G(A)(x) \wedge \nu_G(B)(x) \} \]

\[ G(A \cup B) = \{ \mu_G(A)(x) \vee \mu_G(B)(x), \sigma_G(A)(x) \vee \sigma_G(B)(x), \nu_G(A)(x) \vee \nu_G(B)(x) \} \]

\[ G(A \cap B) = \{ \mu_G(A)(x) \wedge \mu_G(B)(x), \sigma_G(A)(x) \wedge \sigma_G(B)(x), \nu_G(A)(x) \wedge \nu_G(B)(x) \} \]

\[ G(A \cup B) = \{ \mu_G(A)(x) \vee \mu_G(B)(x), \sigma_G(A)(x) \vee \sigma_G(B)(x), \nu_G(A)(x) \vee \nu_G(B)(x) \} \]

a) \( \cap \neg G A \) may be defined as:

- Type I:

\[ G(A \cap B) = \{ \mu_G(A)(x) \wedge \mu_G(B)(x), \sigma_G(A)(x) \wedge \sigma_G(B)(x), \nu_G(A)(x) \wedge \nu_G(B)(x) \} \]

- Type II:

\[ G(A \cap B) = \{ \mu_G(A)(x) \wedge \mu_G(B)(x), \sigma_G(A)(x) \wedge \sigma_G(B)(x), \nu_G(A)(x) \wedge \nu_G(B)(x) \} \]

- Type III:

\[ G(A \cap B) = \{ \mu_G(A)(x) \wedge \mu_G(B)(x), \sigma_G(A)(x) \wedge \sigma_G(B)(x), \nu_G(A)(x) \wedge \nu_G(B)(x) \} \]

b) \( \cup \neg G A \) may be defined as:

- Type I:

\[ G(A \cup B) = \{ \mu_G(A)(x) \wedge \mu_G(B)(x), \sigma_G(A)(x) \wedge \sigma_G(B)(x), \nu_G(A)(x) \wedge \nu_G(B)(x) \} \]

- Type II:

\[ G(A \cup B) = \{ \mu_G(A)(x) \wedge \mu_G(B)(x), \sigma_G(A)(x) \wedge \sigma_G(B)(x), \nu_G(A)(x) \wedge \nu_G(B)(x) \} \]

- Type III:

\[ G(A \cup B) = \{ \mu_G(A)(x) \wedge \mu_G(B)(x), \sigma_G(A)(x) \wedge \sigma_G(B)(x), \nu_G(A)(x) \wedge \nu_G(B)(x) \} \]
2) \[ G(\cup A_j) = \left\{ \bigvee \mu_{G(A_j)}(x), \bigvee \sigma_{G(A_j)}(x), \bigvee V_{G(A_j)}(x) \right\}. \]

### 3.4 Definition

Let \( f : X \rightarrow Y \) be a mapping.

(i) The image of a neutrosophic set \( A \) generated by \( \mathcal{G}_A \) on \( X \) under \( f \) is a neutrosophic set \( B \) on \( Y \) generated by \( \mathcal{G}_B \), denoted by \( f(A) \), whose reality function \( \lambda_1 \left\{ \mathcal{N}_{\mathcal{G}_A}(\lambda) \land \lambda \right\} \)

\[ \mu_{G(B)} = \sup \lambda_1 \left\{ \mathcal{N}_{\mathcal{G}_A}(\lambda) \land \lambda \right\} \]

\[ \sigma_{G(B)} = \sup \lambda_2 \left\{ \mathcal{N}_{\mathcal{G}_A}(\lambda) \land \lambda \right\} \]

\[ V_{G(B)} = \sup \lambda_3 \left\{ \mathcal{N}_{\mathcal{G}_A}(\lambda) \land \lambda \right\} \]

(ii) The preimage of a neutrosophic set \( B \) on \( Y \) generated by \( \mathcal{G}_B \) under \( f \) is a neutrosophic set \( A \) on \( X \) generated by \( \mathcal{G}_A \), denoted by \( f^{-1}(B) \), whose reality function \( \mu_{f^{-1}(B)}(x) \), whose reality function \( \mathcal{G}_A(x) \), satisfies the property \( G(A) = G(B) \circ f \).

### 3.5 Proposition

Let \( A \) and \( B \) be neutrosophic sets on \( X \) and \( Y \) generated by \( \mathcal{G}_A \) and \( \mathcal{G}_B \), respectively. Then, for a mapping \( f : X \rightarrow Y \), the following properties hold:

(i) If \( A_j \subseteq_{\mathcal{N}_G} A_k \), then \( f(A_j) \subseteq_{\mathcal{N}_G} f(A_k) \).

(ii) If \( B_j \subseteq_{\mathcal{N}_G} B_k \), then \( f^{-1}(B_j) \subseteq_{\mathcal{N}_G} f^{-1}(B_k) \).

(iii) \( f^{-1} \left( \bigcup_{j \in J} B_j \right) = \bigcap_{j \in J} f^{-1}(B_j) \).

### 3.6 Definition

Let \( X = \{ x \in \Omega \} \) be a nonempty set, \( \mathcal{G} \) a family of neutrosophic sets generated by \( \mathcal{G}_A \) and \( \mathcal{G}_B \), respectively. Then for a mapping \( f : X \rightarrow Y \), we have:

(i) \( f \) is injective if and only if \( f^{-1}(A) = \{ x \in X : f(x) \in A \} \).

(ii) \( f \) is surjective if and only if \( \bigcup_{A \in \mathcal{G}} f^{-1}(A) = X \).

### 3.7 Definition

Let \( X = \{ x \in \Omega \} \) be a nonempty set, \( \mathcal{G} \) a family of neutrosophic sets generated by \( \mathcal{G}_A \) and \( \mathcal{G}_B \), respectively. Then for a mapping \( f : X \rightarrow Y \), we have:

(i) \( \mathcal{G}_A \) is the interior of \( \mathcal{G}_B \) if \( f^{-1}(\mathcal{G}_A) \subseteq \mathcal{G}_B \).

(ii) \( \mathcal{G}_B \) is the closure of \( \mathcal{G}_A \) if \( f^{-1}(\mathcal{G}_B) \subseteq \mathcal{G}_A \).

### 3.8 Definition

Let \( X = \{ x \in \Omega \} \) be a nonempty set, \( \mathcal{G} \) a family of neutrosophic sets generated by \( \mathcal{G}_A \) and \( \mathcal{G}_B \), respectively. Then for a mapping \( f : X \rightarrow Y \), we have:

(i) \( \mathcal{G}_A \) is the interior of \( \mathcal{G}_B \) if \( f^{-1}(\mathcal{G}_A) \subseteq \mathcal{G}_B \).

(ii) \( \mathcal{G}_B \) is the closure of \( \mathcal{G}_A \) if \( f^{-1}(\mathcal{G}_B) \subseteq \mathcal{G}_A \).
3.6 **Proposition**. For any neutrosophic set $A$ generated by a NTS $(X, \Psi)$, we have

(i) $\text{cl}_{N^c} A = \text{Ng}_{(\text{int} A)}^{N^c}$

(ii) $\text{Int}_{N^c} A = \text{Ng}_{(\text{cl} A)}^{N^c}$

**References**


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