Interval Valued Q-Fuzzy Multiparameterized Soft Set and its Application

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ABSTRACT

Molodtsov’s soft set theory is a newly emerging mathematical tool for handling uncertainty. However, classical multiparameterized soft sets are not appropriate for imprecise and Q-fuzzy parameters. This work aims to extend the classical multiparameterized soft sets to interval-valued Q-fuzzy multiparameterized soft (IVQFMP -soft) sets by giving an importance degree for each element in the multiparameterized sets. The equality, subset, complement, union, intersection, ∧ and ∨-product operations are defined for the IVQFMP-soft sets. Finally, the proposed IVQFMP-soft set is applied to a decision-making problem, and its effectiveness is demonstrated through a numerical example.

Keywords: Approximate function, decision making, membership, multi Q-fuzzy set.
1. Introduction

Most real-life problems in the medical, engineering, management, environmental, and social sciences involve various uncertainties. Thus, the inherent data for these problems is not crisp, precise, or deterministic. Such uncertainties are usually handled using probabilistic techniques, fuzzy sets Zadeh (1965), rough sets Pawlak (1982) and other well-known mathematical tools. However, all of these theories have limitations. To overcome these weaknesses, Molodtsov (1999) proposed a novel approach for modelling vagueness and uncertainty. This so-called soft set theory has potential applications in many different fields, such as in genetics by Varnamkhasti and Hassan (2012, 2013). Ali et al. (2009) introduced some new operations and discussed their basic properties.

Babitha and Sunil (2010) introduced the concept of soft set relation and function and discussed many related concepts. Moreover, Babitha and Sunil (2011) further worked on soft set relation and ordering by introducing the concept of anti-symmetric relation and transitive closure of a soft set relation. Cagman and Enginoglu (2010) developed soft matrix theory and successfully applied it to a decision making problem. Sezgin and Atagun (2011) and Ge and Yang (2011) gave some modifications of the work by Maji et al. (2003) and also established some new results. Singh and Onyeozili (2012) proved that the operations defined on soft sets are equivalent to the corresponding operations defined on their soft matrices.

Although Zhu and Wen (2013) proposed some operations in soft sets, their effort was inadequate for dealing with complex problems. Cagman (2014) added contributions to the theory of soft sets and made modifications to soft set operations that serve as the foundations of further research. Earlier, Majumdar and Samanta (2013) proposed the softness of soft sets. Before that, Maji et al. (2003) studied the theory of fuzzy soft sets. Chen et al. (2005) presented a new definition of soft set parametrization reduction so as to improve the soft set based decision making. Cagman et al. (2010, 2011) further developed a form of parameterized fuzzy soft set theory and its applications. Yang et al. (2009) introduced interval-valued fuzzy soft sets which realize a common extension of both Molodtsov’s soft sets and interval-valued fuzzy sets.

Alkhazaleh et al. (2011) introduced fuzzy parameterized interval-valued fuzzy soft set theory followed by soft intuitionistic fuzzy sets by Alhazaymeh et al. (2012), while Alhazaymeh and Hassan (2012a, 2012b, 2012c) applied vague soft sets to decision making, and further explored interval-valued vague soft set (2013a, 2013c), generalized vague soft expert set (2013b, 2014a, 2014b,
Recently, in order to establish the degree of multi-membership of elements in Q-fuzzy sets, Adam and Hassan (2014a, 2014b, 2014c, 2014d, 2015, 2016) proposed the concept of multi-Q-fuzzy soft set. These concepts will be extended further to a multiparameterized form to use fuzziness advantages to represent descriptions of objects and give an importance degree for each element in the multiparameterized set. Its properties, applications and algorithm will also be studied.

The rest of this paper is organized as follows. The background on multiparameterized soft set and multi-Q-fuzzy sets will initially be reviewed. Then, concepts and operations of interval-valued Q-fuzzy sets are proposed. Next, interval valued Q-fuzzy multiparameterized soft (IVQFMP-soft) set is introduced and the properties of union and intersection, \( \wedge \) and \( \vee \)-product are discussed. The proposed IVQFMP-soft set is later applied to a decision-making problem, and an explicit algorithm is designed, before the remarks of the conclusion of this study.

2. Preliminaries

In this section the basic definitions of multiparameterized soft set theory and multi Q-fuzzy set required as preliminaries are presented.

Let \( U \) be a universe set and \( E_i \) be a set of parameters for all \( i \in I \) such that \( \bigcap_{i \in I} E_i = \emptyset \), \( E = \bigcup_{i \in I} E_i \). Let \( P(U) \) denote the power set of \( U \), \( P(E) \) denotes the power set of \( E \) and \( A \subset P(E) \).

The following three definitions on multiparameterized soft set were proposed by Salleh et al. (2012).

**Definition 2.1.** \((F,A)\) is called a multiparameterized soft set over \( U \) where \( F \) is a mapping given by \( F : A \rightarrow P(U) \).

In other words, a multiparameterized soft set over \( U \) is a parameterized family of subsets of the universe \( U \). For all \( e \in A \), \( F(e) \) may be considered as the set
of e-approximate sets of the multiparameterized soft set \((F, A)\).

**Definition 2.2.** The intersection of two multiparameterized soft sets \((F, A)\) and \((G, B)\) over a common universe set \(U\) is the multiparameterized soft set \((H, C)\), where \(C = A \cap B\), and \(\forall e \in C, H(e) = F(e)\) or \(G(e)\), (as both are in the same set). We write \((F, A) \cap (G, B) = (H, C)\).

**Definition 2.3.** The union of two multiparameterized soft sets of \((F, A)\) and \((G, B)\) over the common universe \(U\) is the multiparameterized soft set \((H, C)\), where \(C = A \cup B\), and \(\forall e \in C, H(e) = \begin{cases} F(e), & \text{if } e \in A - B; \\ G(e), & \text{if } e \in B - A; \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}\) We write \((F, A) \cup (G, B) = (H, C)\).

The following definition on multi Q-fuzzy set was proposed by Adam and Hassan (2014d).

**Definition 2.4.** Let \(I\) be a unit interval \([0, 1]\), \(k\) be a positive integer, \(U\) be a universal set and \(Q\) be a non-empty set. A multi Q-fuzzy set \(\tilde{A}_Q\) in \(U\) and \(Q\) is a set of ordered sequences \(\tilde{A}_Q = \{(u, q), (\mu_1(u, q), \mu_2(u, q), ..., \mu_k(u, q)) : u \in U, q \in Q\}\), where \(\mu_i(u, q) \in I, \text{ for all } i = 1, 2, ..., k\).

The function \((\mu_1(u, q), \mu_2(u, q), ..., \mu_k(u, q))\) is called the membership function of multi Q-fuzzy set \(\tilde{A}_Q\); and \(\mu_1(u, q) + \mu_2(u, q) + ... + \mu_k(u, q) \leq 1\), \(k\) is called the dimension of \(\tilde{A}_Q\). In other words, if the sequences of the membership functions have only \(k\)-terms (finite number of terms) the multi Q-fuzzy set is a function from \(U \times Q\) to \(I^k\) such that for all \((u, q) \in U \times Q\), \(\mu_{\tilde{A}_Q} = (\mu_1(u, q), \mu_2(u, q), ..., \mu_k(u, q))\). The set of all multi Q-fuzzy sets of dimension \(k\) in \(U\) and \(Q\) is denoted by \(M^kQF(U)\).

### 3. Interval Valued Q-Fuzzy Set

In this section the authors define the concept of an interval valued Q-fuzzy set and its basic operations namely subset, equality, complement, union and intersection.
**Definition 3.1.** An interval valued $Q$-fuzzy set on the universe $U$ and non-empty set $Q$ is a mapping such that

$$F_Q : U \times Q \to \text{Int}([0,1])$$

where $\text{Int}([0,1])$ stands for all closed subintervals of $[0,1]$, while the set of all interval valued $Q$-fuzzy sets on $U$ and $Q$ is denoted by $\text{IVQF}(U)$.

Suppose that $F_Q \in \text{IVQF}(U) \quad \forall x \in U, \ q \in Q$, then $\mu_{F_Q}(x, q) = [\mu_{F_Q}^-(x, q), \mu_{F_Q}^+(x, q)]$ is called the degree of membership of an element $(x, q)$ to $F_Q$. $\mu_{F_Q}^-(x, q)$ and $\mu_{F_Q}^+(x, q)$ are referred to as the lower and upper degrees of membership of $(x, q)$ to $F_Q$ where $0 \leq \mu_{F_Q}^-(x, q) \leq \mu_{F_Q}^+(x, q) \leq 1$.

**Example 3.1.** Let $U = \{u_1, u_2, u_3\}$ be a universal set, $Q = \{p, q, r\}$ be a non-empty set. The set $A_Q = \{(u_1, p), [0.1, 0.3]), ((u_1, q), [0.4, 0.7]), ((u_1, r), [0.6, 0.8]), (u_2, p), [0.1, 0.3]), ((u_2, q), [0.2, 0.4]), ((u_2, r), [0.0, 1]), ((u_3, p), [0.1, 0.7]), ((u_3, q), [0.4, 0.6]), ((u_3, r), [0.8])\}$ is an interval valued $Q$-fuzzy set.

**Definition 3.2.** Let $A_Q$ and $B_Q$ be two interval valued $Q$-fuzzy sets,

$A_Q = \{(u, q), [\mu_{A_Q}^-(u, q), \mu_{A_Q}^+(u, q)] : u \in U, \ q \in Q\}$, and

$B_Q = \{(u, q), [\nu_{B_Q}^-(u, q), \nu_{B_Q}^+(u, q)] : u \in U, \ q \in Q\}$. Then we have the following relations and operations for all $u \in U$ and $q \in Q$.

1. $A_Q \subseteq B_Q$ if and only if $\mu_{A_Q}^-(u, q) \leq \nu_{B_Q}^-(u, q)$, and $\mu_{A_Q}^+(u, q) \leq \nu_{B_Q}^+(u, q)$.

2. $A_Q = B_Q$ if and only if $\mu_{A_Q}^-(u, q) = \nu_{B_Q}^-(u, q)$, and $\mu_{A_Q}^+(u, q) = \nu_{B_Q}^+(u, q)$.

3. $A_Q \cup B_Q = \{(u, q), [\sup(\mu_{A_Q}^-(u, q), \nu_{B_Q}^-(u, q)), \sup(\mu_{A_Q}^+(u, q), \nu_{B_Q}^+(u, q))]\}$.

4. $A_Q \cap B_Q = \{(u, q), [\inf(\mu_{A_Q}^-(u, q), \nu_{B_Q}^-(u, q)), \inf(\mu_{A_Q}^+(u, q), \nu_{B_Q}^+(u, q))]\}$.

**Example 3.2.** Let $U = \{u_1, u_2, u_3\}$ be a universal set, $Q = \{p, q, r\}$ be a non-empty set. Suppose $A_Q = \{(u_1, p), [0.2, 0.3]), ((u_1, q), [0.2, 0.5]), ((u_1, r), [0.3, 0.5]), (u_2, p), [0.1, 0.3]), (u_2, q), (u_2, r), [0.0, 1]), ((u_3, p), [0.1, 0.6]), ((u_3, q), [0.1, 0.4]), ((u_3, r), [0.4])\}$ and $B_Q = \{(u_1, p), [0.3, 0.5]), ((u_1, q), [0.3, 0.7]), ((u_1, r), [0.4, 0.6]), ((u_2, p), [0.1, 0.4]), ((u_2, q), [0.4, 0.5]), ((u_2, r), [0.1, 0.3]), ((u_3, p), [0.2, 0.7]), ((u_3, q), [0.2, 0.6]), ((u_3, r), [0.1, 0.6])\}$ are interval valued $Q$-fuzzy sets, then
Definition 4.1. Let $A_Q$ be an interval valued $Q$-fuzzy set. The complement of interval valued $Q$-fuzzy set $A_Q$ is an interval-valued $Q$-fuzzy set denoted by $C(A_Q)$ and defined by

$$C(A_Q) = \{(u,q) : 1 - \mu_{\tilde{A}_Q}(u,q), 1 - \mu_{\tilde{A}_Q}(u,q) : u \in U, q \in Q\}.$$  

Example 3.3. From Example 3.1. the complement of interval valued $Q$-fuzzy set $A_Q$ is $C(A_Q) = \{((u_1,p),[0.7,0.9]),((u_1,q),[0.3,0.6]),((u_1,r),[0.2,0.4]),((u_2,p),[0.7,0.9]),((u_2,q),[0.6,0.8]),((u_2,r),[0.9,1]),((u_3,p),[0.3,0.9]),((u_3,q),[0.4,0.6]),((u_3,r),[0.2,1])\}.$

4. Interval Valued Q-Fuzzy Multiparameterized Soft Set

In this section the authors introduce the concept of an interval valued Q-fuzzy multiparameterized soft set and define some operations on an interval-valued Q-fuzzy multiparameterized soft set, namely subset, equality, null, absolute, complement, union, intersection, $\wedge$ and $\vee$-product.

Definition 4.1. The interval-valued $Q$-fuzzy multiparameterized soft set (IVQFMP-soft set) $F_{Qx}$ on the universe $U$ is defined by the set of ordered pairs

$$F_{Qx} = \{((x,q),\mu_{\tilde{Q}_x}(x,q),\mu_{\tilde{Q}_x}(x,q)), f_{Qx}(x)) : x \in A, f_{Qx} \in P(U)\}$$

where the function $f_{Qx} : A \rightarrow P(U)$ is called the approximate function such that $f_{Qx}(x) = \emptyset$, if $[\mu_{\tilde{Q}_x}(x,q),\mu_{\tilde{Q}_x}(x,q)] = \emptyset$, and the function $\mu_{\tilde{Q}_x}(x,q),\mu_{\tilde{Q}_x}(x,q) : A \times Q \rightarrow \text{int}([0,1])$ is called the membership function of IVQFMP-soft set $F_{Qx}$. The value of $[\mu_{\tilde{Q}_x}(x,q),\mu_{\tilde{Q}_x}(x,q)]$ is the degree of importance of the parameter $x$, and depends on the decision maker’s requirements.

Note that from now on the sets of all IVQFMP-soft sets over $U$ will be denoted by $\text{IVQFMP}_{Qx}(U)$. 

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Let us consider a multiparameterized soft set which describes the attractiveness of houses that a person is considering to purchase. Suppose that there are seven houses in the universe \( U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\} \), \( Q = \{q, r\} \) be a set of cities under consideration and that \( E_i = \{E_1, E_2\} \) is a set of decision parameters. Let \( E_1 \) be a set of cost parameters given by \( E_1 = \{e_{1,1} = \text{expensive}, e_{1,2} = \text{cheap}, e_{1,3} = \text{very expensive}\} \), \( E_2 \) is a set of location parameters given by \( E_2 = \{e_{2,1} = \text{Piazza}, e_{2,2} = \text{Valencia}, e_{2,3} = \text{Atrium}\} \), let \( E = \bigcup_i E_i \) and \( A \subset P(E) \) such that \( A = \{x_1 = \{e_{1,1}, e_{2,1}\}, x_2 = \{e_{1,2}, e_{2,2}\}, x_3 = \{e_{1,3}, e_{2,3}\}\} \).

Let \( X = \{(x_1, q), [0.1, 0.4]), ((x_1, r), [0.3, 0.7]), ((x_2, q), [0.3, 0.6]), ((x_2, r), [0.2, 0.8]), ((x_3, q), [0.6, 0.7])\} \), and \( f_{Q^X}(x_1) = \{u_1, u_3, u_4\}, f_{Q^X}(x_2) = \{u_2, u_5\}, \text{ and } f_{Q^X}(x_3) = \{u_6, u_7\} \), then \( F_{Q^X} = \{((x_1, q), [0.1, 0.4]), ((x_1, r), [0.3, 0.7]), (u_1, u_3, u_4)\}, \{((x_2, q), [0.3, 0.6]), ((x_2, r), [0.1, 0.5]), (u_2, u_5)\}, \{((x_3, q), [0.2, 0.8]), ((x_3, r), [0.6, 0.7]), (u_6, u_7)\}\} \).

Thus an interval valued Q-fuzzy multiparameterized soft set which gives us a collection of approximate descriptions of an object is as follows.

The houses \( u_1, u_3 \) and \( u_4 \) are at Piazza and expensive with membership \([0.1, 0.4]\) and \([0.3, 0.7]\) in cities \( q \) and \( r \) respectively, while the houses \( u_2 \) and \( u_5 \) are at Valencia and cheap with membership \([0.3, 0.6]\) and \([0.1, 0.5]\) in cities \( q \) and \( r \) respectively. The houses \( u_6 \) and \( u_7 \) are at Atrium and very expensive with membership \([0.2, 0.8]\) and \([0.3, 0.7]\) in cities \( q \) and \( r \) respectively.

In the following, we introduce the definitions of the empty IVQFMP-soft set and the absolute IVQFMP-soft set.

**Definition 4.2.** Let \( F_{Q^X} \in IVQFMP\{U\} \). If \( f_{Q^X}(x) = \emptyset \), for all \( x \in A \), then \( F_{Q^X} \) is called an empty IVQFMP-soft set denoted by \( F_{Q^\emptyset} \).

**Definition 4.3.** Let \( F_{Q^X} \in IVQFMP\{U\} \). If \( f_{Q^X}(x) = U \), for all \( x \in A \), then \( F_{Q^X} \) is called an absolute IVQFMP-soft set, denoted by \( F_{Q^U} \).

The following are examples related to the empty IVQFMP-soft set and the absolute IVQFMP-soft set.

**Example 4.2.** Consider \( U = \{u_1, u_2, u_3, u_4, u_5, u_6\} \) as a universal set, \( Q = \{q, p\} \) be a non-empty set, \( E_i = \{E_1, E_2, E_3\} \) be a set of decision parameters,
such that $E_1 = \{x_{1,1}, x_{1,2}, x_{1,3}\}$, $E_2 = \{x_{2,1}, x_{2,2}, x_{2,3}\}$, $E_3 = \{x_{3,1}, x_{3,2}, x_{3,3}\}$, $A \subset P(E)$ such that $A = \{x_1 = \{x_{1,1}, x_{1,2}, x_{1,3}\}, x_2 = \{x_{2,1}, x_{2,2}, x_{1,3}\}, x_3 = \{x_{2,2}, x_{3,2}, x_{3,3}\}\}$. Let $X = \{((x_1, q), [0, 1, 0, 3]), ((x_1, p), [0, 2, 0, 8]), ((x_2, q), [0, 0, 1]), ((x_2, p), [0, 3, 0, 6]), ((x_3, q), [0, 2, 0, 8]), ((x_3, p), [0, 4, 0, 5])\}$. If $f_{Q_x}(x_1) = \emptyset$, $f_{Q_x}(x_2) = \emptyset$, and $F_{Q_x} = \{(((x_1, q), [0, 1, 0, 3]), ((x_1, p), [0, 2, 0, 8]), \emptyset), ((x_2, q), [0, 0, 1]), ((x_2, p), [0, 3, 0, 6]), \emptyset), ((x_3, q), [0, 2, 0, 8]), ((x_3, p), [0, 4, 0, 5])\}$, then $F_{Q_x} = F_{Q_x}$.

Definition 4.4. Let $F_{Q_x} \in IVQFMP(U)$. The complement of $F_{Q_x}$, denoted by $F_{Q_x}$, is an IVQFMP-soft set defined by the approximate and membership functions as

$$\mu_{Q_x}(x, q) = [1 - \mu_{Q_x}^+(x, q), 1 - \mu_{Q_x}^-(x, q)]$$

and $f_{Q_x}(x) = U - f_{Q_x}(x)$.

Proposition 4.1. Let $F_{Q_X} \in IVQFMP(U)$. Thus

1. $(F_{Q_X}^c)^c = F_{Q_X}$.
2. $F_{Q_X}^c = F_{Q_X}$.

Proof. By using the approximate and membership functions of the IVQFMP-soft sets, the proof is straightforward. □

In the following, the concept of the subset of two IVQFMP-soft sets is introduced.

Definition 4.5. Let $F_{Q_X}, F_{Q_Y} \in IVQFMP(U)$. $F_{Q_X}$ is a IVQFMP-soft subset of $F_{Q_Y}$, denoted by $F_{Q_X} \subseteq F_{Q_Y}$, if $\mu_{Q_X}(x, q) \leq \mu_{Q_Y}(x, q)$, and $\mu_{Q_X}^+(x, q) \leq \mu_{Q_Y}^+(x, q)$, and $f_{Q_X}(x) \subseteq f_{Q_Y}(x)$ for all $x \in A$ and $q \in Q$.

In the following, the proposition on the IVQFMP-soft subset is provided.

Proposition 4.2. Let $F_{Q_X}, F_{Q_Y} \in IVQFMP(U)$. Therefore

1. $F_{Q_X} \subseteq F_{Q_Y}$.
2. $F_{Q_X} \subseteq F_{Q_Y}$.
3. If $F_{Q_X} \subseteq F_{Q_Y}$ and $F_{Q_Y} \subseteq F_{Q_H}$, then $F_{Q_X} \subseteq F_{Q_H}$.
Proof. These can be proven easily by using the approximate and membership functions of the IVQFMP-soft sets.

In the following, the equality of two IVQFMP-soft sets is introduced.

**Definition 4.6.** Let $F_{Qx}, F_{Qy} \in IVQFPS(U)$. $F_{Qx}$ and $F_{Qy}$ are equal, written as $F_{Qx} = F_{Qy}$, if and only if $f_{Qx}(x) = f_{Qy}(x)$ and $\mu_{Qx}^-(x, q) = \mu_{Qy}^-(x, q)$, and $\mu_{Qx}^+(x, q) = \mu_{Qy}^+(x, q)$, for all $x \in A$, and $q \in Q$.

5. Operations on interval valued Q-fuzzy multiparameterized soft set

In this section the authors introduce the union, intersection, $\land$ and $\lor$-product of interval valued Q-fuzzy multiparameterized soft set.

**Definition 5.1.** Let $F_{Qx}, F_{Qy} \in IVQFPS(U)$. The union of two IVQFMP-soft sets $F_{Qx}$ and $F_{Qy}$ is the IVQFMP-soft set denoted by $F_{Qx} \cup F_{Qy}$ and defined by $\mu_{Qx} \cup \mu_{Qy} = \sup(\mu_{Qx}^-(x, q), \mu_{Qy}^-(x, q))$, $\mu_{Qx} \cap \mu_{Qy} = \inf(\mu_{Qx}^+(x, q), \mu_{Qy}^+(x, q))$, and $f_{Qx} \cup f_{Qy} = f_{Qx}(x) \lor f_{Qy}(x)$ for all $x \in A$, and $q \in Q$.

An example and a proposition on the union of two IVQFMP-soft sets are as follows.

**Example 5.1.** Consider $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ as a universal set, $Q = \{q, p\}$ be a non-empty set, $E_i = \{E_1, E_2, E_3\}$ be a set of decision parameters, $E_1 = \{x_{1.1}, x_{1.2}, x_{1.3}\}$, $E_2 = \{x_{2.1}, x_{2.2}, x_{2.3}\}$, $E_3 = \{x_{3.1}, x_{3.2}, x_{3.3}\}$, $A \subset P(E)$ such that $A = \{x_1 = \{x_{1.1}, x_{1.2}, x_{1.3}\}, x_2 = \{x_{1.2}, x_{2.1}, x_{1.3}\}, x_3 = \{x_{1.3}, x_{2.3}, x_{3.3}\}\}$.

Let $X = \{((x_1, q), [0.2, 0.8]), ((x_1, p), [0.1, 0.2]), ((x_2, q), [0.4, 0.6]), ((x_2, p), [0.3, 0.5]), ((x_3, q), [0.5, 0.5]), ((x_3, p), [0, 0.4])\}$. $f_{Qx}(x_1) = \{u_1, u_3, u_5\}$, $f_{Qx}(x_2) = \{u_1, u_6, u_7\}$, $f_{Qx}(x_3) = \{u_4, u_2, u_4, u_5\}$, and $Y = \{((x_1, q), [0.3, 0.7]), ((x_1, p), [0.4, 0.5]), ((x_2, q), [0.5, 0.7]), ((x_2, p), [0.3, 0.6]), ((x_3, q), [0.1]), ((x_3, p), [0.1, 0.9])\}$. $f_{Qy}(x_1) = \{u_1, u_2, u_7\}$, $f_{Qy}(x_2) = \{u_3, u_4, u_7\}$, $f_{Qy}(x_3) = \{u_4, u_8\}$.

Suppose $F_{Qx} = \{((x_1, q), [0.2, 0.8]), ((x_1, p), [0.1, 0.2]), \{u_1, u_3, u_5\}\}, ((x_2, q), [0.4, 0.6]), ((x_2, p), [0.3, 0.5]), \{u_1, u_6, u_7\}\}, ((x_3, q), [0.5, 0.5]), ((x_3, p), [0, 0.4]), \{u_1, u_2, u_4, u_5\}\}$, and $F_{Qy} = \{((x_1, q), [0.3, 0.7]), ((x_1, p), [0.4, 0.5]), \{u_1, u_2, u_7\}\}, ((x_2, q), [0.5, 0.7]), ((x_2, p), [0.3, 0.6]), \{u_3, u_4, u_7\}\}, ((x_3, q), [0.1]), ((x_3, p), [0.1, 0.9]), \{u_4, u_8\}\}$.
Then $F_{Q_1} \cup F_{Q_2} = \{(((x_1, q), [0,3, 0.8]), ((x_1, p), [0.4, 0.5]), \{u_1, u_2, u_3, u_5, u_7\}),
((x_2, q), [0.5, 0.7]), ((x_2, p), [0.3, 0.6]), \{u_1, u_3, u_4, u_6, u_7\}), ((x_3, q), [0.5, 1]), ((x_3, p),
[0.1, 0.9], \{u_1, u_2, u_4, u_5, u_6\})\}.$

**Proposition 5.1.** Let $F_{Q_1}, F_{Q_2}, F_{Q_3} \in IVQFMPS(U)$. Therefore

1. $F_{Q_1} \cap F_{Q_2} = F_{Q_1}.$
2. $F_{Q_1} \cap F_{Q_3} = F_{Q_2}.$
3. $F_{Q_2} \cap F_{Q_3} = F_{Q_3}.$
4. $(F_{Q_1} \cap F_{Q_2}) \cap F_{Q_3} = F_{Q_1} \cap (F_{Q_2} \cap F_{Q_3}).$

**Proof.** The proof can be easily obtained from Definition 5.1.\qed

**Definition 5.2.** Let $F_{Q_1}, F_{Q_2} \in IVQFMPS(U)$. The intersection of two
IVQFMP-soft sets $F_{Q_1}$ and $F_{Q_2}$, is the IVQFMP-soft set denoted by $F_{Q_1} \cap F_{Q_2}$
and defined by $\mu_{Q_1} \cap Q_2 = \inf(\mu_{Q_1}(x, q) \cdot \mu_{Q_2}(x, q)), \text{inf}(\mu_{Q_1}(x, q) \cdot \mu_{Q_2}(x, q)),$
and $f_{Q_1} \cap Q_2 = f_{Q_1}(x) \cap f_{Q_2}(x)$ for all $x \in A$.

**Example 5.2.** From Example 5.1, suppose $F_{Q_1} = \{(((x_1, q), [0,2, 0.8]), ((x_1, p),
[0.1, 0.2]), \{u_1, u_3, u_5\}), ((x_2, q), [0.4, 0.6]), ((x_2, p), [0.3, 0.5]), \{u_1, u_6, u_7\}), ((x_3, q),
[0.5, 0.5]), ((x_3, p), [0,0.4]), \{u_1, u_2, u_4, u_5\})\}, and $F_{Q_2} = \{(((x_1, q), [0,3, 0.7]), ((x_1, p),
[0.4, 0.5]), \{u_1, u_2, u_7\}), ((x_2, q), [0.5, 0.7]), ((x_2, p), [0.3, 0.6]), \{u_3, u_4, u_7\}), ((x_3, q), [0,1]), ((x_3, p), [0.1, 0.9]), \{u_4, u_8\})\}.$

Then $F_{Q_1} \cap F_{Q_2} = \{(((x_1, q), [0,2, 0.7]), ((x_1, p), [0.1, 0.2]), \{u_1\}), ((x_2, q),
[0.4, 0.6]), ((x_2, p), [0.3, 0.5]), \{u_7\}), ((x_3, q), [0,0.5]), ((x_3, p), [0.0, 0.4]), \{u_4\})\}.$

Now we give some propositions on the union and intersection of IVQFMP-
soft sets.

**Proposition 5.2.** Let $F_{Q_1}, F_{Q_2}, F_{Q_3} \in IVQFMPS(U)$. Therefore

1. $F_{Q_1} \cap F_{Q_2} = F_{Q_1}.$
2. $F_{Q_1} \cap F_{Q_3} = F_{Q_2}.$
3. $F_{Q_2} \cap F_{Q_3} = F_{Q_3}.$
4. $(F_{Q_1} \cap F_{Q_2}) \cap F_{Q_3} = F_{Q_1} \cap (F_{Q_2} \cap F_{Q_3}).$

**Proof.** Result follows trivially from Definition 5.2.\qed
Proposition 5.3. Let $F_{Q_X}, F_{Q_Y}, F_{Q_H} \in IVQFMPS(U)$. Therefore

1. $(F_{Q_X} \cap (F_{Q_Y} \cup F_{Q_H})) = (F_{Q_X} \cap F_{Q_Y}) \cup (F_{Q_X} \cap F_{Q_H})$.
2. $(F_{Q_X} \cup (F_{Q_Y} \cap F_{Q_H})) = (F_{Q_X} \cup F_{Q_Y}) \cap (F_{Q_X} \cup F_{Q_H})$.

Proof. Let $[\mu_{Q_X}^{-1}(x, q), \mu_{Q_X}^+(x, q)] = \mu_{Q_X}(x, q)$, $[\mu_{Q_Y}^-(x, q), \mu_{Q_Y}^+(x, q)] = \mu_{Q_Y}(x, q)$, and $[\mu_{Q_H}^{-1}(x, q), \mu_{Q_H}^+(x, q)] = \mu_{Q_H}(x, q)$, for all $x \in A$ and $q \in Q$.

1. Assume that $\mu_{Q_X}(x, q), \mu_{Q_Y}(x, q)$ and $\mu_{Q_H}(x, q)$ are elements of $Q_X, Q_Y$ and $Q_H$ respectively, then we have

\[\mu_{Q_X}(x, q) = \mu_{Q_X}(x, q) \cap (\mu_{Q_Y}(x, q) \cup \mu_{Q_H}(x, q)),\]
\[= \inf(\mu_{Q_X}(x, q), \sup(\mu_{Q_Y}(x, q), \mu_{Q_H}(x, q))),\]
\[= \sup(\inf(\mu_{Q_X}(x, q), \mu_{Q_Y}(x, q)), \inf(\mu_{Q_X}(x, q), \mu_{Q_H}(x, q))),\]
\[= \mu_{Q_X \cap (Q_Y \cup Q_H)}(x, q).\]

This implies $f((Q_X \cap (Q_Y \cup Q_H))(x) = f_{Q_X}(x) \cap (f_{Q_Y}(x) \cup f_{Q_H}(x))$, therefore
\[= (f_{Q_X}(x) \cap f_{Q_Y}(x)) \cup (f_{Q_H}(x) \cap f_{Q_H}(x)).\]

Thus $(F_{Q_X} \cup (F_{Q_Y} \cap F_{Q_H})) = (F_{Q_X} \cap F_{Q_Y}) \cap (F_{Q_X} \cap F_{Q_H})$.

Assertion 2 can be proven in a similar fashion. \hfill \Box

Proposition 5.4. (De Morgan’s Law). Let $F_{Q_X}, F_{Q_Y}, F_{Q_H} \in IVQFMPS(U)$. Therefore

1. $(F_{Q_X} \cup F_{Q_Y})^c = F_{Q_X}^c \cup F_{Q_Y}^c$.
2. $(F_{Q_X} \cap F_{Q_Y})^c = F_{Q_X}^c \cap F_{Q_Y}^c$.

Proof.

Let $[\mu_{Q_X}^{-1}(x, q), \mu_{Q_X}^+(x, q)] = \mu_{Q_X}(x, q)$, $[\mu_{Q_Y}^-(x, q), \mu_{Q_Y}^+(x, q)] = \mu_{Q_Y}(x, q)$, and $[\mu_{Q_H}^{-1}(x, q), \mu_{Q_H}^+(x, q)] = \mu_{Q_H}(x, q)$, for all $x \in A$ and $q \in Q$.

1. By Definitions 5.1 and 5.2 we have

\[(\mu_{Q_X}(x, q) \cup \mu_{Q_Y}(x, q))^c = (\sup(\mu_{Q_X}(x, q), \mu_{Q_Y}(x, q)))^c,\]
\[= 1 - \sup(\mu_{Q_X}(x, q), \mu_{Q_Y}(x, q)),\]
\[= \inf(1 - \mu_{Q_X}(x, q), 1 - \mu_{Q_Y}(x, q)),\]
Definition 5.3. Let $F_{Q_X}$ and $F_{Q_Y}$ be interval valued Q-fuzzy multiparameterized soft sets over $U$. The $\wedge$-product (also called AND operation) of interval valued Q-fuzzy multiparameterized soft sets $F_{Q_X}$ and $F_{Q_Y}$ is an interval valued Q-fuzzy multiparameterized soft sets defined by $F_{Q_X} \wedge F_{Q_Y} = F_{Q_{X,Y}}$ where $X \times Y = \min((\mu_{Q_X}(x,q), \mu_{Q_Y}(x,q))$ for all $(x,q) \in A \times Q$, and $f_{Q_{X,Y}}(a,b) = f_{Q_X}(a) \cap f_{Q_Y}(b)$ for all $(a,b) \in A \times A$.

The following example illustrates the $\wedge$-product of interval valued Q-fuzzy multiparameterized soft sets.

Example 5.3. Let us consider a staff selection problem to fill a position in a private company. Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be the universe set consisting of five candidates, $Q = \{r\}$ be a qualification set. Let us consider $E_i = \{E_1, E_2, E_3\}$ the set of decision parameters where $E_1 = \{x_{1,1} = \text{experience}, x_{1,2} = \text{computer knowledge}\}$, $E_2 = \{x_{2,1} = \text{young}, x_{2,2} = \text{charming}\}$, and $E_3 = \{x_{3,1} = \text{general}, x_{3,2} = \text{fluent}\}$, and $A \in P(E)$ such that $A = \{x_1 = \{x_{1,1}, x_{2,1}\}, x_2 = \{x_{1,2}, x_{2,2}\}, x_3 = \{x_{1,1}, x_{1,3}\}, x_4 = \{x_{2,2}, x_{3,3}\}\}$.

Let $X = \{((x_1, r), [0.1, 0.3]), ((x_2, r), [0.1, 0.2]), ((x_3, r), [0.2, 0.3]), ((x_4, r), [0, 0.1])\}$. Let $f_{Q_X}(x_1) = \{u_{1, u_2, u_3}\}, f_{Q_X}(x_2) = \{u_{1, u_4, u_6}\}, f_{Q_X}(x_3) = \{u_{1, u_2, u_4, u_5}\}$, and $f_{Q_X}(x_4) = \{u_{1, u_2}\}$. Suppose $Y = \{((x_1, r), [0.3, 0.9]), ((x_2, r), [0.4, 0.5]), ((x_3, r), [0.3, 0.5]), ((x_4, r), [0.2, 0.3])\}$, $f_{Q_Y}(x_1) = \{u_{1, u_2, u_4, u_5}\}$, $f_{Q_Y}(x_2) = \{u_{3, u_4, u_5}\}, f_{Q_Y}(x_3) = \{u_{4, u_5}\}$ and $f_{Q_Y}(x_4) = \{u_{1, u_2}\}$.

The representation of these sets $F_{Q_X}$ and $F_{Q_Y}$ are as follows:

$F_{Q_X} = \{(((x_1, r), [0.1, 0.3]), \{u_1, u_2, u_3\}), (((x_2, r), [0.1, 0.2]), \{u_{1, u_4, u_6}\}), (((x_3, r), [0.3, 0.2]), \{u_{1, u_2, u_4, u_5}\}), (((x_4, r), [0, 0.1]), \{u_{1, u_2}\})\}$, and $F_{Q_Y} = \{(((x_1, r), [0.3, 0.9]), \{u_{1, u_2, u_4, u_5}\}), (((x_2, r), [0.4, 0.5]), \{u_{3, u_4, u_5}\}), (((x_3, r), [0.3, 0.5]), \{u_{4, u_5}\}), (((x_4, r), [0.2, 0.3]), \{u_{1, u_2}\})\}$.
Then the ∧-product of interval valued Q-fuzzy multiparameterized soft sets $F_{QX}$ and $F_{QY}$ are as follows

\[ X \times Y = \{ (((x_1, r), (x_1, r)), [0.1, 0.3]), (((x_1, r), (x_2, r)), [0.1, 0.3]), (((x_1, r), (x_3, r)), [0.1, 0.3]), (((x_1, r), (x_4, r)), [0.1, 0.3]), (((x_2, r), (x_1, r)), [0.1, 0.2]), (((x_2, r), (x_2, r)), [0.1, 0.2]), (((x_2, r), (x_3, r)), [0.1, 0.2]), (((x_2, r), (x_4, r)), [0.1, 0.2]), (((x_3, r), (x_1, r)), [0.2, 0.3]), (((x_3, r), (x_2, r)), [0.2, 0.3]), (((x_3, r), (x_3, r)), [0.2, 0.3]), (((x_3, r), (x_4, r)), [0.2, 0.3]), (((x_4, r), (x_1, r)), [0.0, 0.1]), (((x_4, r), (x_2, r)), [0.0, 0.1]), (((x_4, r), (x_3, r)), [0.0, 0.1]), (((x_4, r), (x_4, r)), [0.0, 0.1]) \}, \]

and

\[ f_{QX \times Y}(x_1, x_1) = \{ u_1, u_2, u_3 \}, f_{QX \times Y}(x_1, x_2) = \{ u_5 \}, f_{QX \times Y}(x_1, x_3) = \{ u_5 \}, f_{QX \times Y}(x_1, x_4) = \{ u_1, u_2, u_3 \}, f_{QX \times Y}(x_2, x_1) = \{ u_1, u_4 \}, f_{QX \times Y}(x_2, x_2) = \{ u_4 \}, f_{QX \times Y}(x_2, x_3) = \{ u_4 \}, f_{QX \times Y}(x_2, x_4) = \{ u_4 \} \]

Similarly we can get $f_{QX \times Y}(x_3, x_1), f_{QX \times Y}(x_3, x_2), f_{QX \times Y}(x_3, x_3), f_{QX \times Y}(x_3, x_4), f_{QX \times Y}(x_4, x_1), f_{QX \times Y}(x_4, x_2), f_{QX \times Y}(x_4, x_3), f_{QX \times Y}(x_4, x_4)$.

Thus $F_{QX} \wedge F_{QY} = \{ (((x_1, r), (x_1, r)), [0.1, 0.3]), \{ u_1, u_2, u_5 \} \}, (((x_1, r), (x_2, r)), [0.1, 0.3], \{ u_5 \}), (((x_1, r), (x_3, r)), [0.1, 0.3], \{ u_5 \}), (((x_1, r), (x_4, r)), [0.1, 0.3], \{ u_5 \}), (((x_2, r), (x_1, r)), [0.1, 0.2], \{ u_1, u_4 \}), (((x_2, r), (x_2, r)), [0.1, 0.2], \{ u_4 \}), (((x_2, r), (x_3, r)), [0.1, 0.2], \{ u_4 \}), (((x_2, r), (x_4, r)), [0.1, 0.2], \{ u_1 \}), \ldots, (((x_4, r), (x_4, r)), [0.0, 0.1], \{ u_1, u_2 \}) \}.$

**Definition 5.4.** Let $F_{QX}$ and $F_{QY}$ be interval valued Q-fuzzy multiparameterized soft sets over $U$. The ∨-product (also called AND operation) of interval valued Q-fuzzy multiparameterized soft sets $F_{QX}$ and $F_{QY}$ is an interval valued Q-fuzzy multiparameterized soft sets defined by $F_{QX} \vee F_{QY} = F_{QX \times Y}$ where $X \times Y = \max((\mu_{QX}(x, q), \mu_{QY}(x, q)))$ for all $(x, q) \in A \times Q$, and $f_{QX \vee QY}(a, b) = f_{QX}(a) \cup f_{QY}(b)$ for all $(a, b) \in A \times A$.

**Example 5.4.** From Example 5.3., the ∨-product of interval valued Q-fuzzy multiparameterized soft sets $F_{QX}$ and $F_{QY}$ are as follows: $X \times Y = \{ (((x_1, r), (x_1, r)), [0.3, 0.9]), (((x_1, r), (x_2, r)), [0.4, 0.5]), (((x_1, r), (x_3, r)), [0.3, 0.5]), (((x_1, r), (x_4, r)), [0.2, 0.3]), (((x_2, r), (x_1, r)), [0.3, 0.9]), (((x_2, r), (x_2, r)), [0.4, 0.5]), (((x_2, r), (x_3, r)), [0.3, 0.5]), (((x_2, r), (x_4, r)), [0.2, 0.3]), (((x_3, r), (x_1, r)), [0.3, 0.5]), (((x_3, r), (x_2, r)), [0.4, 0.5]), (((x_3, r), (x_3, r)), [0.3, 0.5]), (((x_3, r), (x_4, r)), [0.2, 0.3]), (((x_4, r), (x_1, r)), [0.3, 0.9]), (((x_4, r), (x_2, r)), [0.4, 0.5]), (((x_4, r), (x_3, r)), [0.3, 0.5]), (((x_4, r), (x_4, r)), [0.2, 0.3]) \},$ and $f_{QX \times Y}(x_1, x_1) = \{ u_1, u_2, u_4, u_5 \}, f_{QX \times Y}(x_1, x_2) = \{ u_1, u_2, u_3, u_4, u_5 \}, f_{QX \times Y}(x_1, x_3) = \{ u_1, u_2, u_4, u_5 \}, f_{QX \times Y}(x_2, x_1) = \{ u_1, u_2, u_4, u_5, u_6 \}, f_{QX \times Y}(x_2, x_2) = \{ u_1, u_3, u_4, u_5, u_6 \}, f_{QX \times Y}(x_2, x_3) = \{ u_1, u_4, u_5, u_6 \},$ and $f_{QX \times Y}(x_2, x_4) = \{ u_1, u_2, u_4, u_6 \}.$
Similarly we can get \( f_{Q_{x 	imes y}}(x_3, x_1), f_{Q_{x 	imes y}}(x_3, x_2), f_{Q_{x 	imes y}}(x_3, x_3), f_{Q_{x 	imes y}}(x_3, x_4), f_{Q_{x 	imes y}}(x_4, x_1), f_{Q_{x 	imes y}}(x_4, x_2), f_{Q_{x 	imes y}}(x_4, x_3), \) and \( f_{Q_{x 	imes y}}(x_4, x_4). \) Then \( F_{Q_x} \cup F_{Q_y} = \{ (((x_1, r), (x_1, r)), [0.3, 0.9]), \{ u_1, u_2, u_4, u_5 \}) \}, (((x_1, r), (x_2, r)), [0.4, 0.5]), \{ u_1, u_2, u_3, u_4, u_5 \}), (((x_1, r), (x_3, r)), [0.3, 0.5]), \{ u_1, u_2, u_4, u_5 \}), (((x_1, r), (x_4, r)), [0.2, 0.3]), \{ u_1, u_2, u_3 \}), (((x_2, r), (x_1, r)), [0.3, 0.9]), \{ u_1, u_2, u_4, u_5 \}), (((x_2, r), (x_3, r)), [0.3, 0.5]), \{ u_1, u_4, u_5, u_6 \}), (((x_2, r), (x_4, r)), [0.2, 0.3]), \{ u_1, u_2, u_4, u_6 \}), \ldots, (((x_4, r), (x_4, r)), [0.2, 0.3]), \{ u_1, u_2 \}) \}.

Now we give some propositions on the \( \wedge \) and \( \vee \)-product of interval valued Q-fuzzy multiparameterized soft sets

**Proposition 5.5.** Let \( F_{Q_x} \) and \( F_{Q_y} \) be interval valued Q-fuzzy multiparameterized soft sets over \( U. \) Then we have

1. \( F_{Q_x} \subseteq F_{Q_x} \cup F_{Q_y}. \)
2. \( F_{Q_y} \subseteq F_{Q_x} \cup F_{Q_y}. \)
3. \( F_{Q_x} \wedge F_{Q_y} \subseteq F_{Q_x}. \)
4. \( F_{Q_x} \wedge F_{Q_y} \subseteq F_{Q_y}. \)

**Proof.** (1) Let \( [\mu_{Q_x}^- (u, q), \mu_{Q_x}^+ (u, q)] = \mu_{Q_x} (u, q), [\mu_{Q_y}^- (u, q), \mu_{Q_y}^+ (u, q)] = \mu_{Q_y} (u, q). \) Assume that \( \mu_{Q_x} (u, q) \) and \( \mu_{Q_y} (u, q) \) are elements of \( Q_X \) and \( Q_Y \) respectively.

Then from Definition 5.4 we have \( \mu_{Q_X \cup Q_Y} (x, q) = \max \{ \mu_{Q_X} (u, q), \mu_{Q_Y} (u, q) \}, \mu_{Q_X} (u, q) \leq \max \{ \mu_{Q_X} (u, q), \mu_{Q_Y} (u, q) \}. \)

This implies \( f_{Q_X \cup Q_Y} (x) = f_{Q_X} (x) \cup f_{Q_Y} (x), \) therefore \( f_{Q_X} (x) \subseteq f_{Q_X} (x) \cup f_{Q_Y} (x). \)

Thus \( F_{Q_X} \subseteq F_{Q_X} \cup F_{Q_Y}. \)

Assertion (2) can be proven in a similar fashion.

(3). Assume \( F_{Q_X} \wedge F_{Q_Y} = (\mu_{Q_X} (u, q) \wedge \mu_{Q_Y} (u, q)), \) and from Definition 5.3 we have \( \mu_{Q_X \wedge Q_Y} (x, q) = \min \{ \mu_{Q_X} (u, q), \mu_{Q_Y} (u, q) \}, \min \{ \mu_{Q_X} (u, q), \mu_{Q_Y} (u, q) \} \subseteq \mu_{Q_X} (u, q). \)

This implies \( f_{Q_X \wedge Q_Y} (x) = f_{Q_X} (x) \cap f_{Q_Y} (x), \) therefore \( f_{Q_X} (x) \cap f_{Q_Y} (x) \subseteq f_{Q_X} (x). \)
Thus $F_{Q_X} \land F_{Q_Y} \subseteq F_{Q_X}$. The proof of (4) can be obtained in a similar fashion.

**Proposition 5.6.** Let $F_{Q_X}, F_{Q_Y}$ and $F_{Q_H}$ be interval valued $Q$-fuzzy multiparameterized soft sets over $U$. If $F_{Q_X} \subseteq F_{Q_Y}$, then we have

1. $F_{Q_X} \land F_{Q_H} \subseteq F_{Q_Y} \land F_{Q_H}$.
2. $F_{Q_X} \land F_{Q_H} \subseteq F_{Q_Y} \land F_{Q_H}$.

**Proof.** (1) Let $[\mu_{Q_X}(u,q), \mu_{Q_X}^+(u,q)] = \mu_{Q_X}(u,q), [\mu_{Q_Y}(u,q), \mu_{Q_Y}^+(u,q)] = \mu_{Q_Y}(u,q)$ and $[\mu_{Q_H}(u,q), \mu_{Q_H}^+(u,q)] = \mu_{Q_H}(u,q)$, Assume that $\mu_{Q_X}(u,q), \mu_{Q_Y}(u,q)$ and $\mu_{Q_H}(u,q)$ are elements of $Q_X, Q_Y$ and $Q_H$ respectively. Thus

(i) $\mu_{Q_X} \land Q_H(x,q) = \min(\mu_{Q_X}(u,q), \mu_{Q_H}(u,q))$, and

(ii) $\mu_{Q_Y} \land Q_H(x,q) = \min(\mu_{Q_Y}(u,q), \mu_{Q_H}(u,q))$.

There are three cases to be considered.

1. If the minimum membership of (i) is $\mu_{Q_X}(u,q)$, and for (ii) is $\mu_{Q_Y}(u,q)$, then $F_{Q_X} \subseteq F_{Q_Y}$. Therefore $F_{Q_X} \land F_{Q_H} \subseteq F_{Q_Y} \land F_{Q_H}$.

2. If the minimum membership of (i) is $\mu_{Q_X}(u,q)$, and for (ii) is $\mu_{Q_H}(u,q)$, then $\mu_{Q_X}(u,q) \leq \mu_{Q_H}(u,q)$, implies $f_{Q_X} \subseteq f_{Q_H}$ and consequently $F_{Q_X} \subseteq F_{Q_H}$. Thus $F_{Q_X} \land F_{Q_H} \subseteq F_{Q_Y} \land F_{Q_H}$.

3. If the minimum membership of (i) is $\mu_{Q_H}(u,q)$, and for (ii) is $\mu_{Q_Y}(u,q)$, then $\mu_{Q_H}(u,q) \leq Q_X(u,q)$, implies $f_{Q_H} \subseteq f_{Q_X}$, and $F_{Q_H} \subseteq F_{Q_X}$. Consequently if $F_{Q_H} \subseteq F_{Q_X}$ and $F_{Q_X} \subseteq F_{Q_Y}$, then $F_{Q_H} \subseteq F_{Q_Y}$.

Therefore $F_{Q_X} \land F_{Q_H} \subseteq F_{Q_Y} \land F_{Q_H}$.

Assertion (2) can be proven in a similar fashion.

**Proposition 5.7.** Let $F_{Q_X}, F_{Q_Y}$ and $F_{Q_H}$ be interval valued $Q$-fuzzy multiparameterized soft sets over $U$. Therefore

1. $F_{Q_X} \lor (F_{Q_X} \land F_{Q_Y}) \subseteq F_{Q_X} \lor F_{Q_Y}$.
2. $F_{Q_X} \land F_{Q_Y} \subseteq F_{Q_X} \land (F_{Q_X} \lor F_{Q_Y})$.

**Proof.** (1) Let us write $F_{Q_X \land Y} = F_{Q_X} \land F_{Q_Y}$, where $F_{Q_X \land Y}(a,b) = F_{Q_X}(a) \lor F_{Q_Y}(b)$, for all $(a,b) \in A \times A$. Then let $F_{Q_X} \lor F_{Q_X \land Y} = F_{Q_X \lor (X \land Y)}$ where
$F_{QX \lor (X \land Y)} (a, (a, b)) = F_{QX} (a) \cup (F_{QX} (a) \cap F_{QY} (b))$, for all $(a, (a, b)) \in A \times (A \times A)$. Let $F_{QX \lor X} = F_{QX} \lor F_{QX}$ where $F_{QX \lor X} (a_1, a_2) = F_{QX} (a) \cup F_{QX} (a)$, for all $(a_1, a_2) \in A \times A$. Note also that $F_{QX} (a) \lor (F_{QX} (a) \cap F_{QY} (b)) = (F_{QX} (a) \cap F_{QX} (a)) \cup (F_{QX} (a) \cap F_{QY} (b))$. Hence we deduce that $F_{QX \lor (X \land Y)} (a, (a, b)) \subseteq (F_{QX} (a) \cup F_{QX} (a)) \cup (F_{QX} (a) \cap F_{QY} (b))$. This shows that $F_{QX \lor (X \land Y)} \subseteq F_{QX \land X}$ as required.

Assertion (2) can be proven in a similar fashion.

6. Interval Valued Q-Fuzzy Decision Set of an IVQFMP-Soft Set

In this section, the authors define an interval value Q-fuzzy decision set of an IVQFMP-soft set to construct a decision method by which approximate functions of a soft set are combined to produce a single Q-fuzzy set that can be used to evaluate each alternative.

**Definition 6.1.** Let $F_{QX} \in IVQFMP(U)$. An interval valued Q-fuzzy decision set of $F_{QX}$, denoted by $F_{dQX}^{d}$ is defined by

$F_{dQX}^{d} = \{(u,q) : \mu_{F_{dQX}^{d}} (u,q) : u \in U, q \in Q\}$,

which is Q-fuzzy set over $U$, whereby its membership function $\mu_{F_{dQX}^{d}} (u,q)$ is defined by $\mu_{F_{dQX}^{d}} : U \times Q \to [0,1]$ and

$\mu_{F_{dQX}^{d}} (u,q) = 1/|\text{supp}(X)| \sum_{(x,q) \in \text{supp}(X)} \frac{[\mu_{QX}^- (x,q) + \mu_{QX}^+ (x,q)]}{2} \chi_{f_{QX}(x)}(u,q)$,

where $\text{supp}(X)$ is the support set of $X$, $f_{QX}(x)$ is the crisp subset determined by the parameter $x$ and

$f_{QX}(x) = \begin{cases} 1, & \text{if } u \in f_{QX} \text{ and } q \in X \\ 0, & \text{if } u \not\in f_{QX} \text{ or } q \not\in X \end{cases}$.

Using the definition of interval valued Q-fuzzy decision set of an IVQFMP-soft set, a decision method by the following algorithm can be constructed.
Step 1: Construct a $F_{QX}$ over $U$.

Step 2: Compute the interval valued $Q$-fuzzy decision set $F^d_{QX}$.

Step 3: Select the largest membership grade $\max \mu_{F^d_{QX}}(u, q)$.

The following example will illustrate the idea of the algorithm given above.

Example 6.1. Let us assume that a person goes to buy a car from a set of five cars $U = \{u_1, u_2, u_3, u_4, u_5\}$, $Q = \{q, p\}$ be a set years of car made and $E = \{E_1, E_2\}$ be a set of parameters where $E_1$ is a set of cost parameters given by $\{x_{1.1} = \text{cheap}, x_{1.2} = \text{expensive}, x_{1.3} = \text{very expensive}, x_{1.4} = \text{very cheap}\}$, $E_2$ is a set of color parameters given by $\{x_{2.1} = \text{red}, x_{2.2} = \text{black}, x_{2.3} = \text{white}\}$, with $A \subseteq P(E)$ such that $A = \{x_1 = \{x_{1.1}, x_{2.1}\}, x_2 = \{x_{1.4}, x_{2.2}\}, x_3 = \{x_{1.3}, x_{2.3}\}\}$. The person considers two multi parameters namely, (cheap, red color) and (very cheap, white color). That is, the subset of parameters is $X = \{(x_1, q), [0.6, 0.8]), ((x_1, p), [0.6, 0.9]), ((x_2, q), [0.4, 0.7]), ((x_2, p), [0.3, 0.5])\}$.

The steps below can be used to find a suitable car for the person to buy.

Step 1: Construct $F_{QX}$ over $U$,

$$F_{QX} = \{(((x_1, q), [0.6, 0.8]), \{u_1, u_2\}), (((x_1, p), [0.6, 0.9]), \{u_1, u_5\}), (((x_2, q), [0.4, 0.7]), \{u_2, u_4, u_3\}), (((x_2, p), [0.3, 0.5]), \{u_1, u_4, u_5\})\}.$$

Step 2: The $Q$-fuzzy decision set of $F_{QX}$ can be found as

$$F^d_{QX} = \{((u_1, q), 0.1), ((u_1, p), 0.29), ((u_2, q), 0.31), ((u_2, p), 0), ((u_3, q), 0.14), ((u_3, p), 0), ((u_4, q), 0.14), ((u_4, p), 0.1), ((u_5, q), 0.19), ((u_5, p), 0.1)\}.$$

Step 3: Finally, the largest membership grade can be chosen by

$$\max \mu_{F^d_{QX}}(u, q) = 0.31.$$

This implies that the candidate $(u_2, q)$ has the largest membership grade, hence the car $u_2$ made in year $q$ is most suitable to be bought.
The following example will illustrate that IVQFMP-soft set can better reflect the decision maker’s preferences compared to the multi Q-fuzzy set.

Example 6.2. Suppose that a university wants to select a student for the "Best outgoing student award" and there are nominees who form the universe $U = \{p_1, p_2, p_3, p_4, p_5\}$, $Q = \{q, p\}$ be the nationality and $E_i = \{E_1, E_2, E_3\}$, be a set of parameters where $E_1$ is a set of fields of study parameters $E_1 = \{x_{1,1} = \text{Biochemistry}, x_{1,2} = \text{Biology}, x_{1,3} = \text{Ecology}\}$, $E_2$ is a set of achievement of the students parameters given by $E_2 = \{x_{2,1} = \text{academic performance}, x_{2,2} = \text{co-curricular activities}, x_{2,3} = \text{good conduct}\}$, and $E_3$ is a set of programme of study parameters given by $E_3 = \{x_{3,1} = \text{graduate students}, x_{3,2} = \text{undergraduate students}\}$. Consider a set of parameters, $\{x_{1,1}, x_{2,1}\}$, $\{x_{1,3}, x_{2,2}, x_{3,2}\}$, $\{x_{1,2}, x_{2,3}\}$, and $\{x_{1,1}, x_{3,1}\}$.

In this example it will be difficult to explain the universal $U$ with two or three membership function using multi Q-fuzzy set, especially when there are different numbers of parameters involved. However, IVQFMP-soft set can reflect the decision maker’s preferences. Thus, the expert committee can consider sets of multi parameters namely,

$x_1 = \{\text{Biochemistry, high academic performance}\}$,
$x_2 = \{\text{Ecology, co-curricular activities, undergraduate students}\}$,
$x_3 = \{\text{Biology, good conduct}\}$, and
$x_4 = \{\text{Biochemistry, graduate students}\}$.

That is, the subset of parameter and IVQFMP-soft sets are as follows.

$$X = \{((x_1, q), [0.3, 0.5]), ((x_1, p), [0.2, 0.9]), ((x_2, q), [0.4, 0.8]), ((x_2, p), [0.3, 0.5]), ((x_3, q), [0.3, 0.6]), ((x_3, p), [0.1, 0.7]), ((x_4, q), [0.1, 0.3]), ((x_4, p), [0.5, 0.7])]\}.$$  

$$F_{QX} = \{(((x_1, q), [0.3, 0.5]), \{u_1, u_5\})), (((x_1, p), [0.2, 0.9]), \{u_3, u_6\}), (((x_2, q), [0.4, 0.8]), \{u_2, u_3, u_4\}), (((x_2, p), [0.3, 0.5]), \{u_1, u_4, u_5\}), (((x_3, p), [0.3, 0.6]), \{u_1, u_4, u_5\}), (((x_3, q), [0.1, 0.7]), \{u_3, u_4, u_5\}), (((x_4, q), [0.3, 0.6]), \{u_4, u_5\}), (((x_4, p), [0.1, 0.7]), \{u_1\})}\}.$$  

7. Conclusion

The authors introduced the concepts of an interval-valued Q-fuzzy set and IVQFMP-soft set and described their equality, subset, union, intersection, $\wedge$ and $\vee$-product. An algorithm for a decision method using an IVQFMP-soft set was constructed. The proposed IVQFMP-soft set is then successfully applied to a decision making problem which shows that it can more accurately reflect the decision maker’s hesitancy in stating his preferences over choice alternatives,
as compared to the multi $Q$-fuzzy set.

References


Interval Valued Q-Fuzzy Multiparameterized Soft Set and its Application


