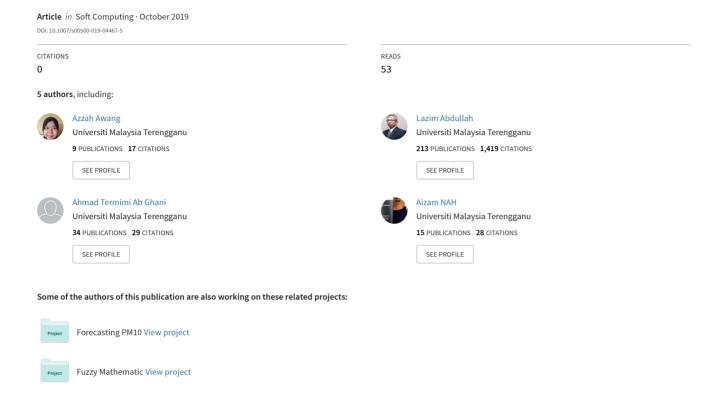
A fusion of decision-making method and neutrosophic linguistic considering multiplicative inverse matrix for coastal erosion problem



METHODOLOGIES AND APPLICATION



A fusion of decision-making method and neutrosophic linguistic considering multiplicative inverse matrix for coastal erosion problem

Azzah Awang¹ · Lazim Abdullah¹ · Ahmad Termimi Ab Ghani¹ · Nur Aidya Hanum Aizam¹ · Mohammad Fadhli Ahmad²

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Abstract

The recent boom of decision-making under uncertain information has attracted many researchers to the field of integrating various types of sets with decision-making methods. In this paper, a combined decision-making trial and evaluation laboratory (DEMATEL) with single-valued neutrosophic sets is proposed to solve the decision problem. This new model combines the advantages of multiplicative inverse of decision matrix in DEMATEL and neutrosophic numbers in linguistic variables, which can find the interrelationship among factors of decision problem. Differently from the typical multiplicative inverse of DEMATEL, which directly used inverse of matrix using real numbers, this method introduces the concept of inverse of matrix using the proposed left–right neutrosophic numbers. This step will enhance the validity of multiplicative inverse of decision matrix in the DEMATEL with neutrosophic numbers. The proposed neutrosophic DEMATEL is also be compared with the DEMATEL and fuzzy DEMATEL. This paper includes a case study that demonstrates the applicability of the neutrosophic DEMATEL in establishing the relationship among influential factors of coastal erosion. Extensive empirical studies using 12 factors of coastal erosion were presented to study the benefits of the proposed method. The result unveils the cause-and-effect relationships among the factors, where seven factors are identified as cause factors and five factors are grouped as effect factors. It is discovered that the factor 'imbalance sediment supply' gives a significant influence to coastal erosion. It is also shown that the degree of importance of the factors is almost consistent with the other two methods despite differences in type of numbers used in defining linguistic variables.

Keywords Decision-making · DEMATEL · Inverse matrix · Neutrosophic set · Coastal erosion

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1 Introduction

The decision-making trial and evaluation laboratory (DEMATEL) method was originally developed by the Science and Human Affairs Program of the Battelle Memorial Institute of Geneva. The DEMATEL approach has been considered as one of the best methods to deal with the degree of importance of evaluation criteria and more importantly to build the cause-effect relationships among the evaluation criteria (Hsu et al. 2013). Recently, many innovations and hybrid methods promoting the DEMATEL method have been explored. For example, the combination of analytic hierarchy process (AHP) and DEMATEL methods can be seen in Lin et al. (2015); Roy et al. (2012) and Sara et al. (2015). The DEMATEL method has also been combined with the analytic networks process (ANP) method (Chen et al. 2011; Liou et al. 2007; Tsai and Hsu 2010). Apart from that, other combinations



DEMATEL-ANP-VIKOR methods (Wang and Tzeng 2012) and fuzzy Delphi-DEMATEL-ANP method (Shen et al. 2011). Although the DEMATEL method alone or combination with other methods has been widely discussed, the DEMATEL method is unable to capture the uncertain elements that exist in the problem since it uses zero to four real numbers in defining linguistic variables during evaluation process. In order to overcome this issue, fuzzy set theory is applied to deal with the ambiguity of human judgments and evaluations. Chang et al. (2011) were the first to apply a fuzzy version of DEMATEL in supplier selection for enterprises. Other applications of fuzzy DEMATEL can be retrieved from Abdullah and Zulkifli (2015), Alam-Tabriz et al. (2014), Cebi (2013), Baykasoğlu et al. (2013) and Karasan et al. (2018). Very recently, Can and Delice (2018) developed an integrated fuzzy set-DEMATEL and fuzzy set-multi-objective optimization on the basis of ratio analysis for shopping mall selection.

The literature shows that fuzzy set theory is very successful in its respective domains. However, it is still unable to deal with any indeterminate and inconsistent information that exists in the real world. Therefore, Smarandache (1999) proposed the concept of neutrosophic set to overcome that issue. The neutrosophic set generalizes the concepts of the classic set, fuzzy set, intuitionistic fuzzy sets, and interval-valued intuitionistic fuzzy sets (Smarandache 1999). The neutrosophic values can be dealt effectively and efficiently with vague, uncertain, and incomplete information. In the neutrosophic set, there are three components in which these three components are called as truth-membership, indeterminacy-membership, and falsity membership. All memberships are assumed to be represented independently. However, since the neutrosophic sets are based on philosophical point of view and its functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets, it is difficult to apply in real scientific and engineering areas. Therefore, Wang et al. (2005, 2010) introduced a single-valued neutrosophic set (SVNS) and an interval neutrosophic set (INS), which are the subclass of a neutrosophic set.

Successful applications of SVNS have been developed in multiple attribute decision-making (MADM) and multiple criteria decision-making (MCDM). Many researchers have shown great interest in the SVNS theory and applied it to the field of decision-making (see Ye 2013, 2014; Peng et al. 2014, 2015; Awang et al. 2019). Liu and Wang (2014) defined a single-valued neutrosophic normalized weighted Bonferroni mean operator. Wang et al. (2018) expressed Maclaurin symmetric mean aggregation operators using single-valued neutrosophic linguistic environments to solve a decision problem. Şahin and Küçük (2015) proposed the concept of neutrosophic subset hood

based on distance measure for SVNSs. Majumdar and Samanta (2014) studied the notions of distance and several similarity measures between two SVNSs as well as entropy of a SVNS. However, little information is available on either the direct fusion of SVNS or through SVNS linguistic variables in the DEMATEL method. Recently, there was one research conducted by Abdel-Basset et al. (2018), in which trapezoidal neutrosophic numbers were hybridized with the DEMATEL and applied to supplier selection problem. However, trapezoidal neutrosophic numbers with four numbers were employed to define three memberships of truth, indeterminacy, and falsity membership of SVNS. Consequently, it undermined the concept of three memberships of SVNS. In response to these issues, we intend to propose a fusion model where linguistic variables defined in three memberships of SVNS and the DEMATEL method are combined. The new fusion uses five linguistic scales in SVNS instead of five linguistic real numbers of the DEMATEL. Nevertheless, our knowledge about how to propose computational procedures of the DEMATEL method based on SVNS is far less clear, especially in finding the multiplicative inverse of decision matrix because the elements in the decision matrix are not written in real numbers. This issue is elaborated further in the subsequent paragraph.

In the fuzzy DEMATEL method where the fuzzy numbers are predominantly used, there is an issue in finding the multiplicative inverse of fuzzy matrix. In the DEMATEL method, the elements of (I-X) matrix are real numbers where I is identity matrix and X is a decision matrix with real number elements. Thus, the multiplicative inverse of (I - X) matrix is easily computed. Contrarily, in fuzzy DEMATEL method, instead of real number elements, the matrix entails triangular fuzzy number elements in which the computation could not be similar as the computation of a matrix with real number elements. This issue has been commented by Pandey and Kumar (2017) in which they claimed that the multiplicative inverse of matrix in Chou et al. (2012) is invalid. The similar invalid computation of multiplicative inverse of fuzzy numbers was found in Cebi (2013), Karasan et al. (2018), Buyukozkan and Cifci (2012), Lin and Wu (2008) and Liu et al. (2014). Pandey and Kumar (2017) provided a comprehensible example and justifications of why Chou et al. (2012) multiplicative inverse of triangular fuzzy matrix was invalid. According to Pandey and Kumar (2017), this problem could be resolved by using the method proposed by Dehghan et al. (2007), where the concept of left inverse and right inverse are introduced. This concept of inverse for fuzzy numbers is supported by a convincing mathematical proof but is yet to be explored for the neutrosophic numbers. In this paper, we intend to implement the concept of left inverse and right inverse of fuzzy matrix in the



context of the multiplicative inverse of neutrosophic matrix.

Responding to these necessities, this paper aims to introduce a new DEMATEL where several new innovations would emerge out of this combination. Firstly, we introduce a linguistic scale with single-valued neutrosophic numbers (SVNNs) in the DEMATEL framework. Secondly, single-valued neutrosophic-weighted averaging aggregation operator is used to aggregate the evaluations provided by decision makers. Thirdly, we propose the multiplicative inverse of identity matrix minus decision matrix with the concept of left inverse and right inverse. Another innovative feature of the proposed DEMATEL is the validity of the multiplicative inverse. A mathematical proof based on the famous definition of the inverse of $n \times n$ matrix $XX^{-1} = I$ is also proposed. To demonstrate the feasibility of the proposed method, we apply it to a coastal erosion problem. The rest of the paper is organized as follows. Section 2 introduces some prerequisite concepts of neutrosophic set and definition of inverse matrix. Section 3 describes the proposed neutrosophic DEMATEL method. In Sect. 4, the state of the art of the proposed method is discussed concisely. Section 5 demonstrates the coastal erosion problem which was adopted as a case study to verify the proposed method. Finally, Sect. 6 provides the concluding remarks.

2 Preliminaries

In this section, the necessary definitions for the proposed approach are presented. The definition of neutrosophic set introduced by Smarandache (1999) is provided as follows.

Definition 1 (Smarandache, 1999) Let X be a space of points (objects) with generic elements in X denoted by x. A neutrosophic set A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, and falsity membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is, $T_A(x) \to]0^-, 1^+[$ and $F_A(x) \to]0^-, 1^+[$. Thus, there is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \le \sup T_A(x) + \sup I_A(x) + \sup I_A(x) \le 3^+$.

Obviously, it is difficult to apply in real scientific and engineering areas. Hence, Wang et al. (2010) introduced the definition of a SVNS.

Definition 2 (Wang et al. 2010) Let X be a space of points (objects) with generic elements in X denoted by x. A SVNS \tilde{A} in X is characterized by truth-membership function $T_{\tilde{A}}(x)$, indeterminacy-membership function $I_{\tilde{A}}(x)$, and falsity membership function $F_{\tilde{A}}(x)$. Then, a SVNS \tilde{A} can be

denoted by $\tilde{A} = \left\{ \left\langle x, T_{\bar{A}}(x), I_{\bar{A}}(x), F_{\bar{A}}(x) \right\rangle x \in X \right\}$, where $T_{\bar{A}}(x), I_{\bar{A}}(x), F_{\bar{A}}(x) \in [0,1]$ for each point x in X. Therefore, the sum of $T_{\bar{A}}(x), I_{\bar{A}}(x)$ and $F_{\bar{A}}(x)$ satisfies the condition $0 \leq T_{\bar{A}}(x) + I_{\bar{A}}(x) + F_{\bar{A}}(x) \leq 3$.

The importance of each decision maker is considered with linguistic variables and expressed in SVNNs. The rating of the decision makers can be computed as the definition given as follows.

Definition 3 (Biswas et al. 2015) Let $E_k = \langle T_k, I_k, F_k \rangle$ be a neutrosophic number defined for the rating of kth decision maker. Then, the weight of the kth decision maker can be written as:

$$\psi_{k} = \frac{1 - \sqrt{\left\{ (1 - T_{k}(x))^{2} + (I_{k}(x))^{2} + (F_{k}(x))^{2} \right\} / 3}}{\sum_{k=1}^{p} \left(1 - \sqrt{\left\{ (1 - T_{k}(x))^{2} + (I_{k}(x))^{2} + (F_{k}(x))^{2} \right\} / 3} \right)}$$
(1)

In any group decision-making, all the individual assessments need to be fused into an aggregated group opinion representing by one aggregated neutrosophic decision matrix. The single-valued neutrosophic-weighted averaging (SVNWA) aggregation operator defined by Sahin and Liu (2015) is used in this paper. The definition is given as follows.

Definition 4 (Sahin and Liu 2015) Let $D^{(k)} = \left(d_{ij}^{(k)}\right)_{m \times n}$ be the single-valued neutrosophic decision matrix of the kth decision maker and $\psi = \left(\psi_1, \psi_2, \ldots, \psi_p\right)^T$ be the weight vector of decision maker such that each $\psi_k \in [0,1]$. Then,

$$d_{ij} = SVNWA_{\psi}\left(d_{ij}^{(1)}, d_{ij}^{(2)}, \dots, d_{ij}^{(p)}\right)$$

$$= \psi_{1}d_{ij}^{(1)} \oplus \psi_{2}d_{ij}^{(2)} \oplus \dots \oplus \psi_{p}d_{ij}^{(p)}$$

$$= \left\langle 1 - \prod_{k=1}^{p} \left(1 - T_{ij}^{(k)}\right)^{\psi_{k}}, \prod_{k=1}^{p} \left(I_{ij}^{(k)}\right)^{\psi_{k}}, \prod_{k=1}^{p} \left(F_{ij}^{(k)}\right)^{\psi_{k}}\right\rangle$$
(2)

Deneutrosophication is the process to obtain crisp numbers from the neutrosophic numbers. It can be obtained by implementing the centroid method as follows:

$$T + [(I - T) + (F - T)]/3 \tag{3}$$

where $\langle T, I, F \rangle$ is an SVNN.

The inverse of $n \times n$ matrix is defined as follows.



Definition 5 (Poole 2006) If X is an $n \times n$ matrix, an inverse of X is also an $n \times n$ matrix X^{-1} with the property that

$$XX^{-1} = I \text{ and } X^{-1}X = I \tag{4}$$

where $I = I_n$ is the $n \times n$ matrix. If such an X^{-1} exists, then X is invertible (Poole, 2006).

To facilitate the multiplicative inverse, the concept of left-right (LR) fuzzy number and operations of fuzzy numbers are defined as follows:

Definition 6 (Dubois and Prade 1978) A fuzzy number \tilde{a} is said to be an LR fuzzy number if

$$\mu_{\tilde{a}}(x) = \begin{cases} L\left(\frac{a-x}{\alpha}\right), & x \le a, \ \alpha > 0\\ R\left(\frac{x-a}{\beta}\right), & x \ge a, \ \beta > 0 \end{cases}$$
 (5)

where a is the mean value of \tilde{a} and α and β are the left and right spreads, respectively. $L(\cdot)$ and $R(\cdot)$ are symmetric and non-increasing on $[0,\infty)$ with respect to the left and right shape functions: L(0)=R(0)=1. Such LR fuzzy number \tilde{a} is symbolically written

$$\tilde{a} = (a, \alpha, \beta)_{LR} \tag{6}$$

The following algebraic operations of addition, scalar multiplication, subtraction, and approximate multiplication and division are presented (Roy et al. 2012).

$$(a, \alpha, \beta)_{IR} \oplus (b, \gamma, \delta)_{IR} = (a + b, \alpha + \gamma, \beta + \delta)_{IR} \tag{7}$$

$$\lambda \otimes (a, \alpha, \beta)_{LR} = \begin{cases} (\lambda a, \lambda \alpha, \lambda \beta)_{LR}, & \lambda > 0\\ (\lambda a, -\lambda \beta, -\lambda \alpha)_{LR}, & \lambda < 0 \end{cases}$$
(8)

$$(a, \alpha, \beta)_{LR} - (b, \gamma, \delta)_{LR} = (a - b, \alpha + \delta, \beta + \gamma)_{LR}$$
 (9)

$$(a, \alpha, \beta)_{IR} \otimes (b, \gamma, \delta)_{IR} \cong (ab, a\gamma + b\alpha, a\delta + b\beta)_{IR}$$
 (10)

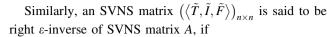
$$(a, \alpha, \beta)_{LR} \div (b, \gamma, \delta)_{LR} \cong \left(\frac{a}{b}, \frac{\delta a + \alpha b}{b^2}, \frac{\gamma a + \beta b}{b^2}\right)_{LR}$$
 (11)

where $\tilde{a} > 0$ and $\tilde{b} > 0$.

The concept of the left–right (LR) fuzzy numbers and ε -inverse of a fuzzy matrix introduced by Dehghan et al. (2007) is implemented to develop a multiplicative inverse of neutrosophic matrix as in Definition 7.

Definition 7 Let ε be a positive crisp number. An SVNS matrix $A = (\langle T, I, F \rangle)_{n \times n}$ is said to be the left ε -inverse of SVNS matrix $\tilde{A} = (\langle \tilde{T}, \tilde{I}, \tilde{F} \rangle)_{n \times n}$, if

$$(\langle T, I, F \rangle)_{n \times n} \otimes (\langle \tilde{T}, \tilde{I}, \tilde{F} \rangle)_{n \times n} = (\langle I, \varepsilon I, \varepsilon I \rangle)_{n \times n}$$
(12)



$$(\langle \tilde{T}, \tilde{I}, \tilde{F} \rangle)_{n \times n} \otimes (\langle T, I, F \rangle)_{n \times n} = (\langle I, \varepsilon I, \varepsilon I \rangle)_{n \times n} \tag{13}$$

To further understand the ε -inverse of SVNS matrix, a related theorem and proof are given in Appendix A. All these definitions, the theorem, and proof are directly used in the proposed method.

3 Proposed method

The procedure of DEMATEL method basically consists of four steps. The steps include the construction of decision matrix, normalizing the direct-relation matrix, acquire the total relation matrix, and draw the cause–effect diagram. In this section, we present an improvement to the DEMATEL where the SVNS is the prominent element in the proposed method. It is structured in three phases, and the systematic flow of the proposed method is illustrated as in Fig. 1.

In the initial phase, direct-relation matrix for individual decision maker is constructed where the elements of this matrix are the evaluation scores that are given in SVNNs. Three main computational procedures occur in aggregation and multiplicative inverse matrix phase. Individual evaluation scores of decision makers are now aggregated to construct aggregated neutrosophic matrix and normalized direct-relation matrix. Furthermore, this matrix is utilized to construct total relation matrix where multiplicative inverse matrix is implemented. In the output phase, a cause–effect diagram is plotted where two groups of factors are identified. Detailed descriptions of the proposed method are given as follows:

Step 1: Identify goal and factors.

Identify the decision goal and find out the influencing factors of the problem.

Step 2: Identify the decision makers.

A group of decision makers is chosen from the experts of the related fields. The importance of each decision maker is expressed using linguistic variables, and the weight of each decision makers can be computed using Eq. (1).

Step 3: Preference evaluation.

A survey form is distributed among decision makers to investigate the interrelationship between factors of decision problem.

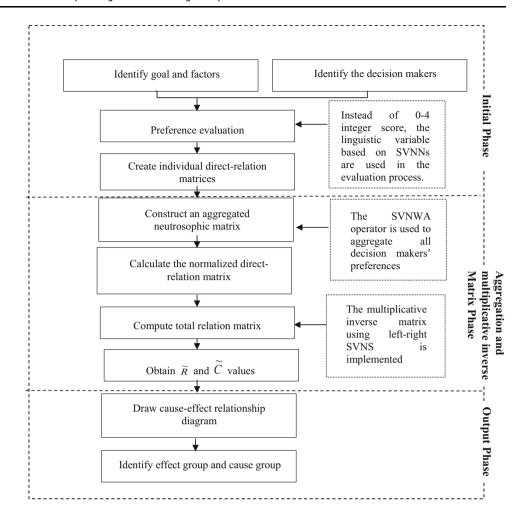
Step 4: Create individual direct-relation matrices.

The direct-relation matrix is constructed based on the individual decision makers' preferences which can be formed as follows:

$$\tilde{X}^k = \left[\tilde{x}_{ij}\right]_{n \times n},$$



Fig. 1 Flow chart of the proposed method



where x_{ij} is the preference made by kth decision maker for ith factor in comparison with the jth factor and n represents the number of factors. \tilde{x}_{ij} is the neutrosophic number represented by three elements, $\langle \tilde{T}, \tilde{I}, \tilde{F} \rangle$.

Step 5: Construct the aggregated neutrosophic matrix.

The SVNWA aggregation operator in Eq. 2 is used to get the aggregated neutrosophic matrix, \tilde{A} .

Step 6: Calculate the normalized direct-relation matrix.

In the DEMATEL procedure, the aggregated direct-relation matrix needs to be normalized. The normalized direct-relation matrix, D is obtained in which all the diagonal elements are zero (Jassbi et al. 2011). The normalized direct-relation matrix can be obtained as follows:

$$D = s \times \tilde{A},\tag{14}$$

where

$$s = \frac{1}{\max_{ij} \left[\max_{1 \le i \le n} \sum_{j=1}^{n} \tilde{T}_{ij}, \max_{1 \le j \le n} \sum_{i=1}^{n} \tilde{T}_{ij}, \right]}$$
(15)

Step 7: Compute the total relation matrix.

The total relation matrix T can be obtained by the following equation:

$$matrix T = D(I - D)^{-1}, (16)$$

where I is the identity matrix.

Step 8: Obtain \tilde{R} and \tilde{C} values.

 \tilde{R} and \tilde{C} values can be obtained by summing up rows and columns of total relation matrix T, respectively. Then, the neutrosophic \tilde{R} and \tilde{C} values are deneutrosophied to get the crisp numbers by Eq. (3).

Step 9: Draw cause–effect relationship diagram.

Draw the cause–effect relationship diagram and obtain the structural model based on the obtained \tilde{R} and \tilde{C} values. The diagram can be acquired by mapping the data set of $(\tilde{R} + \tilde{C}, \tilde{R} - \tilde{C})_{crisp}$

Step 10: Identify effect group and cause group

Based on the cause–effect diagram, we can interpret and discuss the output. The $(\tilde{R}+\tilde{C})_{\rm crisp}$ indicates the importance of each factors, while $(\tilde{R}-\tilde{C})_{\rm crisp}$ values can



categorize the factors into the net cause or effect group. Generally, when the $(\tilde{R}-\tilde{C})_{\rm crisp}$ axis is positive, the factors belong to the cause group. Otherwise, the factors belong to the effect group if the $(\tilde{R}-\tilde{C})_{\rm crisp}$ axis is negative.

The following MCDM problem which is adopted from Ji et al. (2018) is illustrated to show the usefulness of the proposed method.

Example An electronic commerce retailer aims to identify the importance and interrelationship between criteria of third-party provider. Suppose that there are three experts,

 $\{E_1, E_2, E_3\}$, that were invited to rate the importance of four criteria: the cost of service (C_1) , operational experience in the industry (C_2) , customer satisfaction (C_3) , and market reputation (C_4) . Given the weights of experts, $\psi_k = (0.3, 0.25, 0.45)^T$. The evaluation information is represented in the form of SVNSs. Then, according to the proposed MCDM approach, we have three direct-relation matrices given by three experts (see Tables 1, 2, 3).

Aggregated matrix and normalized aggregated matrix are given in Tables 4 and 5.

By using theorem in Appendix A, we can obtain ε -inverse of matrix D. We have,

$$\text{Matrix } (I-D) = \begin{pmatrix} \langle 1,1,1 \rangle & \langle -0.31,-0.07,-0.22 \rangle & \langle -0.33,-0.1,-0.19 \rangle & \langle -0.12,-0.08,-0.09 \rangle \\ \langle -0.27,-0.29,-0.13 \rangle & \langle 1,1,1 \rangle & \langle -0.37,0,-0.29 \rangle & \langle -0.36,0,-0.23 \rangle \\ \langle -0.06,-0.33,-0.2 \rangle & \langle -0.33,-0.09,-0.14 \rangle & \langle 1,1,1 \rangle & \langle -0.34,-0.14,-0.13 \rangle \\ \langle -0.08,-0.29,-0.38 \rangle & \langle -0.17,-0.13,-0.36 \rangle & \langle -0.21,-0.24,-0.21 \rangle & \langle 1,1,1 \rangle \end{pmatrix} .$$

Table 1 Direct-relation matrix evaluated by E_1

Criteria	C_1	C_2	C ₃	C ₄
$\overline{C_1}$	$\langle 0, 0, 0 \rangle$	(0.5, 0.1, 0.4)	(0.5, 0.1, 0.2)	(0.3, 0.2, 0.1)
C_2	$\langle 0.3, 0.3, 0.4 \rangle$	$\langle 0, 0, 0 \rangle$	$\langle 0.7, 0, 0.1 \rangle$	$\langle 0.5, 0.3, 0.2 \rangle$
C_3	$\langle 0.1, 0.7, 0.4 \rangle$	$\langle 0.5, 0.1, 0.3 \rangle$	$\langle 0, 0, 0 \rangle$	$\langle 0.6, 0.2, 0.2 \rangle$
C_4	$\langle 0.2, 0.2, 0.6 \rangle$	$\langle 0.2,0.2,0.5\rangle$	$\langle 0.4, 0.3, 0.2 \rangle$	$\langle 0, 0, 0 \rangle$

Table 2 Direct-relation matrix evaluated by E_2

Criteria	C_1	C_2	C_3	C ₄
C_1	$\langle 0, 0, 0 \rangle$	$\langle 0.6,0.2,0.4 \rangle$	$\langle 0.6, 0.2, 0.3 \rangle$	(0.2, 0.1, 0.1)
C_2	$\langle 0.4,\ 0.7,\ 0.1 \rangle$	$\langle 0, 0, 0 \rangle$	$\langle 0.6, 0.1, 0.3 \rangle$	$\langle 0.6, 0.3, 0.2 \rangle$
C_3	$\langle 0.1, 0.6, 0.6 \rangle$	$\langle 0.6, 0.1, 0.2 \rangle$	$\langle 0, 0, 0 \rangle$	(0.5, 0.3, 0.2)
C_4	$\langle 0.1,0.8,0.6 \rangle$	$\langle 0.3,0.2,0.6\rangle$	$\langle 0.3,0.3,0.2\rangle$	$\langle 0, 0, 0 \rangle$

Table 3 Direct-relation matrix evaluated by E_3

Criteria	C_1	C_2	C ₃	C ₄
$\overline{C_1}$	$\langle 0, 0, 0 \rangle$	(0.4, 0.1, 0.3)	$\langle 0.5, 0.2, 0.4 \rangle$	(0.1, 0.1, 0.2)
C_2	$\langle 0.5, 0.5, 0.2 \rangle$	$\langle 0, 0, 0 \rangle$	$\langle 0.5, 0.1, 0.2 \rangle$	$\langle 0.6, 0, 0.1 \rangle$
C_3	$\langle 0.1, 0.4, 0.2 \rangle$	$\langle 0.5,0.2,0.2\rangle$	$\langle 0, 0, 0 \rangle$	(0.5, 0.2, 0.2)
C_4	$\langle 0.1, 0.6, 0.6 \rangle$	$\langle 0.3, 0.2, 0.6 \rangle$	$\langle 0.3, 0.5, 0.6 \rangle$	$\langle 0, 0, 0 \rangle$

 Table 4 Aggregated

 neutrosophic matrix, \tilde{A}

Criteria	C_1	C_2	C ₃	C ₄
C_1	$\langle 0, 0, 0 \rangle$	(0.49, 0.12, 0.35)	(0.53, 0.16, 0.30)	(0.19, 0.12, 0.14)
C_2	$\langle 0.42, 0.47, 0.21 \rangle$	$\langle 0, 0, 0 \rangle$	$\langle 0.59, 0, 0.18 \rangle$	$\langle 0.57, 0, 0.15 \rangle$
C_3	$\langle 0.1, 0.52, 0.32 \rangle$	$\langle 0.53, 0.14, 0.23 \rangle$	$\langle 0, 0, 0 \rangle$	$\langle 0.53, 0.22, 0.2 \rangle$
C_4	$\langle 0.13,0.46,0.6\rangle$	$\langle 0.27,0.2,0.57\rangle$	$\langle 0.33, 0.38, 0.33\rangle$	$\langle 0, 0, 0 \rangle$



Table 5 Normalized neutrosophic matrix, *D*

Criteria	C_1	C_2	C ₃	C ₄
$\overline{C_1}$	$\langle 0, 0, 0 \rangle$	(0.31, 0.07, 0.22)	(0.33, 0.1, 0.19)	(0.12, 0.08, 0.09)
C_2	$\langle 0.27, 0.29, 0.13 \rangle$	$\langle 0, 0, 0 \rangle$	$\langle 0.37, 0, 0.29 \rangle$	$\langle 0.36, 0, 0.23 \rangle$
C_3	$\langle 0.06,0.33,0.2\rangle$	$\langle 0.33, 0.09, 0.14 \rangle$	$\langle 0, 0, 0 \rangle$	(0.34, 0.14, 0.13)
C_4	$\langle 0.08,0.29,0.38\rangle$	$\langle 0.17,0.13,0.36\rangle$	$\langle 0.21,0.24,0.21\rangle$	$\langle 0, 0, 0 \rangle$

Then, by using

$$\begin{split} \varepsilon &= \left[\max \left\{ \max_{ij} \left\{ \frac{\tilde{F}_{ij}}{\tilde{T}_{ij}} \right\}, \ \max_{ij} \left\{ \frac{\tilde{I}_{ij}}{\tilde{T}_{ij}} \right\} \right\}, \ 1 + \max_{ij} \left\{ \frac{\tilde{I}_{ij}}{\tilde{T}_{ij}} \right\} \right], \\ \varepsilon &= [4.57, 5.24]. \end{split}$$

Using $\varepsilon = 4.57$, the inverse of neutrosophic matrix (I - D) is obtained. The following formula written in Appendix A is used to get the inverse of neutrosophic matrix:

$$(\tilde{T}^{-1}, \tilde{T}^{-1}(\varepsilon I - \tilde{I}\tilde{T}^{-1}), \tilde{T}^{-1}(\varepsilon I - \tilde{F}\tilde{T}^{-1})).$$

Then, we have

 $(\tilde{R} + \tilde{C})_{\text{crisp}}$ values. The criteria of third-party provider can be divided into two groups: the effect group (C1, C2, and C3) and the cause group (C4).

4 State of the art

This section presents the state of the art in method for decision-making and neutrosophic set. The first and important feature of the development of the proposed method begins with the idea of improving the traditional DEMATEL with the latest sets that purposedly dealt with vague and indeterminacy in decision-making. For example,

$$\text{Matrix } (I-D)^{-1} = \begin{pmatrix} \langle 1.35, 4.65, 4.85 \rangle & \langle 0.86, 1.67, 2.63 \rangle & \langle 0.93, 1.79, 2.77 \rangle & \langle 0.78, 1.21, 2.11 \rangle \\ \langle 0.61, 2.09, 2.22 \rangle & \langle 1.72, 4.79, 5.81 \rangle & \langle 1.07, 2.12, 3.32 \rangle & \langle 1.05, 1.83, 3.01 \rangle \\ \langle 0.39, 1.59, 1.6 \rangle & \langle 0.8, 1.89, 2.54 \rangle & \langle 1, 1, 1 \rangle & \langle 0.88, 1.88, 2.51 \rangle \\ \langle 0.3, 1.34, 1.63 \rangle & \langle 0.53, 1.48, 2.31 \rangle & \langle 0.6, 1.72, 2.43 \rangle & \langle 1.43, 4.52, 5.22 \rangle \end{pmatrix}$$

Then, using Eq. (16), the total relation matrix T is obtained as Table 6.

Table 7 shows the obtained \tilde{R} and \tilde{C} values from summing up rows and columns of matrix T.

Based on the information in Table 7, the importance of criteria can be ranked as C4 > C3 > C2 > C1 based on the

Baykasoğlu et al. (2013) integrated fuzzy sets, DEMATEL, and hierarchical TOPSIS in solving truck selection problem of a land transportation company. Recently, Abdullah and Zulkifli (2019) developed a new DEMATEL approach based on the interval type-2 fuzzy set and applied it to identify the causal relationship of knowledge management

Table 6 Total relation matrix, T

Criteria	C ₁	C_2	C ₃	C ₄
C_1	(0.35, 0.41, 0.94)	(0.86, 0.64, 1.97)	(0.93, 0.77, 1.98)	(0.78, 0.68, 1.61)
C_2	$\langle 0.61, 1.35, 1.47 \rangle$	$\langle 0.72, 0.48, 1.61 \rangle$	$\langle 1.07, 0.52, 2.5 \rangle$	$\langle 1.05, 0.35, 2.2 \rangle$
C_3	$\langle 0.39, 1.19, 1.64 \rangle$	$\langle 0.61, 1.02, 1.33 \rangle$	$\langle 0.61, 1.02, 1.33 \rangle$	$\langle 0.88, 1.2, 1.52 \rangle$
C_4	$\langle 0.3, 2, 2.98 \rangle$	$\langle 0.53, 1.56, 3.62\rangle$	$\langle 0.6, 1.95, 3.39\rangle$	$\langle 0.43, 1.04, 2.41 \rangle$

Table 7 The \tilde{R} and \tilde{C} values

	\tilde{R}	$ ilde{C}$	$\left(\tilde{R}+\tilde{C}\right)$	$(\tilde{R}-\tilde{C})$	$\left(ilde{R} + ilde{C} ight)_{crisp}$	$\left(\tilde{R}-\tilde{C}\right)_{crisp}$
C_1	(2.92, 2.5, 6.5)	(1.65, 5.67, 6.88)	⟨4.57, 8.17, 13.38⟩	$\langle 1.27, -3.17, -0.38 \rangle$	8.71	- 0.76
C_2	$\langle 3.45, 2.7, 7.78 \rangle$	$\langle 2.91, 3.87, 8.84 \rangle$	(6.36, 6.57, 16.62)	$\langle 0.54, -1.17, -1.06 \rangle$	9.85	- 0.56
C_3	(2.68, 5.32, 5.98)	$\langle 3.21, 4.26, 9.2 \rangle$	(5.89, 9.58, 15.18)	$\langle -0.53, 1.06, -3.22 \rangle$	10.22	- 0.9
C_4	$\langle 1.86,6.55,12.4\rangle$	$\langle 3.14, 3.27, 7.74 \rangle$	$\langle 5, 9.82, 20.14 \rangle$	$\langle -1.28, 3.28, 4.66 \rangle$	11.65	2.22



criteria. Karasan et al. (2018) proposed an integrated DEMATEL-AHP-TOPSIS based on intuitionistic fuzzy sets and applied it in evaluation of locations for electric vehicles charge stations. Abdel-Basset et al. (2018) developed a new DEMATEL method based on trapezoidal neutrosophic numbers and applied it to identify the importance of supplier selection criteria. The DEMATEL method has been improvised by using the linguistic scales of neutrosophic sets instead of real or fuzzy numbers. In this paper, five SVNS linguistic scales were proposed for the evaluation stage of DEMATEL method. Consequently, SVNS provides an appropriate way to handle the indeterminacy and fuzziness information in human judgment.

The second feature of the proposed method is the weights of the decision makers. Alam-Tabriz et al. (2014) assumed the weights of four decision makers are equal. In an experiment, Shieh et al. (2010) collected data from 19 experts to identify the key success factors of hospital service quality and the weight of each expert is equal. Hsu et al. (2013) took an average score in the aggregation process of experts' judgments which indicates that the weight of each expert is equal. Differently, from the above literature, in this study, the weight of each decision maker is introduced based on their knowledge and experience on the related decision-making problem. Each decision maker is rated based on the SVNS linguistic variables, namely 'no important', 'low important', 'medium important', 'high important', and 'very high important'. In addition, this study utilized the single-valued neutrosophic-weighted averaging (SVNWA) operator to aggregate all the individual direct-relation matrices. Accordingly, the indeterminacy and uncertainty elements are taken care of.

The third feature of this study is the improvement to a multiplicative inverse matrix computation in the DEMA-TEL method. In the traditional DEMATEL method, the computation of inverse matrix can be done using $A \times A^{-1} = I$, where A^{-1} is the inverse of matrix A and I is the identity matrix. These matrices are dealing with real numbers in which inverse matrix can be computed straightforwardly. In case of fuzzy DEMATEL, Chirra and Kumar (2018) employed the direct-relation matrix where fuzzy numbers are defuzzified to the real numbers before continuing with the next steps of computational procedures of DEMATEL method including the inverse computation of real numbers matrix. Chou et al. (2012) computed the multiplicative inverse of (I - X) fuzzy matrix individually by its lower, mean, and upper boundaries of triangular fuzzy numbers matrix. According to Pandey and Kumar (2017), the computation of multiplicative inverse of fuzzy matrix cannot be done separately as the lower, mean, and upper boundaries of triangular fuzzy numbers are dependent on each other. Dehghan et al. (2007) proposed two

notions to get the inverse of fuzzy matrix where the entries are triangular fuzzy numbers. The first notion is called the 'scenario-based', while the second one is called the 'arithmetic-based'. In this paper, the second notion which is the 'arithmetic-based' is implemented to generate a multiplicative inverse of neutrosophic matrix. We assume that the multiplicative inverse of fuzzy matrix and multiplicative inverse of neutrosophic matrix can be computed in the similar manner since the triangular fuzzy number and SVNNs have several common features. This notion is basically rooted from the famous definition inverse of matrix, $A \times A^{-1} = I$. If $\tilde{A} \times \tilde{B} = \tilde{I}$, where \tilde{A} is a neutrosophic matrix, then \tilde{B} is the inverse neutrosophic matrix of \tilde{A} , and \tilde{I} is the identity neutrosophic matrix. In addition, the concept of ε-inverse of fuzzy matrix proposed by Dehghan et al. (2007) is implemented to obtain the multiplicative inverse of neutrosophic matrix. These are among the new innovations that we included in the proposed method. It is hoped that this proposed method would enrich a new knowledge in decision-making and neutrosophic sets.

5 A case of coastal erosion

In this section, we implement the proposed method to the case study of the coastal erosion problem. Ratings of criteria can be expressed using linguistic variables such as very poor (VP), poor (P), good (G), very good (VG), and excellent (EX). Linguistic variables can be transformed into single-valued neutrosophic numbers as given in Table 8.

Step 1: Identify the goal and influencing factors.

The factors of the coastal erosion are adapted from Luo et al. (2013). The final list of factors has been identified and was accepted by a group of experts. There are 12 factors of the coastal erosion as shown in Table 9.

Step 2: Identify group of decision makers

A group of decision makers has been formed to evaluate the interrelationship among factors of the coastal erosion. They are DM1: a coastal engineer at Dr. Nik Association

Table 8 Linguistic variables and single-valued neutrosophic numbers (Biswas et al. 2015)

Linguistic variable	SVNNs		
No influence/not important (NI)	$\langle 0.1,0.8,0.9\rangle$		
Low influence/important (LI)	$\langle 0.35, 0.6, 0.7 \rangle$		
Medium influence/important (MI)	$\langle 0.5, 0.4, 0.45 \rangle$		
High influence/important (HI)	$\langle 0.8, 0.2, 0.15 \rangle$		
Very high influence/important (VHI)	$\langle 0.9,0.1,0.1\rangle$		



Table 9 Factors of the coastal erosion

Factors

C1: Hydrodynamic wave and current

C2: Imbalance sediment supply

C3: Storm surge

C4: Tidal range

C5: Global warming

C6: Bottom beach profile and shoreline instability

C7: Sea level rise

C8: Sand mining activities

C9: Coastal development

C10: Coastal protection

C11: Budgetary revenue

C12: Coastal zone management and policy

Sdn. Bhd., DM2: a professor attached to the Department of Hydraulic and Hydrology, Universiti Teknologi Malaysia and DM3: the Director of National Hydraulic Research Institute of Malaysia. Table 10 presents the importance of decision makers which are expressed in linguistic term and its corresponding SVNNs.

The weight of decision makers in crisp value can be determined using Eq. (1) and the weightage of DM1 in

$$\psi_1 = \frac{1 - \sqrt{\left\{ (1 - 0.9)^2 + (0.1)^2 + (0.1)^2 \right\} / 3}}{\left(3 - \sqrt{0.03/3} - \sqrt{0.1025/3} - \sqrt{0.6125/3} \right)}$$
= 0.398

Similarly, we can obtain the other decision makers' weights by the similar calculation. Thus, the weights of all three decision makers are obtained as follows:

$$\psi = (0.398, 0.36, 0.242)$$

Step 3: Preference evaluation.

Three decision makers who have more than five years of experience in coastal erosion problems were asked to compare the cause–effect relationship between two factors based on their experience and knowledge in the coastal related problem. They were asked to provide their opinion based on the linguistic variable abbreviations given in Table 8.

Step 4: Create the individual direct-relation matrix.

All the individual decision makers' preferences are transferred as the $n \times n$ matrix. For example, the matrix given as follows is the preferences provided by DM1.

crisp value is shown as follows.

The linguistic terms are then transformed to SVNNs (see Table 8).

Table 10 Importance of decision makers with SVNNs

	DM1	DM2	DM3
Linguistic variable	Very high important	High important	Medium important
Weight in SVNN	$\langle 0.9, 0.1, 0.1 \rangle$	$\langle 0.8,0.2,0.15 \rangle$	$\langle 0.5,\ 0.4,\ 0.45\rangle$



Table 11 The aggregated neutrosophic matrix

	C1	C2	C3	C4	C5	C6
C1	$\langle 0, 0, 0 \rangle$	(0.85, 0.15, 0.13)	(0.65, 0.3, 0.29)	(0.71, 0.29, 0.28)	(0.41, 0.54, 0.63)	(0.88, 0.12, 0.11)
C2	$\langle 0.62, 0.42, 0.47 \rangle$	$\langle 0, 0, 0 \rangle$	$\langle 0.34, 0.63, 0.74 \rangle$	$\langle 0.17, 0.76, 0.87 \rangle$	$\langle 0.34, 0.63, 0.74 \rangle$	$\langle 0.87, 0.14, 0.12 \rangle$
C3	$\langle 0.8, 0.2, 0.15 \rangle$	$\langle 0.8, 0.2, 0.15 \rangle$	$\langle 0, 0, 0 \rangle$	$\langle 0.8, 0.2, 0.15 \rangle$	$\langle 0.75, 0.25, 0.24 \rangle$	(0.84, 0.16, 0.13)
C4	$\langle 0.71, 0.29, 0.28 \rangle$	$\langle 0.72, 0.26, 0.22, \rangle$	$\langle 0.88, 0.12, 0.11 \rangle$	$\langle 0, 0, 0 \rangle$	$\langle 0.41, 0.51, 0.59 \rangle$	$\langle 0.9, 0.1, 0.1 \rangle$
C5	$\langle 0.84, 0.16, 0.13 \rangle$	$\langle 0.79, 0.24, 0.26 \rangle$	$\langle 0.87, 0.13, 0.12 \rangle$	$\langle 0.88, 0.12, 0.11 \rangle$	$\langle 0, 0, 0 \rangle$	(0.84, 0.16, 0.13)
C6	$\langle 0.71, 0.26, 0.23 \rangle$	$\langle 0.85, 0.16, 0.13 \rangle$	$\langle 0.5, 0.4, 0.45 \rangle$	$\langle 0.47, 0.46, 0.52 \rangle$	$\langle 0.26, 0.7, 0.81 \rangle$	$\langle 0, 0, 0 \rangle$
C7	$\langle 0.85, 0.16, 0.13 \rangle$	$\langle 0.72, 0.28, 0.27 \rangle$	$\langle 0.79, 0.2, 0.19 \rangle$	$\langle 0.6, 0.36, 0.39 \rangle$	$\langle 0.79, 0.24, 0.26 \rangle$	(0.85, 0.16, 0.13)
C8	$\langle 0.44, 0.49, 0.57 \rangle$	$\langle 0.87, 0.14, 0.12 \rangle$	$\langle 0.8, 0.2, 0.15 \rangle$	$\langle 0.3, 0.66, 0.77 \rangle$	$\langle 0.72, 0.26, 0.22 \rangle$	$\langle 0.85, 0.15, 0.13 \rangle$
C9	(0.85, 0.16, 0.13)	$\langle 0.9, 0.1, 0.1 \rangle$	(0.85, 0.16, 0.13)	$\langle 0.41, 0.54, 0.63 \rangle$	$\langle 0.5, 0.4, 0.45 \rangle$	(0.85, 0.16, 0.13)
C10	$\langle 0.72, 0.28, 0.27 \rangle$	$\langle 0.85, 0.15, 0.13 \rangle$	$\langle 0.34, 0.63, 0.74 \rangle$	$\langle 0.47, 0.46, 0.52 \rangle$	$\langle 0.45, 0.48, 0.56 \rangle$	(0.85, 0.16, 0.13)
C11	$\langle 0.45, 0.48, 0.56 \rangle$	$\langle 0.63, 0.39, 0.44 \rangle$	(0.41, 0.53, 0.62)	$\langle 0.35, 0.6, 0.7 \rangle$	$\langle 0.21, 0.74, 0.84 \rangle$	(0.35, 0.6, 0.7)
C12	$\langle 0.35, 0.6, 0.7 \rangle$	$\langle 0.75, 0.25, 0.24 \rangle$	$\langle 0.21, 0.71, 0.81 \rangle$	$\langle 0.17, 0.75, 0.85 \rangle$	$\langle 0.17, 0.76, 0.87 \rangle$	$\langle 0.35,0.6,0.7\rangle$
	C7	C8	C9	C10	C11	C12
C1	(0.6, 0.36, 0.39)	(0.45, 0.48, 0.56)	(0.39, 0.56, 0.65)	(0.77, 0.28, 0.27)	(0.71, 0.29, 0.28)	(0.71, 0.29, 0.28)
C2	$\langle 0.72, 0.28, 0.27 \rangle$	$\langle 0.31, 0.66, 0.77 \rangle$	$\langle 0.88,0.12,0.11\rangle$	$\langle 0.85, 0.16, 0.13 \rangle$	$\langle 0.85, 0.15, 0.13 \rangle$	$\langle 0.88, 0.12, 0.11 \rangle$
C3	$\langle 0.72, 0.28, 0.27 \rangle$	$\langle 0.38, 0.6, 0.7 \rangle$	$\langle 0.75, 0.25, 0.24 \rangle$	$\langle 0.8, 0.2, 0.15 \rangle$	$\langle 0.64, 0.34, 0.36 \rangle$	$\langle 0.71, 0.29, 0.28 \rangle$
C4	$\langle 0.88, 0.12, 0.11 \rangle$	$\langle 0.72, 0.28, 0.27 \rangle$	$\langle 0.87, 0.14, 0.12 \rangle$	$\langle 0.8, 0.2, 0.15 \rangle$	$\langle 0.79, 0.24, 0.26 \rangle$	$\langle 0.72, 0.28, 0.27 \rangle$
C5	$\langle 0.9, 0.1, 0.1 \rangle$	$\langle 0.34, 0.63, 0.74 \rangle$	$\langle 0.84, 0.16, 0.13 \rangle$	$\langle 0.76, 0.26, 0.26 \rangle$	(0.87, 0.14, 0.12)	$\langle 0.82, 0.22, 0.25 \rangle$
C6	$\langle 0.41, 0.54, 0.63 \rangle$	$\langle 0.79, 0.24, 0.26 \rangle$	$\langle 0.71, 0.29, 0.28 \rangle$	$\langle 0.85, 0.16, 0.13 \rangle$	$\langle 0.8, 0.2, 0.15 \rangle$	$\langle 0.72, 0.26, 0.22 \rangle$
C7	$\langle 0, 0, 0 \rangle$	$\langle 0.72, 0.28, 0.27 \rangle$	$\langle 0.85, 0.15, 0.13 \rangle$	(0.76, 0.26, 0.26)	$\langle 0.82, 0.22, 0.25 \rangle$	(0.8, 0.2, 0.15)
C8	(0.71, 0.26, 0.23)	$\langle 0, 0, 0 \rangle$	(0.76, 0.26, 0.26)	(0.8, 0.2, 0.15)	(0.87, 0.14, 0.12)	(0.85, 0.16, 0.13)
C9	(0.85, 0.16, 0.13)	(0.8, 0.2, 0.15)	$\langle 0, 0, 0 \rangle$	(0.85, 0.16, 0.13)	(0.85, 0.16, 0.13)	(0.63, 0.39, 0.44)
C10	(0.79, 0.24, 0.26)	(0.58, 0.36, 0.39)	(0.8, 0.2, 0.15)	$\langle 0, 0, 0 \rangle$	(0.8, 0.2, 0.15)	(0.47, 0.46, 0.5)
C11	(0.41, 0.53, 0.62)	(0.8, 0.2, 0.15)	(0.76, 0.26, 0.25)	(0.72, 0.28, 0.27)	$\langle 0, 0, 0 \rangle$	(0.85, 0.15, 0.13)
C12	(0.41, 0.53, 0.62)	(0.79, 0.24, 0.26)	(0.85, 0.16, 0.13)	(0.85, 0.16, 0.13)	(0.85, 0.15, 0.13)	$\langle 0, 0, 0 \rangle$

Step 5: Construct an aggregated neutrosophic matrix, \tilde{A} .

The individual direct-relation matrices are aggregated using Eq. (2). For instance, the element a_{12} can be obtained as follows.

$$\begin{split} &\left\langle 1 - \left((0.9)^{0.398} \otimes (0.8)^{0.36} \otimes (0.8)^{0.242} \right), \\ &\left((0.1)^{0.398} \otimes (0.2)^{0.36} \otimes (0.2)^{0.242} \right), \\ &\left((0.1)^{0.398} \otimes (0.15)^{0.36} \otimes (0.15)^{0.242} \right) \right\rangle = \left\langle 0.85, 0.15, 0.13 \right\rangle \end{split}$$

Table 11 shows the aggregated neutrosophic matrix.

Step 6: Calculate the normalized direct-relation matrix.

The normalized direct-relation matrix is obtained using Eqs. (14–15) and illustrated in Table 12.

Step 7: Compute the total relation matrix, T.

Here, the inverse of neutrosophic matrix needs to be computed to get the total relation matrix. By using theorem in Appendix A, the range of ε is obtained which is [7.348, 8.263]. Using ε = 7.348, the inverse of (1 - D) neutrosophic matrix is obtained. Then, the total relation matrix T is computed by Eq. (16). Table 13 shows the total relation matrix.

Step 8: Obtain \tilde{R} and \tilde{C} values

Table 14 shows the \tilde{R} and \tilde{C} values, the importance of the coastal erosion factors $(\tilde{R}+\tilde{C})$ and the cause–effect relationship $(\tilde{R}-\tilde{C})$. The deneutrosophication to crisp numbers of $(\tilde{R}+\tilde{C})$ and $(\tilde{R}-\tilde{C})$ values can be obtained by Eq. (3).

Step 9: Draw cause-effect relationship diagram

The obtained \tilde{R} and \tilde{C} values are used to draw the causal diagram which is plotted using a spread sheet software. The digraph of these 12 factors is depicted in Fig. 2.

Step 10: Identify effect group and cause group



Table 12 The normalized direct-relation matrix, D

	C1	C2	C3	C4	C5	C6
C1	$\langle 0, 0, 0 \rangle$	(0.1, 0.03, 0.02)	(0.07, 0.05, 0.05)	(0.08, 0.05, 0.05)	(0.05, 0.1, 0.1)	(0.1, 0.02, 0.02)
C2	$\langle 0.07, 0.08, 0.08 \rangle$	$\langle 0, 0, 0 \rangle$	$\langle 0.04, 0.11, 0.12 \rangle$	$\langle 0.02, 0.14, 0.14 \rangle$	$\langle 0.04, 0.11, 0.12 \rangle$	$\langle 0.1, 0.02, 0.02 \rangle$
C3	$\langle 0.09, 0.04, 0.02 \rangle$	$\langle 0.09, 0.04, 0.02 \rangle$	$\langle 0, 0, 0 \rangle$	$\langle 0.09, 0.04, 0.02 \rangle$	$\langle 0.08, 0.05, 0.04 \rangle$	(0.09, 0.03, 0.02)
C4	$\langle 0.08, 0.05, 0.05 \rangle$	$\langle 0.08, 0.05, 0.04 \rangle$	$\langle 0.1, 0.02, 0.02 \rangle$	$\langle 0, 0, 0 \rangle$	$\langle 0.05, 0.09, 0.09 \rangle$	$\langle 0.1, 0.02, 0.02 \rangle$
C5	$\langle 0.09, 0.03, 0.02 \rangle$	$\langle 0.09, 0.04, 0.04 \rangle$	$\langle 0.1, 0.02, 0.02 \rangle$	$\langle 0.1, 0.02, 0.02 \rangle$	$\langle 0, 0, 0 \rangle$	(0.09, 0.03, 0.02)
C6	$\langle 0.08, 0.05, 0.04 \rangle$	$\langle 0.1, 0.03, 0.02 \rangle$	$\langle 0.06, 0.07, 0.07 \rangle$	$\langle 0.05, 0.08, 0.08 \rangle$	$\langle 0.03, 0.13, 0.13 \rangle$	$\langle 0, 0, 0 \rangle$
C7	$\langle 0.1, 0.03, 0.02 \rangle$	$\langle 0.08, 0.05, 0.04 \rangle$	$\langle 0.09, 0.03, 0.03 \rangle$	$\langle 0.07, 0.06, 0.06 \rangle$	$\langle 0.09, 0.04, 0.04 \rangle$	(0.09, 0.03, 0.02)
C8	$\langle 0.05, 0.09, 0.09 \rangle$	$\langle 0.1, 0.02, 0.02 \rangle$	$\langle 0.09, 0.04, 0.02 \rangle$	(0.03, 0.12, 0.12)	$\langle 0.08, 0.05, 0.04 \rangle$	(0.09, 0.03, 0.02)
C9	$\langle 0.1, 0.03, 0.02 \rangle$	$\langle 0.1, 0.02, 0.02 \rangle$	$\langle 0.1, 0.03, 0.02 \rangle$	$\langle 0.05, 0.1, 0.1 \rangle$	$\langle 0.06, 0.07, 0.07 \rangle$	(0.09, 0.03, 0.02)
C10	(0.08, 0.05, 0.04)	(0.1, 0.03, 0.02)	(0.04, 0.11, 0.12)	$\langle 0.05, 0.08, 0.08 \rangle$	$\langle 0.05, 0.09, 0.09 \rangle$	(0.09, 0.03, 0.02)
C11	(0.05, 0.09, 0.09)	(0.07, 0.07, 0.07)	(0.05, 0.1, 0.1)	(0.04, 0.11, 0.11)	(0.02, 0.13, 0.14)	(0.04, 0.11, 0.11)
C12	(0.04, 0.11, 0.11)	(0.08, 0.05, 0.04)	(0.02, 0.13, 0.13)	(0.02, 0.13, 0.14)	(0.02, 0.14, 0.14)	(0.04, 0.11, 0.11)
	C7	C8	С9	C10	C11	C12
C1	(0.07, 0.06, 0.06)	(0.05, 0.09, 0.09)	(0.04, 0.1, 0.1)	(0.09, 0.05, 0.04)	(0.08, 0.05, 0.05)	(0.08, 0.05, 0.05)
C2	$\langle 0.08, 0.05, 0.04 \rangle$	(0.03, 0.12, 0.12)	$\langle 0.1, 0.02, 0.02 \rangle$	$\langle 0.1, 0.03, 0.02 \rangle$	$\langle 0.1, 0.03, 0.02 \rangle$	$\langle 0.1, 0.02, 0.02 \rangle$
C3	$\langle 0.08, 0.05, 0.04 \rangle$	$\langle 0.04, 0.11, 0.11 \rangle$	$\langle 0.08, 0.05, 0.04 \rangle$	$\langle 0.09, 0.04, 0.02 \rangle$	$\langle 0.07, 0.06, 0.06 \rangle$	(0.08, 0.05, 0.05)
C4	$\langle 0.1, 0.02, 0.02 \rangle$	$\langle 0.08, 0.05, 0.04 \rangle$	$\langle 0.1, 0.03, 0.02 \rangle$	$\langle 0.09, 0.04, 0.02 \rangle$	$\langle 0.09, 0.04, 0.04 \rangle$	(0.08, 0.05, 0.04)
C5	$\langle 0.1, 0.02, 0.02 \rangle$	(0.04, 0.11, 0.12)	$\langle 0.09, 0.03, 0.02 \rangle$	$\langle 0.09, 0.05, 0.04 \rangle$	$\langle 0.1, 0.02, 0.02 \rangle$	(0.09, 0.04, 0.02)
C6	$\langle 0.05, 0.1, 0.1 \rangle$	$\langle 0.09, 0.04, 0.04 \rangle$	$\langle 0.08, 0.05, 0.05 \rangle$	$\langle 0.1, 0.03, 0.02 \rangle$	$\langle 0.09, 0.04, 0.02 \rangle$	(0.08, 0.05, 0.04)
C7	$\langle 0, 0, 0 \rangle$	$\langle 0.08, 0.05, 0.04 \rangle$	$\langle 0.1, 0.03, 0.02 \rangle$	$\langle 0.09, 0.05, 0.04 \rangle$	$\langle 0.09, 0.04, 0.04 \rangle$	(0.09, 0.04, 0.02)
C8	$\langle 0.08, 0.05, 0.04 \rangle$	$\langle 0, 0, 0 \rangle$	$\langle 0.09, 0.05, 0.04 \rangle$	$\langle 0.09, 0.03, 0.02 \rangle$	$\langle 0.1, 0.02, 0.02 \rangle$	(0.1, 0.03, 0.02)
C9	$\langle 0.1, 0.03, 0.02 \rangle$	$\langle 0.09, 0.04, 0.02 \rangle$	$\langle 0, 0, 0 \rangle$	$\langle 0.1, 0.03, 0.02 \rangle$	$\langle 0.1, 0.03, 0.02 \rangle$	(0.07, 0.07, 0.07)
C10	$\langle 0.09, 0.04, 0.04 \rangle$	$\langle 0.06, 0.06, 0.06 \rangle$	(0.09, 0.04, 0.02)	$\langle 0, 0, 0 \rangle$	(0.09, 0.04, 0.02)	(0.05, 0.08, 0.08)
C11	$\langle 0.05, 0.1, 0.1 \rangle$	(0.09, 0.04, 0.02)	$\langle 0.09, 0.05, 0.04 \rangle$	$\langle 0.08, 0.05, 0.04 \rangle$	$\langle 0, 0, 0 \rangle$	(0.1, 0.03, 0.02)
		(0.09, 0.04, 0.04)	(0.1, 0.03, 0.02)	(0.1, 0.03, 0.02)	(0.1, 0.03, 0.02)	$\langle 0, 0, 0 \rangle$

Based on Table 14, the importance of the 12 factors can be prioritized as C2 > C11 > C12 > C6 > C10 > C1 >C3 > C7 > C8 > C9 > C4 > C5. The imbalance sediment supply is the most important factor with the importance value of 36.57, while the global warming is the least important criterion with the importance value of 23.81. Figure 2 also depicts the categorization of factors in which these factors are categorized into the effect and cause groups. The cause group includes the hydrodynamic wave and current (C1), imbalance sediment supply (C2), storm surge (C3), bottom beach profile and shoreline instability (C6), coastal protection (C10), budgetary revenue (C11) and coastal zone management and policy (C12). The effect group involves the factors of tidal range (C4), global warming (C5), sea level rise (C7), sand mining activities (C8), and coastal development (C9).

Generally, we should pay more attention to the cause factors (C1, C2, C3, C6, C10, C11, and C12) compared to the effect factors (C4, C5, C7, C8, and C9). Among all the factors in the cause group, coastal zone management and

policy (C12) has the highest $(\tilde{R} - \tilde{C})_{crisp}$ value which is 5.69. This indicates that C12 tends to give more impact on the whole system of coastal erosion. Table 15 shows the net cause and net effect groups to the coastal erosion problem.

The comparative analysis is conducted to compare the degree of importance of criteria obtained using the proposed method with the DEMATEL and fuzzy DEMATEL methods. Table 16 shows the results of comparative analysis and its respective type of number used.

Based on Table 16, the degree of importance of factors obtained using the DEMATEL method and fuzzy DEMATEL method is almost consistent. On the other hand, the degree of importance obtained from the proposed method is slightly inconsistent with the other two methods. This is due to the different types of numbers used in defining the linguistic variables and the procedure of multiplicative inverse of the left–right (LR) SVNS approach. The SVNS has an edge in dealing with a problem



Table 13 The total relation matrix, T

	C1	C2	C3	C4	C5	C6
C1	(0.33, 1.45, 1.29)	(0.49, 1.76, 1.53)	(0.36, 1.71, 1.53)	(0.31, 1.54, 1.4)	(0.27, 1.8, 1.71)	(0.47, 1.69, 1.49)
C2	$\langle 0.38, 2.09, 1.95 \rangle$	(0.38, 1.8, 1.62)	$\langle 0.32, 2.27, 2.17 \rangle$	$\langle 0.25, 2.25, 2.16 \rangle$	$\langle 0.26, 2.08, 2.01 \rangle$	(0.45, 1.92, 1.73)
C3	$\langle 0.46, 1.35, 1.05 \rangle$	$\langle 0.53, 1.48, 8.85 \rangle$	$\langle 0.33, 1.09, 0.88 \rangle$	$\langle 0.35, 1.18, 0.93 \rangle$	(0.34, 1.23, 1.01)	$\langle 0.52, 1.4, 1.1 \rangle$
C4	$\langle 0.47, 1.3, 1.08 \rangle$	$\langle 0.55, 1.37, 1.1 \rangle$	$\langle 0.44, 1.06, 0.88 \rangle$	$\langle 0.28, 0.84, 0.71 \rangle$	(0.32, 1.38, 1.25)	$\langle 0.54, 1.17, 0.96 \rangle$
C5	$\langle 0.5, 1.06, 0.87 \rangle$	$\langle 0.57, 1.26, 1.08 \rangle$	$\langle 0.45, 0.99, 0.83 \rangle$	$\langle 0.39, 0.89, 0.75 \rangle$	$\langle 0.28, 0.75, 0.65 \rangle$	$\langle 0.55, 1.15, 0.94 \rangle$
C6	(0.4, 1.76, 1.51)	$\langle 0.48, 2.33, 2.18 \rangle$	$\langle 0.34, 1.84, 1.68 \rangle$	$\langle 0.28, 1.76, 1.64 \rangle$	$\langle 0.25, 1.99, 1.88 \rangle$	(0.37, 1.58, 1.38)
C7	$\langle 0.49, 1.14, 0.91 \rangle$	$\langle 0.48, 1.78, 1.51 \rangle$	$\langle 0.44, 1.13, 0.94 \rangle$	$\langle 0.35, 1.21, 1.06 \rangle$	$\langle 0.36, 1.07, 0.91 \rangle$	(0.54, 1.22, 0.98)
C8	$\langle 0.42, 1.69, 1.49 \rangle$	$\langle 0.53, 1.42, 1.14 \rangle$	$\langle 0.41, 1.32, 1.06 \rangle$	$\langle 0.3, 1.71, 1.57 \rangle$	(0.33, 1.24, 1.01)	$\langle 0.51, 1.42, 1.14 \rangle$
C9	$\langle 0.48, 1.17, 0.95 \rangle$	(0.56, 1.22, 1)	$\langle 0.34, 2.04, 1.92 \rangle$	(0.32, 1.44, 1.33)	(0.32, 1.27, 1.14)	(0.53, 1.25, 1.02)
C10	(0.41, 1.72, 1.51)	(0.49, 1.73, 1.49)	$\langle 0.29, 2.38, 2.29 \rangle$	$\langle 0.29, 1.72, 1.61 \rangle$	(0.28, 1.71, 1.62)	$\langle 0.47, 1.7, 1.47 \rangle$
C11	(0.33, 2.43, 2.31)	(0.4, 2.5, 2.33)	(0.29, 2.38, 2.29)	$\langle 0.24, 2.29, 2.23 \rangle$	(0.22, 2.38, 2.32)	(0.36, 2.7, 2.57)
C12	$\langle 0.29, 2.63, 2.5 \rangle$	(0.38, 2.43, 2.19)	(0.24, 2.65, 2.54)	(0.19, 2.52, 2.43)	$\langle 0.19, 2.48, 2.4 \rangle$	(0.32, 2.77, 2.63)
	C7	C8	C9	C10	C11	C12
C1	(0.4, 1.85, 1.68)	(0.36, 1.91, 1.77)	(0.38, 2.17, 2.01)	(0.47, 1.9, 1.67)	(0.48, 1.91, 1.69)	(0.45, 1.84, 1.63)
C2	(0.39, 1.96, 9.2)	(0.34, 2.26, 2.15)	$\langle 0.36, 1.9, 1.7 \rangle$	$\langle 0.47, 1.98, 1.75 \rangle$	$\langle 0.47, 1.97, 1.75 \rangle$	$\langle 0.45, 1.87, 1.68 \rangle$
C3	$\langle 0.45, 1.46, 1.18 \rangle$	$\langle 0.4, 1.75, 1.56 \rangle$	$\langle 0.43, 1.52, 1.21 \rangle$	$\langle 0.54, 1.49, 1.14 \rangle$	(0.52, 1.64, 1.35)	(0.49, 1.53, 1.24)
C4	$\langle 0.49, 1.13, 0.92 \rangle$	(0.45, 1.24, 1.03)	$\langle 0.45, 1.22, 0.98 \rangle$	$\langle 0.55, 1.31, 1.03 \rangle$	(0.56, 1.36, 1.13)	(0.52, 1.36, 1.12)
C5	$\langle 0.5, 1.01, 0.86 \rangle$	(0.43, 1.55, 1.44)	$\langle 0.47, 1.16, 0.94 \rangle$	$\langle 0.56, 1.28, 1.08 \rangle$	$\langle 0.58, 1.15, 0.95 \rangle$	(0.54, 1.2, 1.04)
C6	$\langle 0.37, 2.06, 1.9 \rangle$	(0.39, 1.65, 1.46)	$\langle 0.37, 1.89, 1.64 \rangle$	$\langle 0.48, 1.79, 1.51 \rangle$	(0.48, 1.83, 1.54)	(0.44, 1.83, 1.56)
C7	$\langle 0.4, 0.98, 0.8 \rangle$	(0.45, 1.23, 1.02)	$\langle 0.45, 1.22, 0.98 \rangle$	(0.55, 1.36, 1.11)	(0.57, 1.32, 1.11)	(0.53, 1.25, 0.98)
C8	$\langle 0.45, 1.45, 1.18 \rangle$	$\langle 0.35, 1.1, 0.9 \rangle$	$\langle 0.42, 1.52, 1.25 \rangle$	$\langle 0.52, 1.49, 1.17 \rangle$	(0.54, 1.42, 1.14)	(0.5, 1.39, 1.11)
C9	$\langle 0.48, 1.19, 0.97 \rangle$	$\langle 0.45, 1.18, 0.94 \rangle$	$\langle 0.45, 1.1, 0.9 \rangle$	$\langle 0.55, 1.29, 1.04 \rangle$	(0.56, 1.29, 1.04)	(0.5, 1.49, 1.29)
C10	(0.42, 1.69, 1.51)	(0.38, 1.74, 1.57)	(0.38, 1.75, 1.49)	(0.4, 1.57, 1.37)	(0.49, 1.78, 1.51)	(0.43, 1.98, 1.8)
C11	(0.33, 2.48, 2.37)	(0.35, 2.01, 1.81)	(0.32, 2.32, 2.12)	(0.41, 2.4, 2.17)	(0.34, 2.09, 1.92)	(0.4, 2.17, 1.97)
C12	(0.29, 2.55, 2.43)	(0.28, 2.12, 1.96)	(0.28, 2.28, 2.05)	(0.38, 2.34, 2.09)	(0.39, 2.33, 2.1)	(0.28, 2.08, 1.9)

Table 14 The \tilde{R} and \tilde{C} values

	\tilde{R}	$ ilde{C}$	$\left(ilde{R}+ ilde{C} ight)$	$\left(ilde{R}- ilde{C} ight)$	$\left(ilde{R} + ilde{C} ight)_{crisp}$	$\left(\tilde{R}-\tilde{C}\right)_{crisp}$
C1	⟨4.78, 21.54, 19.4⟩	(4.96, 19.79, 17.42)	(9.74, 41.33, 36.82)	⟨ − 0.18, 1.75, 1.98⟩	29.30	1.18
C2	(4.5, 24.35, 29.87)	(5.9, 20.12, 24.96)	$\langle 10.4, 44.47, 54.83 \rangle$	$\langle -1.4, 4.23, 4.91 \rangle$	36.57	2.58
C3	(5.36, 17.13, 21.5)	(4.41, 19.6, 17.64)	(9.77, 36.73, 39.15)	$\langle 0.96, -2.47, 3.86 \rangle$	28.55	0.78
C4	(5.61, 14.74, 12.2)	(3.55, 19.35, 17.82)	(9.16, 34.09, 30.02)	$\langle 2.06, -4.62, -5.61 \rangle$	24.42	- 2.72
C5	(5.83, 13.44, 11.42)	(3.42, 19.38, 17.91)	(9.25, 32.83, 29.34)	$\langle 2.4, -5.94, -6.49 \rangle$	23.81	-3.34
C6	(4.67, 21.73, 19.22)	(5.62, 19.97, 17.4)	(10.29, 41.7, 36.62)	$\langle -0.95, 1.76, 1.82 \rangle$	29.54	0.88
C7	(5.69, 14.53, 11.93)	(4.97, 19.81, 25.01)	(10.66, 34.34, 36.94)	$\langle 0.72, -5.94, -6.49 \rangle$	27.31	- 5.88
C8	(5.28, 17.17, 14.16)	(4.62, 19.75, 17.63)	(9.9, 36.92, 31.78)	$\langle 0.65, -2.57, -3.47 \rangle$	26.20	- 1.80
C9	(5.61, 15.04, 12.55)	(4.76, 20.06, 17.28)	(10.37, 36.09, 29.83)	$\langle 0.86, -5.02, -4.73 \rangle$	25.10	-2.97
C10	(4.77, 21.13, 18.88)	(5.88, 20.2, 17.14)	(10.65, 41.34, 36.02)	⟨− 1.11, 0.93, 1.75⟩	29.33	0.52
C11	(3.98, 28.14, 26.41)	(5.97, 20.1, 17.23)	(9.47, 49.28, 44.45)	⟨− 1.99, 8.04, 9.17⟩	34.40	5.08
C12	(3.51, 29.18, 27.22)	(5.52, 19.98, 17.33)	(9.03, 49.16, 44.55)	$\langle -2.01, 9.19, 9.89 \rangle$	34.25	5.69



Fig. 2 Causal and effect diagram

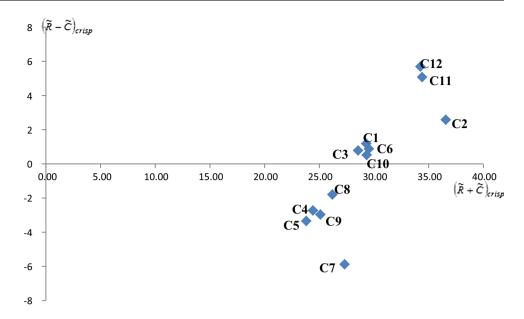


Table 15 Net cause and net effect grouping

Net cause	Net effect	
Hydrodynamic wave and current (C1)	Tidal range (C4)	
Imbalance sediment supply (C2)	Global warming (C5)	
Storm surge (C3)	Sea level rise (C7)	
Bottom beach profile and shoreline instability (C6)	Sand mining activities (C8)	
Coastal protection(C10)	Coastal development (C9)	
Budgetary revenue (C11)		
Coastal zone management and policy (C12)		

Table 16 The results of coastal erosion study using different methods

Evaluation method	Degree of importance	Type of number used
DEMATEL	C9 > C10 > C7 > C2 > C6 > C11 > C8 > C1 > C3 > C12 > C5 > C4	Real number
Fuzzy DEMATEL	C9 > C7 > C10 > C2 > C6 > C11 > C8 > C1 > C3 > C5 > C12 > C4	Triangular fuzzy number
Neutrosophic DEMATEL	C2 > C11 > C12 > C6 > C10 > C1 > C3 > C7 > C8 > C9 > C4 > C5	Single-valued neutrosophic number

characterized by not only uncertainty but also the truth, indeterminacy, and falsity elements.

6 Conclusion

This paper introduced the neutrosophic numbers into the DEMATEL method. This model has several innovations which make it more reliable compared to the existing methods. Firstly, the neutrosophic numbers were applied in the evaluation process where the linguistic variables of SVNNs are used instead of real or fuzzy numbers. Secondly, the SVNWA operator is utilized in aggregating all the preferences given by decision makers instead of normal

averaging operator for real number. Another bold feature of the proposed method is the way the multiplicative inverse of decision matrix is computed. Considering the invalid computation reported in the previous studies, the proposed method utilized the concept of left–right fuzzy numbers with a constant epsilon in finding the multiplicative inverse of the SVNN matrix. Since the inverse of matrix is one of the vital steps in the DEMATEL method, this paper shows the valid method with algebraic proof to compute the multiplicative inverse of neutrosophic matrix. The proposed model has overcome several shortcomings in the existing models.

In this paper, the proposed method has been applied to the coastal erosion problem where the cause-effect



relationship among factors of coastal erosion was established. A set of 12 complex influencing factors has been divided into the cause-and-effect groups. The most important criterion towards coastal erosion is the imbalance sediment supply (C1), but the most influencing criterion is the coastal zone management and policy (C12). The results would be very useful for policy makers to visualize the clusters of factors that influence coastal erosion. The comparative study was also conducted to show the applicability of the proposed method compared to the existing methods that used real and fuzzy numbers. It is unveiled that the degree of importance of the results obtained from the proposed method is slightly different from the DEMATEL and fuzzy DEMATEL methods. However, the results obtained in this study are more reliable since it considers the indeterminacy information of the problem. The proposed approach can be applied in dealing with group decision-making problems (Morente-Molinera et al. 2019), decision support system (Gonzalez-Ferrer et al. 2017), and information fusion system (Fritze et al. 2016). In short, the proposed method is applicable to all decision problems that require segmenting indeterminate factors.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Appendix A

Theorem Consider the nonnegative SVNS matrix $\tilde{A} = (\langle \tilde{T}, \tilde{I}, \tilde{F} \rangle)_{n \times n}$ and let \tilde{A} be the element of the class of all nonnegative matrices with nonnegative inverses. Moreover, suppose that $\tilde{F}_{ii} = 0$ if \tilde{T}_{ij} is zero; then,

$$\left(\tilde{T}^{-1},\tilde{T}^{-1}\left(\varepsilon I-\tilde{I}\tilde{T}^{-1}\right),\tilde{T}^{-1}\left(\varepsilon I-\tilde{F}\tilde{T}^{-1}\right)\right),$$

is the left ε -inverse of \tilde{A} , where $\varepsilon > 0$ is chosen in the following interval,

$$\begin{bmatrix} \max \left\{ \max_{ij} \left\{ \frac{\tilde{F}_{ij}}{\tilde{T}_{ij}} \right\}, & \max_{ij} \left\{ \frac{\tilde{I}_{ij}}{\tilde{T}_{ij}} \right\} \right\}, & 1 + \max_{ij} \left\{ \frac{\tilde{I}_{ij}}{\tilde{T}_{ij}} \right\} \end{bmatrix},$$
where $\tilde{T}_{ij} \neq 0$

This theorem can be proved using the concept of inverse matrix, $\tilde{A} \times \tilde{B} = I$, where \tilde{B} is the inverse of \tilde{A} .

Proof We must prove

$$\begin{array}{l} \left(\tilde{T}, \tilde{I}, \tilde{F}\right) \otimes \left(\tilde{T}^{-1}, \tilde{T}^{-1} \left(\varepsilon I - \tilde{I}\tilde{T}^{-1}\right), \tilde{T}^{-1} \left(\varepsilon I - \tilde{F}\tilde{T}^{-1}\right)\right) \\ = \left(I, \varepsilon I, \varepsilon I\right) \end{array}$$

Based on the above assumption on ε , the SVNS matrix

$$\tilde{B} = (\tilde{T}^{-1}, \tilde{T}^{-1}(\varepsilon I - \tilde{I}\tilde{T}^{-1}), \tilde{T}^{-1}(\varepsilon I - \tilde{F}\tilde{T}^{-1}))$$

is positive. Thus, it can be rewritten as

$$\begin{split} & \left(\tilde{T}\tilde{T}^{-1}, \tilde{T}\tilde{T}^{-1} \left(\varepsilon I - \tilde{I}\tilde{T}^{-1} \right) + \tilde{I}\tilde{T}^{-1}, \tilde{T}\tilde{T}^{-1} \left(\varepsilon I - \tilde{F}\tilde{T}^{-1} \right) + \tilde{F}\tilde{T}^{-1} \right) \\ &= \left(I, \varepsilon I, \varepsilon I \right) \end{split}$$

Or equivalently

$$\begin{cases} \tilde{T}\tilde{T}^{-1} = I \\ \tilde{T}\tilde{T}^{-1}\left(\varepsilon I - \tilde{I}\tilde{T}^{-1}\right) + \tilde{I}\tilde{T}^{-1} = \varepsilon I \\ \tilde{T}\tilde{T}^{-1}\left(\varepsilon I - \tilde{F}\tilde{T}^{-1}\right) + \tilde{F}\tilde{T}^{-1} = \varepsilon I \end{cases}$$

which is a valid system.

References

Abdel-Basset M, Manogaran G, Gamal A, Smarandache F (2018) A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. Des Autom Embed Syst 22(3):257–278

Abdullah L, Zulkifli N (2015) Integration of fuzzy AHP and interval type-2 fuzzy DEMATEL: an application to human resource management. Expert Syst Appl 42(9):4397–4409

Abdullah L, Zulkifli N (2019) A new DEMATEL method based on interval type-2 fuzzy sets for developing causal relationship of knowledge management criteria. Neural Comput Appl 31(8):4095–4111

Alam-Tabriz A, Rajabani N, Farrokh M (2014) An integrated fuzzy DEMATEL-ANP-TOPSIS methodology for supplier selection problem. Glob J Manag Stud Res 1(2):85–99

Awang A, Aizam N, Abdullah L (2019) An integrated decision-making method based on neutrosophic numbers for investigating factors of coastal erosion. Symmetry 11(3):328

Baykasoğlu A, Kaplanoğlu V, Durmuşoğlu Z, Şahin C (2013) Integrating fuzzy DEMATEL and fuzzy hierarchical TOPSIS methods for truck selection. Expert Syst Appl 40(3):899–907

Biswas P, Pramanik S, Giri B (2015) TOPSIS method for multiattribute group decision-making under single-valued neutrosophic environment. Neural Comput Appl 27(3):727–737

Büyüközkan G, Çifçi G (2012) A novel hybrid MCDM approach based on fuzzy DEMATEL, fuzzy ANP and fuzzy TOPSIS to evaluate green suppliers. Expert Syst Appl 39(3):3000–3011

Can GF, Delice KE (2018) A task-based fuzzy integrated MCDM approach for shopping mall selection considering universal design criteria. Soft Comput 22(22):7377–7397

Cebi S (2013) A quality evaluation model for the design quality of online shopping websites. Electron Commer Res Appl 12(2):124–135



- Chang B, Chang C, Wu C (2011) Fuzzy DEMATEL method for developing supplier selection criteria. Expert Syst Appl 38(3):1850–1858
- Chen F, Hsu T, Tzeng G (2011) A balanced scorecard approach to establish a performance evaluation and relationship model for hot spring hotels based on a hybrid MCDM model combining DEMATEL and ANP. Int J Hosp Manag 30(4):908–932
- Chirra S, Kumar D (2018) Evaluation of supply chain flexibility in automobile industry with fuzzy DEMATEL approach. Glob J Flex Syst Manag 19(4):305–319
- Chou Y, Sun C, Yen H (2012) Evaluating the criteria for human resource for science and technology (HRST) based on an integrated fuzzy AHP and fuzzy DEMATEL approach. Appl Soft Comput 12(1):64–71
- Dehghan M, Ghatee M, Hashemi B (2007) Inverse of a fuzzy matrix of fuzzy numbers. Int J Comput Math 86(8):1433–1452
- Dubois D, Prade H (1978) Operations on fuzzy numbers. Int J Syst Sci 9(6):613–626
- Fritze A, Mönks U, Lohweg V (2016) A support system for sensor and information fusion system design. Procedia Technol 26:580–587
- González-Ferrer A, Seara G, Cháfer J, Mayol J (2017) Generating big data sets from knowledge-based decision support systems to pursue value-based healthcare. Int J Interact Multimedia Artif Intell 4(7):42–46
- Hsu CY, Kuo KT, Chen S, Hu A (2013) Using DEMATEL to develop a carbon management model of supplier selection in green supply chain management. J Clean Prod 56:164–172
- Jassbi J, Mohamadnejad F, Nasrollahzadeh H (2011) A fuzzy DEMATEL framework for modeling cause and effect relationships of strategy map. Expert Syst Appl 38(5):5967–5973
- Ji P, Wang JQ, Zhang HY (2018) Frank prioritized Bonferroni mean operator with single-valued neutrosophic sets and its application in selecting third-party logistics providers. Neural Comput Appl 30(3):799–823
- Karaşan A, Kaya İ, Erdoğan M (2018) Location selection of electric vehicles charging stations by using a fuzzy MCDM method: a case study in Turkey. Neural Comput Appl. https://doi.org/10. 1007/s00521-018-3752-2
- Lin C, Wu W (2008) A causal analytical method for group decisionmaking under fuzzy environment. Expert Syst Appl 34(1):205–213
- Lin LK, Den W, Chou Y, Yen H, Lu C (2015) A study on developing the indicators of energy conservation and carbon reduction for the business. In: IEEE international conference on industrial engineering and engineering management, pp 1491–1495
- Liou JJ, Tzeng GH, Chang HC (2007) Airline safety measurement using a hybrid model. J Air Transp Manag 13(4):243–249
- Liu P, Wang Y (2014) Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean. Neural Comput Appl 25(7–8):2001–2010
- Liu H, You J, Lin Q, Li H (2014) Risk assessment in system FMEA combining fuzzy weighted average with fuzzy decision-making trial and evaluation laboratory. Int J Comput Integr Manuf 28(7):701–714
- Luo S, Wang H, Cai F (2013) An integrated risk assessment of coastal erosion based on fuzzy set theory along Fujian coast, southeast China. Ocean Coast Manag 84:68–76
- Majumdar P, Samanta SK (2014) On similarity and entropy of neutrosophic sets. J Intell Fuzzy Syst 26(3):1245–1252
- Morente-Molinera JA, Ríos-Aguilar S, González-Crespo R (2019) Dealing with group decision-making environments that have a

- high amount of alternatives using card-sorting techniques. Expert Syst Appl 127:187–198
- Pandey A, Kumar A (2017) Commentary on "Evaluating the criteria for human resource for science and technology (HRST) based on an integrated fuzzy AHP and fuzzy DEMATEL approach". Appl Soft Comput 51:351–352
- Peng JJ, Wang JQ, Wu X, Wang J, Chen X (2014) Multi-valued neutrosophic sets and power aggregation operators with their applications in multi-criteria group decision-making problems. Int J Comput Intell Syst 8(2):345–363
- Peng JJ, Wang JQ, Wang J, Zhang H, Chen XH (2015) Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. Int J Syst Sci 47(10):2342–2358
- Poole D (2006) Linear algebra: a modern introduction. Thompson Brooks/Cole, Belmont
- Roy B, Misra SK, Gupta P, Neha AG (2012) An integrated DEMATEL and AHP approach for personnel estimation. IJCSITS 2:1206–1212
- Şahin R, Küçük A (2015) Subsethood measure for single valued neutrosophic sets. J Intell Fuzzy Syst 29(2):525–530
- Şahin R, Liu P (2015) Maximizing deviation method for neutrosophic multiple attribute decision making with incomplete weight information. Neural Comput Appl 27(7):2017–2029
- Sara J, Stikkelman RM, Herder PM (2015) Assessing relative importance and mutual influence of barriers for CCS deployment of the ROAD project using AHP and DEMATEL methods. Int J Greenhouse Gas Control 41:336–357
- Shen Y, Lin G, Tzeng G (2011) Combined DEMATEL techniques with novel MCDM for the organic light emitting diode technology selection. Expert Syst Appl 38(3):1468–1481
- Shieh J, Wu H, Huang K (2010) A DEMATEL method in identifying key success factors of hospital service quality. Knowl-Based Syst 23(3):277–282
- Smarandache F (1999) A unifying field in logics: neutrosophy: neutrosophic probability, set and logic. American Research Press, Rehoboth
- Tsai W, Hsu W (2010) A novel hybrid model based on DEMATEL and ANP for selecting cost of quality model development. Total Qual Manag Bus Excell 21(4):439–456
- Wang Y, Tzeng G (2012) Brand marketing for creating brand value based on a MCDM model combining DEMATEL with ANP and VIKOR methods. Expert Syst Appl 39(5):5600–5615
- Wang H, Smarandache FY, Zhang Q, Sunderraman R (2005) Interval neutrosophic sets and logic: theory and applications in computing. Hexis, Phoenix
- Wang H, Smarandache FY, Zhang Q, Sunderraman R (2010) Single valued neutrosophic sets. Multispace Multistruct 4:410–413
- Wang J, Yang Y, Li L (2018) Multi-criteria decision-making method based on single-valued neutrosophic linguistic Maclaurin symmetric mean operators. Neural Comput Appl 30:1529
- Ye J (2013) Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. Int J Gen Syst 42(4):386–394
- Ye J (2014) A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. J Intell Fuzzy Syst 26(5):2459–2466

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