MULTI – CRITERIA GROUP DECISION MAKING MODEL IN NEUTROSOPIHC REFINED SET AND ITS APPLICATION

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DOI: 10.5281/zenodo.55307

KEYWORDS: Neutrosoophic set, Single valued neutrosoophic set, Neutrosoophic refined sets, Tangent similarity measure, Multi criteria decision making.

ABSTRACT
In the paper, multi criteria group decision making model has been presented based on tangent similarity measure of neutrosoophic refined set. Simplified form of tangent similarity measure in neutrosoophic refined set has been presented. New ranking method has been proposed based on refined tangent similarity measure. The proposed approach has been illustrated by solving a teacher selection problem in neutrosoophic refined set environment.

INTRODUCTION
The term “neutrosoophy” has been coined by Smarandache [1] in his study of new branch of philosophy. According to Smarandache neutrosoophy means knowledge of neutral thoughts. From neutrosoophy, neutrosoophic set has been derived [1] which is the generalization of fuzzy set [2] and intuitionistic fuzzy set [3]. Wang et al. [4] studied single valued neutrosoophic sets to deal realistic problems. Neutrosoophic set and single valued neutrosoophic sets have been applied in different areas of research such as medical diagnosis [5], decision making [6, 7], social problems [8, 9], conflict resolution [10], etc.

Similarity measure is a very important tool to determine the degree of similarity between two objects. Many researchers have studied different similarity measures and their properties. Broumi and Smarandache [11] studied Hausdorff distance between neutrosoophic sets. Broumi and Smarandache [12] also studied correlation coefficient between interval neutrosoophic sets. Broumi and Smarandache [13] also proposed cosine similarity measure and its properties based on Bhattacharye’s distance [14]. Using distances, a matching function, membership grades Majumder and Samanta [15] established different similarity measures of single valued neutrosoophic sets (SVNS) and proposed an entropy measure for SVNS. Ye and Zhang [16] described similarity measures using minimum and maximum operators and proposed a multi criteria decision making method based on the weighted similarity measure for SVNS. Ye [17] presented clustering method for single valued neutrosoophic sets and a clustering algorithm for single valued neutrosoophic data. Multi criteria group decision making approach under simplified neutrosoophic environment has been proposed by Mondal and Pramanik [18]. They have used hybrid score accuracy function to make decision and apply their approach in teacher selection in higher education.

Biswa et al. [19] studied cosine similarity measure based on multi attribute decision making with trapezoidal fuzzy neutrosoophic numbers. Pramanik and Mondal [20] applied cosine similarity measure of rough neutrosoophic set in medical diagnosis. Decision making based on some similarity measures under interval rough neutrosoophic environment has been studied by Mondal and Pramanik [21]. Mondal and Pramanik [22] further developed some rough neutrosoophic similarity measures and their application to multi attribute decision making.

Yager [23] introduced the concept of multiset. Sebastian and Ramakrishnan [24] developed the concept and properties of multi fuzzy set. This concept has been extended to intuitionistic fuzzy multiset by Shinoj and John [25]. Smarandache [26] proposed n- valued refined neutrosoophic logic and its application. Broumi and Smarandache [27] defined neutrosoophic refined similarity measure based on cosine function. Dice similarity
measure between single valued neutrosophic multi sets and its application have been established by Ye and Ye [28].


We have presented simplified form of tangent similarity measure. We have developed multi criteria group decision making model based on tangent similarity measure of neutrosophic refined set.

Rest of the paper has been designed as follows: Some relevant definitions are presented in the section 2. Tangent similarity measure for neutrosophic refined sets and some of its properties have been stated in the next section. Section 4 is devoted to present decision making model based on refined tangent similarity measure. The application of refined tangent similarity measure to the problem on teacher section is shown in the next section. Finally, section 6 presents the conclusion and future works of the proposed approach.

SOME RELEVANT DEFINITIONS

Neutrosophic Set [1]
Let X be an universe of discourse. Then the neutrosophic set P is of the form P = {<x: T_P(x), I_P(x), F_P(x)> | x \in X}, where the functions T, I, F: X \rightarrow [0,1] are defined respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element x \in X to the set N satisfying the following the condition.

0 \leq \sup T_P(x) + \sup I_P(x) + \sup F_P(x) \leq 3

For two neutrosophic sets, P = {<x: T_P(x), I_P(x), F_P(x)> | x \in X} and Q = {<x, T_Q(x), I_Q(x), F_Q(x)> | x \in X} the two relations are defined as follows:

1) P \subseteq Q if and only if T_P(x) \leq T_Q(x), I_P(x) \geq I_Q(x), F_P(x) \geq F_Q(x)
2) P = Q if and only if T_P(x) = T_Q(x), I_P(x) = I_Q(x), F_P(x) = F_Q(x)

Single valued neutrosophic set [4]
Let X be a space of points with generic elements in X denoted by x. A SVNS P in X is characterized by a truth-membership function T_P(x), an indeterminacy-membership function I_P(x), and a falsity membership function F_P(x), for each point x in X. T_P(x), I_P(x), F_P(x) \in [0, 1]. When X is continuous, a SVNS P can be written as:

P = \{<x: T_P(x), I_P(x), F_P(x)> | x \in X\}

When X is discrete, a SVNS P can be written as:

P = \{<x: T_P(x), I_P(x), F_P(x)> | x \in X\}

For two SVNSs, P_{SVNS} = {<x: T_P(x), I_P(x), F_P(x)> | x \in X} and Q_{SVNS} = {<x, T_Q(x), I_Q(x), F_Q(x)> | x \in X} the two relations are defined as follows:

1) P_{SVNS} \subseteq Q_{SVNS} if and only if T_P(x) \leq T_Q(x), I_P(x) \geq I_Q(x), F_P(x) \geq F_Q(x)
2) P_{SVNS} = Q_{SVNS} if and only if T_P(x) = T_Q(x), I_P(x) = I_Q(x), F_P(x) = F_Q(x) for any x \in X

Neutrosophic refined set [29]
Let P be a neutrosophic refined set (NRS)
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Let $P = \{x \in (T^*_1(x), T^*_2(x), \ldots, T^*_r(x))$; $(I^*_1(x), I^*_2(x), \ldots, I^*_r(x)) ; (1^*_1(x), 1^*_2(x), \ldots, 1^*_r(x))\}$ such that $0 \leq \sup T^*_i(x) + \sup I^*_i(x) + \sup F^*_i(x) \leq 3$, for $j = 1, 2, \ldots, r$ for any $x \in X$. Now, $(T^*_1(x), T^*_2(x), \ldots, T^*_r(x))$, $(I^*_1(x), I^*_2(x), \ldots, I^*_r(x))$, $(F^*_1(x), F^*_2(x), \ldots, F^*_r(x))$ are respectively called the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element $x$. Also, $r$ is called the dimension of neutrosophic refined sets $P$.

TANGENT SIMILARITY MEASURE FOR SINGLE VALUED REFINED NEUTROSOPHIC SETS [31]

Let $P$ and $Q$ be two neutrosophic refined sets where $P = \{x \in (T^*_1(x), T^*_2(x), \ldots, T^*_r(x))$; $(I^*_1(x), I^*_2(x), \ldots, I^*_r(x)) ; (1^*_1(x), 1^*_2(x), \ldots, 1^*_r(x))\}$ such that $0 \leq \sup T^*_i(x) + \sup I^*_i(x) + \sup F^*_i(x) \leq 3$ and $Q = \{x \in (T^*_0(x), T^*_0(x), \ldots, T^*_0(x))$; $(I^*_0(x), I^*_0(x), \ldots, I^*_0(x)) ; (1^*_0(x), 1^*_0(x), \ldots, 1^*_0(x))\}$ such that $0 \leq \sup T^*_0(x) + \sup I^*_0(x) + \sup F^*_0(x) \leq 3$, for $j = 1, 2, \ldots, r$, for any $x \in X$. Now refined tangent similarity function between $P$ and $Q$ can be presented as:

$$T_{NRS}(P, Q) = \frac{1}{r} \sum_{i=1}^{r} \left[ \frac{1}{n} \sum_{j=1}^{n} \left( -\tan \left( \frac{\pi}{12} \left( |T^*_i(x) - T^*_j(x)| + |I^*_i(x) - I^*_j(x)| + |F^*_i(x) - F^*_j(x)| \right) \right) \right] \right]$$

In the paper, we redefine the above in more simplified form as

$$T_{NRS}(P, Q) = 1 - \frac{1}{nr} \sum_{i=1}^{r} \sum_{j=1}^{n} \tan \left( \frac{\pi}{12} \left( |T^*_i(x) - T^*_j(x)| + |I^*_i(x) - I^*_j(x)| + |F^*_i(x) - F^*_j(x)| \right) \right)$$

Proposition 3.1

The defined refined tangent similarity measure $T_{NRS}(P, Q)$ between NRSs $P$ and $Q$ satisfies the following properties:

1. $0 \leq T_{NRS}(P, Q) \leq 1$
2. $T_{NRS}(P, Q) = 1$ if $P = Q$
3. $T_{NRS}(Q, P) = T_{NRS}(P, Q)$
4. If $R$ is a NRS in $X$ and $P \subseteq Q \subseteq R$ then $T_{NRS}(P, R) \leq T_{NRS}(Q, R)$ and $T_{NRS}(R, P) \leq T_{NRS}(Q, R)$

For proofs of the propositions in 3.1, see the paper [31].

DECISION MAKING UNDER SINGLE VALUED REFINED NEUTROSOPHIC SET BASED ON TANGENT SIMILARITY MEASURE

Let $E_1, E_2, \ldots, E_m$ be a discrete set of experts, $A_1, A_2, \ldots, A_m$ be the set of attributes and $C_1, C_2, \ldots, C_k$ be the set of candidates. The step by step process of decision making under refined neutrosophic tangent similarity measure are presented as follows.

Step 1: Determination the relation between experts $(E_0)$ and attributes $(A_j)$

Each expert $E_i$ ($i = 1, 2, \ldots, m$) with the attribute $A_j$ ($j = 1, 2, \ldots, n$) is presented in the tale 1 as follows.
The average of each column is presented as follows:

\[ \Omega_t = \frac{\sum C_{j}}{m} \text{ where } t = 1, 2, ..., k. \]
Step 4: Ranking of the candidates
Ranking of candidates is prepared based on the descending order average of each column \( \Omega \), of the table 3.

Step 5: End.

EXAMPLE ON TEACHER SELECTION PROCESS
Suppose that a university is going to recruit an assistant professor in mathematics. A selection board has been formed consisting of four experts (\( E_1, E_2, E_3, E_4 \)). After primary selection five candidates (\( C_1, C_2, C_3, C_4, C_5 \)) appear before the board. Six attributes (criteria) (demonstration (\( A_1 \)), pedagogical knowledge (\( A_2 \)), action research (\( A_3 \)), emotional stability (\( A_4 \)), and knowledge of child psychology (\( A_5 \)), and social quality (\( A_6 \)) obtained from experts’ opinions are the parameters for selection. Selection process is divided into three phases represented by neutrosophic refined sets. The relation between experts and attributes has been presented in the table 4.

![Table 4: The relation between experts and criteria](image)

The relation between attributes \( A_j \) \((j = 1, 2, \ldots, n)\) and candidates (alternatives) \( C_t \) \((t = 1, 2, \ldots, k)\) in terms of SVNs is presented in the table 5.

![Table 5: The relation between criterion and candidates](image)

Using equation (2), the tangent similarity measure between the table 4 and the table 5 is presented in table 6.

![Table 6: The tangent similarity measure between experts and candidates](image)
The average of each column is presented by $\Omega_i$ (see the table 6). The merit list on the basis of average has been shown in the table 7.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B_1</td>
</tr>
<tr>
<td>2</td>
<td>B_2</td>
</tr>
<tr>
<td>3</td>
<td>B_3</td>
</tr>
<tr>
<td>4</td>
<td>B_4</td>
</tr>
<tr>
<td>5</td>
<td>B_5</td>
</tr>
</tbody>
</table>

From the merit list, it is observed that B_1 is the most eligible candidate for the post.

CONCLUSION

In the present study, we have developed multi criteria group decision making model in neutrosophic refined set and presented its application in teacher selection. The tangent similarity function has been presented in more simplified form. The ranking of the candidates has been prepared on the basis of descending order average of each column of tangent similarity measure table. The proposed approach can be also applied to other group decision making problems under refined neutrosophic set environment. The proposed approach can be also extended to neutrosophic hybrid environment.

REFERENCES

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