

# A New PCR Combination Rule for Dynamic Frame Fusion\*

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**Abstract** — Dynamic frame fusion which is based on hybrid DSm model is an important problem in information fusion. But the traditional combination rules are mainly under fixed discernment frame (Shafer model and free DSm model) responding to static model. A new method for dynamic proportional conflict redistribution rules (dynamic PCR rules) based on hybrid DSm model is proposed for the shortness of classical dynamic PCR rules. In the new dynamic PCR rule, combination involved with empty set is defined as one kind to obtain more reasonable results. For the redistribution weight, the conjunction Basic belief assignment (BBA) and conflict redistribution BBA are both taken into account to raise the fusion precision. The effectiveness of revised dynamic PCR rule is studied and simulated in both aspect of fusion accuracy and calculation.

**Key words** — Dynamic frame fusion, Hybrid DSm model, Dynamic proportional conflict redistribution rule.

## I. Introduction

Combination rule is the core part of Dezert-Smarandache theory (DSm theory)<sup>[1,2]</sup>. The effectiveness of DSm theory fusion system depends on the properties of combination rule. And the combination rule should satisfy the application requirements of users. In most of classical problem using belief functions, the frame of discernment  $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$  is considered static. This means the set of elements in the frame (assumed to be non-empty and distinct) and the underlying integrity constraints of the frame are fixed and remain the same with time. These fusion problems are called static fusion. In some application however, like in target tracking and battlefield surveillance for example, the use of such invariant frame is not very appropriate because it can truly change with time depending on the changing process of the events. In the dynamic frame fusion, frame of discernment changes

with time taking in account the non-existential integrity constraints to establish new model of frame<sup>[3]</sup>.

In the classical Dempster-Shafer theory (DS theory)<sup>[4–6]</sup>, the main research focus on the static fusion for nonexistence of non-existential integrity constraints concept, as computation problem<sup>[7]</sup>, static combination rule<sup>[8]</sup>, combination of unreliable evidence<sup>[9–11]</sup>, evidential classification method<sup>[12,13]</sup>. But the dynamic frame fusion is more practical than static fusion<sup>[14]</sup>. The theoretical foundation of DSm theory is hybrid DSm model and dynamic constraint condition. So DSm theory is perfect for the definition, representation and procession of dynamic frame fusion problem<sup>[2]</sup>. Constraint condition changes with time to generate new hybrid DSm model. As the example<sup>[3]</sup>, lets consider the set of three targets at a given time  $k$  to be  $\Theta_k = \{\theta_1, \theta_2, \theta_3\}$ . And one receives a new information at time  $k+1$  confirming that one target, say target  $\theta_3$  has been destroyed. The problem one need to solve is how to combine efficiently evidence taking into account this new non-existential integrity constraint  $\theta_3 \equiv \emptyset$  in the new model of the frame to establish the most threatening and surviving targets belonging to  $\Theta_{k+1} = \{\theta_1, \theta_2\}$ .

Smarandache and Dezert propose a series of combination rules to solve the fusion problem under dynamic frame. These combination rules are dynamic proportional conflict redistribution combination rules (dynamic PCR rules)<sup>[15]</sup> and Dezert-Smarandache hybrid combination rules (DSmH rules)<sup>[2]</sup>. The combination rules have three steps in evidence processing. Firstly, these combination rules define non-existent sets caused by the modification of discernment frame. Secondly, the Basic belief assignment (BBA) of the non-existent sets is transferred

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to empty set  $\emptyset$ . Lastly, the BBA is redistributed for the relationship empty set  $\emptyset$  and elements in discernment frame to complete the dynamic frame fusion. The dynamic PCR rules are more effective than dynamic DSmH rules shown in Ref.[15]. But there are two kinds of drawbacks for dynamic PCR rules: *a*) the unreasonable redistribution, the redistribution of empty set  $\emptyset$  is similar with the redistribution of existential sets; *b*) insufficiency use of information, some useful information is not involved in the redistribution weight. The contribution of this paper is to propose a revised method for dynamic PCR rule based on hybrid DSm model. In the revised method, combination involved with empty set  $\emptyset$  is defined as one kind to obtain more reasonable results. For the redistribution weight, the conjunction BBA and conflict redistribution BBA are both taking into account to raise the fusion precision. The revised PCR rule makes better use of existing information without adding more calculation. The effectiveness of revised PCR rule is shown in the simulation.

## II. Hybrid DSm Model

DSm theory is motivated by expanding the frame of discernment to allow for presumed singletons in DS theory to actually have a well-defined intersection<sup>[3]</sup>. The frame of discernment changes with time according to the dynamic constraint condition, which is the foundation of dynamic frame fusion.

DSm theory contains Shafer model, free DSm model and hybrid DSm model. Depending on the intrinsic nature of the elements of the fusion problem under consideration, it can however happen that free DSm model and Shafer model do not fit the reality because some subsets of  $\Theta$  can contain elements known to be truly exclusive but also truly non existing at all at a given time (specially when working on dynamic fusion problem where the frame  $\Theta$  varies with time with the revision of the knowledge available). These integrity constraints are then explicitly and formally introduced into the free DSm model  $M^f(\Theta)$  in order to adapt it properly to fit as close as possible with the reality and permit to construct a hybrid DSm model  $M(\Theta)$ . The hybrid DSm model  $M(\Theta)$  can deal with the situation of sets which might become empty at time  $t_k$  or new sets/elements that might arise in the frame at time  $t_{k+1}$ <sup>[3]</sup>. The model in Fig.1(c) is the hybrid DSm model with  $\theta_1 \cap \theta_3 = \emptyset$  and  $\theta_2 \cap \theta_3 = \emptyset$ . The free DSm model and Shafer model can be considered as two special case of hybrid DSm model.

## III. Combination Rule of Dynamic Frame Fusion

The discernment frames of Shafer model and free DSm model are determined by the elements in the discernment frame. While the discernment frame of hybrid DSm model

is determined by both the elements in the discernment frame and non-existential integrity constraints. The discernment frame changes with the non-existential integrity constraints. Hybrid DSm model fits the reality because some subsets of  $\Theta$  can contain elements known to be truly exclusive but also truly non existing at all at a given time.

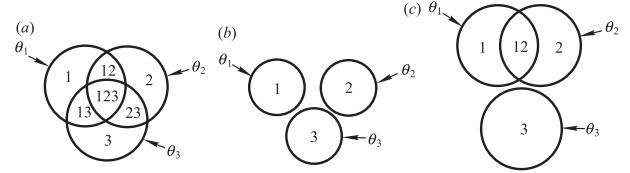


Fig. 1. The model of fusion problems. (a) Free DSm model;  
(b) Shafer model; (c) Hybrid DSm model

The traditional combination rules are under fixed discernment frame (Shafer model and free DSm model) responding to static fusion. But the dynamic frame fusion and hybrid DSm model are consistent with the practical situation. Smarandache and Dezert propose a series of combination rule to solve the problem under dynamic frame fusion. These combination rules are dynamic PCR rules and DSmH rules. Firstly, the dynamic PCRa rule is analyzed in the paper. Then, a new revised PCR rule is proposed to overcome the shortness of PCRa rule. Lastly, the calculation is defined and studied to evaluate the combination rules.

### 1. Revised method of dynamic frame PCR rule

Smarandache and Dezert analyze dynamic PCR rules and DSmH rules in Ref.[15] to draw a conclusion that PCRa rule is the most effective rule. PCRa rule:  $m_{\text{PCRa}}(\emptyset) = 0$  and  $\forall A \in G^\Theta \setminus \emptyset$

$$\begin{aligned} m_{\text{PCRa}}(A) &= m_{12}(A) \\ &+ \sum_{\substack{X \in G^\Theta \setminus \emptyset \\ X \cap A = \emptyset}} \left[ \frac{m_1(A)^2 m_2(X)}{m_1(A) + m_2(X)} + \frac{m_2(A)^2 m_1(X)}{m_2(A) + m_1(X)} \right] \\ &+ \sum_{X \in \emptyset} [m_1(A)m_2(X) + m_2(A)m_1(X)] \quad (1) \\ &+ m_{12}(A) \cdot \frac{\sum_{X, Y \in \emptyset} m_1(X)m_2(Y)}{\sum_{Z \in G^\Theta \setminus \emptyset} m_{12}(Z)} \end{aligned}$$

The redistribution strategy for BBA not involved with empty set  $\emptyset$  is the same as that of classical PCR rule<sup>[16]</sup>. And PCRa redistributes the BBA involved with empty set  $\emptyset$  to non-empty sets, as two kinds. Firstly, the redistribution strategy of intersection between non-empty sets and empty set  $\emptyset$  is defined as  $S_1$ . Secondly, the redistribution strategy of intersection between empty set  $\emptyset$  and empty set  $\emptyset$  is defined as  $S_2$ .

$$S_1 = \sum_{X \in \emptyset} [m_1(A)m_2(X) + m_2(A)m_1(X)] \quad (2)$$

$$S_2 = m_{12}(A) \cdot \frac{\sum_{X,Y \in \emptyset} m_1(X)m_2(Y)}{\sum_{Z \in G^\Theta \setminus \emptyset} m_{12}(Z)} \quad (3)$$

But there are two kinds of problems for dynamic PCR rules: *a*) the unreasonable redistribution, the redistribution of empty set  $\emptyset$  is similar with the redistribution of existential sets in classical PCR rule<sup>[16]</sup>; *b*) insufficiency use of information, some useful information is not involved in the redistribution weight. The PCRa rule redistributes the empty sets via using the same method of processing unknown set  $\Theta$ . The unknown set  $\Theta$  is the universal set of elements in discernment frame suitable for mutual processing. But empty sets  $\emptyset$  are defined as

$$m_{PCRd}(A) = m_t(A) + \frac{m_t(A)}{\sum_{X \in G^\Theta \setminus \emptyset} m_t(X)} \left[ \sum_{X \in \emptyset, A \in G^\Theta \setminus \emptyset} [m_1(A)m_2(X) + m_2(A)m_1(X)] + \sum_{X,Y \in \emptyset} m_1(X)m_2(Y) \right] \quad (4)$$

And

$$m_t(U) = m_{12}(U) + \sum_{\substack{U,P \in G^\Theta \setminus \emptyset \\ P \cap U = \emptyset}} \left[ \frac{m_1(U)^2 m_2(P)}{m_1(U) + m_2(P)} + \frac{m_2(U)^2 m_1(P)}{m_2(U) + m_1(P)} \right] \quad (5)$$

Revised PCR rule uses the information in intersection more effective. And the redistribution of empty set  $\emptyset$  is more reasonable. The fusion result of revised PCR rule is better than that of PCRa rule.

## 2. Analysis of calculation

The calculation is taken into account besides the fusion result to evaluate the combination rules. It is lack of applicability for combination rule which has perfect fusion result and high calculation. In the area of BBA combination, evidence combination can be considered as the composition of multiplication operation (division operation) which is atom operation.

**Definition 1** In hybrid DSm model, lets consider  $n$  focal elements  $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$  in the  $k$  evidence, the set of focal elements is  $G^\Theta$ , the number of focal elements is  $|G^\Theta|$ , for the evidence  $m_j(A_{jl})$ ,  $j = 1, 2, \dots, k$ ,  $l = 1, 2, \dots, |G^\Theta|$ ,  $A_{jl} \subseteq \Theta$ , The weighted sum of multiplication operation times (division operation) for certain combination rule is the BBA combination calculation which is expressed as  $O(f(n, k))$ .

The redistribution content and redistribution weight is adjusted in the revised PCR rule. The conjunction BBA and redistribution BBA of conflict set for non-empty sets which have been obtained in the pre-processing are used in revised PCR rule. So the calculation of revised PCR rule is same as that of PCRa rule. The revised PCR rule has better fusion result without more calculation. So the revised PCR rule is more effective in the aspect of calculation analysis.

## IV. Simulation and Analysis

the non-existential sets which are not suitable for mutual processing. In addition, the BBA of conflict redistribution can express the condition of the support to certain element. But the BBA of conflict redistribution leaves out of consideration of PCRa rule.

To overcome the shortness of PCRa rule, a revised PCR rule is proposed. For the problem *a*), intersection between non-empty sets and empty set  $\emptyset$   $S_1$  and intersection between empty set  $\emptyset$  and empty set  $\emptyset$   $S_2$  uniformly process. For the problem *b*), redistribution BBA of conflict set for non-empty sets is added into redistribution weight (besides conjunction BBA in PCRa rule). Revised PCR rule is called PCRd for short:  $m_{PCRd}(\emptyset) = 0$  and  $\forall A \in G^\Theta \setminus \emptyset$ .

$$m_{PCRd}(A) = m_t(A) + \frac{m_t(A)}{\sum_{X \in G^\Theta \setminus \emptyset} m_t(X)} \left[ \sum_{X \in \emptyset, A \in G^\Theta \setminus \emptyset} [m_1(A)m_2(X) + m_2(A)m_1(X)] + \sum_{X,Y \in \emptyset} m_1(X)m_2(Y) \right] \quad (4)$$

The revised PCR rule and classical dynamic PCR rules are simulated and analyzed with two examples. The example 1 is a classical case in Smarandache's paper<sup>[15]</sup>. The example 2 is a dynamic frame example with overall process. The fusion result and calculation of revised PCR rule (PCRd) and dynamic PCR rules (PCRa, PCRb, PCRc) are analyzed below.

### 1. Example 1 (classical case in Smarandache's paper<sup>[15]</sup>)

In Table 1, the revised PCR rule distributes more BBA to single elements than other three methods. The calculation of revised PCR rule and PCRa rule is 46, higher than that of PCRb rule and PCRc rule.

Table 1. Fusion result and calculation of Smarandache's classical case

	$\theta_1$	$\theta_2$	$\theta_3 \equiv \emptyset$	$\theta_1 \cup \theta_2$	Calculation
Prior : $m_1(\cdot)$	0.2	0.4	0.3	0.1	–
Prior : $m_2(\cdot)$	0.3	0.1	0.4	0.2	–
$m_{PCRa}(\cdot)$	0.420	0.452	0	0.128	46
$m_{PCRb}(\cdot)$	0.404	0.436	0	0.160	30
$m_{PCRc}(\cdot)$	0.364	0.396	0	0.240	26
$m_{PCRd}(\cdot)$	0.417	0.457	0	0.126	46

### 2. Example 2 (dynamic frame example with overall process)

Lets assume a fusion system which runs on hybrid DSm model. The system starts at time  $t = 1$ , the discernment frame  $\Theta = \{\theta_1, \theta_2, \theta_3, \theta_2 \cap \theta_3\}$ . There is new information to confirm  $\theta_2 \cap \theta_3 \equiv \emptyset$  in  $t = 3$ , the discernment frame update as  $\Theta = \{\theta_1, \theta_2, \theta_3\}$  ( $m_3(\theta_2 \cap \theta_3)$  also appears in original BBA). There is new information added to confirm  $\theta_3 \equiv \emptyset$  in  $t = 9$ , the discernment frame update

as  $\Theta = \{\theta_1, \theta_2\}$ . We assume that the target is 1 in the simulation. The sensor obtain the incorrect identification result (BBA) caused by interference in time  $t = 4$  and  $t = 8$ . The BBA and non-existential integrity constraints at different moments are shown in Table 2.

**Table 2. BBA and non-existential integrity constraints at different moments**

Time	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_2 \cap \theta_3$	Constraints
$m_{t=1}(\cdot)$	0.5	0.2	0.2	0.1	—
$m_{t=2}(\cdot)$	0.6	0.1	0.2	0.1	—
$m_{t=3}(\cdot)$	0.7	0.1	0.1	0.1	$\theta_2 \cap \theta_3 \equiv \emptyset$
$m_{t=4}(\cdot)$	0.2	0.7	0.1	0	$\theta_2 \cap \theta_3 \equiv \emptyset$
$m_{t=5}(\cdot)$	0.7	0.1	0.1	0.1	$\theta_2 \cap \theta_3 \equiv \emptyset$
$m_{t=6}(\cdot)$	0.6	0.1	0.1	0.2	$\theta_2 \cap \theta_3 \equiv \emptyset$
$m_{t=7}(\cdot)$	0.6	0.2	0.1	0.1	$\theta_2 \cap \theta_3 \equiv \emptyset$
$m_{t=8}(\cdot)$	0.3	0.4	0.2	0.1	$\theta_2 \cap \theta_3 \equiv \emptyset$
$m_{t=9}(\cdot)$	0.6	0.1	0.2	0.1	$\theta_2 \cap \theta_3 \equiv \theta_3 \equiv \emptyset$
$m_{t=10}(\cdot)$	0.8	0.1	0.1	0	$\theta_2 \cap \theta_3 \equiv \theta_3 \equiv \emptyset$

The BBA is fused by three PCR rules and revised PCR rule. The dynamic fusion results are shown in Table 3. The tendency of fusion result BBA for  $\theta_1$  displays in Fig.2. The figure shows four key points of BBA for  $\theta_1$ .

**Table 3. Dynamic fusion result**

Number of fusion		$\theta_1$	$\theta_2$	$\theta_3$	$\theta_2 \cap \theta_3$	$\theta_1 \cup \theta_2 \cup \theta_3$
1	$m_{PCRa}(\cdot)$	0.6862	0.0583	0.0986	0.1569	0
	$m_{PCRb}(\cdot)$	0.6862	0.0583	0.0986	0.1569	0
	$m_{PCRe}(\cdot)$	0.6862	0.0583	0.0986	0.1569	0
	$m_{PCRd}(\cdot)$	0.6862	0.0583	0.0986	0.1569	0
2	$m_{PCRa}(\cdot)$	0.8919	0.0465	0.0616	0	0
	$m_{PCRb}(\cdot)$	0.8820	0.0516	0.0664	0	0
	$m_{PCRe}(\cdot)$	0.8767	0.0463	0.0613	0	0.0157
	$m_{PCRd}(\cdot)$	0.9203	0.0327	0.0470	0	0
3	$m_{PCRa}(\cdot)$	0.6254	0.3499	0.0247	0	0
	$m_{PCRb}(\cdot)$	0.6180	0.3556	0.0264	0	0
	$m_{PCRe}(\cdot)$	0.6153	0.3585	0.0262	0	0
	$m_{PCRd}(\cdot)$	0.6462	0.3338	0.0200	0	0
4	$m_{PCRa}(\cdot)$	0.7881	0.1895	0.0224	0	0
	$m_{PCRb}(\cdot)$	0.7836	0.1935	0.0229	0	0
	$m_{PCRe}(\cdot)$	0.7817	0.1954	0.0229	0	0
	$m_{PCRd}(\cdot)$	0.8179	0.1609	0.0212	0	0
5	$m_{PCRa}(\cdot)$	0.8697	0.1073	0.0230	0	0
	$m_{PCRb}(\cdot)$	0.8669	0.1098	0.0233	0	0
	$m_{PCRe}(\cdot)$	0.8657	0.1111	0.0232	0	0
	$m_{PCRd}(\cdot)$	0.9061	0.0714	0.0225	0	0
6	$m_{PCRa}(\cdot)$	0.8961	0.0841	0.0197	0	0
	$m_{PCRb}(\cdot)$	0.8946	0.0857	0.0197	0	0
	$m_{PCRe}(\cdot)$	0.8939	0.0862	0.0199	0	0
	$m_{PCRd}(\cdot)$	0.9167	0.0651	0.0182	0	0
7	$m_{PCRa}(\cdot)$	0.7781	0.1707	0.0512	0	0
	$m_{PCRb}(\cdot)$	0.7769	0.1717	0.0514	0	0
	$m_{PCRe}(\cdot)$	0.7763	0.1722	0.0515	0	0
	$m_{PCRd}(\cdot)$	0.7800	0.1679	0.0521	0	0
8	$m_{PCRa}(\cdot)$	0.8945	0.1055	0	0	0
	$m_{PCRb}(\cdot)$	0.8867	0.1133	0	0	0
	$m_{PCRe}(\cdot)$	0.8786	0.1059	0	0	0.0155
	$m_{PCRd}(\cdot)$	0.9282	0.0718	0	0	0
9	$m_{PCRa}(\cdot)$	0.9601	0.0399	0	0	0
	$m_{PCRb}(\cdot)$	0.9571	0.0429	0	0	0
	$m_{PCRe}(\cdot)$	0.9568	0.0417	0	0	0.0015
	$m_{PCRd}(\cdot)$	0.9767	0.0233	0	0	0

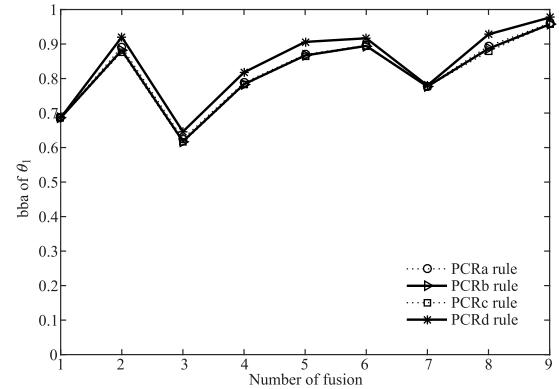


Fig. 2. The tendency of fusion result BBA for  $\theta_1$

The calculation of four combination rules is simulated and analyzed in Table 4.

**Table 4. The calculation of four combination rules**

	1	2	3	4	5	6	7	8	9	Total
$m_{PCRa}$	50	54	54	54	54	54	54	24	24	422
$m_{PCRb}$	50	51	51	51	51	51	51	22	22	400
$m_{PCRe}$	50	46	50	50	50	50	50	26	26	398
$m_{PCRd}$	50	54	54	54	54	54	54	24	24	422

### 3. Analysis of simulation

Example 1 is classical case in Ref.[15]. The character of single element focusing is shown in example 1. A complete simulation with dynamic non-existential integrity constraints is given in example 2. The original BBA is responding to the time BBA obtained (Time). In Table 2, we define the time of combination as number of fusion (Number). Number 1 means the first time of fusion, and so on. The fusion begins in Time 1, the BBA of Time 1 and that of Time 2 are fused to obtain fusion result of Number 1. The constraints of  $\theta_2 \cap \theta_3 \equiv \emptyset$  and  $\theta_3 \equiv \emptyset$  are added in Time 3 and Time 9. The data of Time 4 and Time 8 are conflicting evidence. From Table 2 and Fig.1, revised PCR rule is better than other three combination rule in the overall process. There are not constraints in Number 1. So the fusion results of four rule are same. The Number 6 and Number 9 are the fusion result of low conflicting condition shown in Fig.3(b) and Fig.3(d) respectively. The fusion result of  $\theta_1 m_{PCRa}(\theta_1)$  reaches 0.9167 and 0.9767 in Number 6 and Number 9 better 2.3% and 1.7% than PCRa rule (the best in the three dynamic PCR rules). The Number 2 and Number 7 are the fusion result of high conflicting condition shown in Fig.3(a) and Fig.3(c) respectively. The fusion result of  $\theta_1 m_{PCRa}(\theta_1)$  reach 0.6462 and 0.7780 in Number 2 and Number 7 better 3.3% and 0.25% than PCRa rule (the best in the three dynamic PCR rules). Revised PCR rule has best fusion result in processing low conflicting evidence and high conflicting evidence.

The calculation of four combination rules is shown in Table 4. The PCRe rule has the lowest calculation as 398.

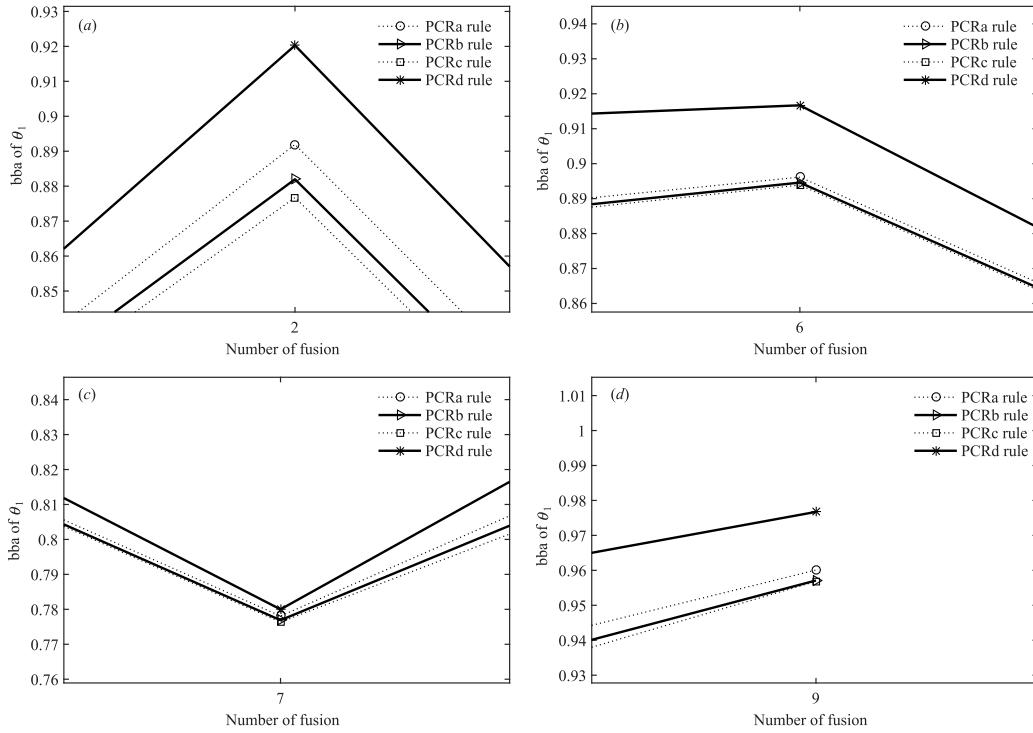


Fig. 3. Key points of BBA for  $\theta_1$  in fusion process. (a) Fusion results of fusion Number 2; (b) Fusion results of fusion Number 6; (c) Fusion results of fusion Number 7; (d) Fusion results of fusion Number 9

The calculation of PCRb rule is larger than 400. The PCRa rule has the largest calculation as 422 in three classical rules. Revised PCR rule is based on the calculated BBA of PCRa rule to obtain the same calculation as PCRa. In 9 times fusion, calculation of revised PCR rule is larger 5.5% and 6% than PCRb rule and PCRc rule, which is acceptable.

In conclusion, revised PCR rule improves the way of distribution based on the using the information of PCRa rule effectively. The fusion result of revised PCR rule is better than PCR rules in both high conflicting condition and low conflicting condition. The calculation remains the same as PCRa rule without extra calculation. The accuracy of fusion result and controllability of calculation show the effectiveness of revised PCR rule.

## V. Conclusions

Dynamic frame fusion satisfies the requirement of practical application. The dynamic frame fusion combination rule based on hybrid DSm model is studied in the paper. A new dynamic frames PCR combination rule is presented to overcome the shortness in processing empty set  $\emptyset$  and redistribution weight of classical dynamic PCR rule. The effectiveness of revised PCR rule is studied and simulated in the both aspect of fusion accuracy and calculation. The fusion weight for different evidence is an important research direction besides dynamic frame. The dynamic weight combination rule and dynamic frame combination rule are specific to the different parts of evidence.

The two kinds of combination rules can be mixed, which is the further research emphasis for our team.

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