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An Improved Focal Element Control Rule

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Abstract

The computational complexity of evidence theory is a hot issue in current research. Dezert-Smarandache theory (DSmT) introduces conflicting focal element, which makes the calculation complexity increases sharply. This paper starts with the focal element control rule used by the approximate calculation method mostly. The examples show that the improved rule is effective and feasible in both Shafer model and DSm model.

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Evidence reasoning theory; Dezert-Smarandache theory; Approximate calculation; Energy function; Focal element

1. Introduction

Evidence reasoning theory is an effective measure to solve uncertainty problem in the multi-sources information fusion system. But the tremendous calculation is the main problem to obstruct the application of evidence reasoning theory. In the Dempster-Shafer theory (DST), there are $\left|2^{\Theta}\right|$ subsets in hypothesis set Θ . The subsets are valued by the belief function. The subsets are all considered in evidence combination. DSmT introduces conflicting focal element. So the number of focal element is bigger than DST. The $\left|D^{\Theta}\right|$ subsets are valued by the belief function, which has larger calculation.

To overcome the limitation, researchers present some approximate calculation methods [1-5]. One of the most important steps is focal element control rule which chooses the focal elements involved in the fusion. There are two kind of control rule as a whole. The first kind is the traditional focal element control

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rule. Only the basic belief assignment (bba) of proposition is considered in this kind of rule, and the relation between the propositions is out of consideration (union focal element and intersection focal element are called composing focal element which is different from single focal element). The second kind is energy function rule[6,7]. This kind of rule takes the relations between the propositions into account. The rule introduces cardinal number of composing focal element and single focal element to solve the drawback of traditional focal element control rule. So the second rule is used in most approximation methods in DST. But DSmT introduces conflicting focal element, which makes the calculation complexity increase sharply and the cardinal number rise. The useful information may be discarded so as to obtain a wrong result. So this paper presents an improved focal element control rule The method introduces a switch factor into traditional energy function and improves the control rule focal element to reduce the calculation and maintain computational precision. The examples are tested and verified the effectiveness and rationality of the method.

2. Analysis of computational complexity

The computational complexity of evidence reasoning theory is mainly the calculation of evidence combination rule. Dempster combination rule has the multiplication number of 2^{kn} and the division number of 1. The computational complexity of Dempster combination rule is $O(2^{kn})$ without consideration the division number. DSmT introduces conflicting focal element, which makes the cardinal number rise. The computational complexity is $O(2^{k(2^n)})$ in DSmT. The hybrid DSm model introduces limitation which has smaller computational complexity than free DSm model.

3. Existing focal element control rule and its limitation

3.1. traditional focal element control rule

Traditional focal element control rule chooses the focal element by the bba. But the different relationship between composing focal element and single focal element is not considered in this kind of control rule. The information of the evidence depends on two factors: one is the bba of the focal element in the evidence, another is the cardinal number. If the bba is the only factor considered, the discard and reservation of focal element is imprecise. And if the composing focal element is chosen in an improper condition, the calculation would lager.

Example 1: Let consider $\Theta = \{\theta_1, \theta_2, \theta_3\}$ the frame of discernment of the fusion problem under Shafer model, there are three pieces of evidence:

$E_1 : m_1(\theta_1) = 0.6$, $m_1(\theta_2) = 0.3$, $m_1(\theta_3) = 0.1$.

$E_2 : m_2(\theta_1) = 0.6$, $m_2(\theta_1 \cup \theta_2) = 0.1$, $m_2(\theta_3) = 0.3$.

$E_3 : m_3(\theta_1) = 0.5$, $m_3(\theta_2) = 0.2$, $m_3(\theta_1 \cup \theta_2 \cup \theta_3) = 0.3$.

The three pieces of evidence are used the traditional focal element control rule to remain two focal elements which have the higher bba. Evidence E_1 remains focal element θ_1 and θ_2, evidence E_2 remains focal element θ_1 and θ_3. The entire focal elements are single focal element in evidence E_1. Evidence E_2 has composing focal element but its bba is not higher than the bba of single focal element. So both evidence E_1 and evidence E_2 obtain reasonable results. But evidence E_3 remains focal element θ_1 and $\theta_1 \cup \theta_2 \cup \theta_3$ discards the focal element θ_2, which is not reasonable.

3.2. Energy function method

Some experts think that cardinal number must be considered for the focal element control rule to overcome the limitation of traditional focal elements control rule. The simple way is to divide cardinal number into bba of each evidence, as $\frac{1}{|A|}m(A)$.This measure is called energy function method.

Define 1: The energy function in focal element control rule is defined as:

$$E(A)=\frac{1}{|A|}m(A) \tag{1}$$

Where A is focal element, and $|A|$ is cardinal number.

Define 2: The average energy function in focal element control rule is defined as:

$$\overline{E}_i=\frac{\left(\sum_{j=1}^{N_j}\frac{1}{|A_j|}m_i(A_j)\right)}{N_j},A_j\subseteq\Theta,i=1,2,\ldots,n \tag{2}$$

The energy function method introduces cardinal number of focal element, the bba and cardinal number should be considered in the method to overcome the limitation of traditional focal element control rule. Evidence E_1 remains focal element θ_1, θ_2 and E_2 remains focal element θ_1, θ_3 by energy function method in example 1, which is the reasonable result as traditional focal element control rule. But Evidence E_3 remains focal element θ_1, θ_2 which corrects the wrong result made by traditional focal element control rule.

Therefore energy function method is a common approximately computational measure of DST. But DSmT introduces conflicting focal elements to deal with high conflicting evidence, but the cardinal number increases in the meantime. The high-valued bba may be discarded for the reason of cardinal number to obtain a wrong result by $\frac{1}{|A|}m(A)$ in the energy function method.

Example 2: Let consider $\Theta=\{\theta_1,\theta_2,\theta_3\}$ the frame of discernment of the fusion problem under free DSm model, there is one piece of evidence:

$m(\theta_1)=0.3$, $m(\theta_1\cap\theta_2)=0.2$, $m(\theta_1\cup\theta_2\cup\theta_3)=0.5$.

The evidence is used energy function method to obtain $\frac{1}{|\theta_1|}m(\theta_1)=0.075$, $\frac{1}{|\theta_1\cap\theta_2|}m(\theta_1\cap\theta_2)=0.1$, $\frac{1}{|\theta_1\cup\theta_2\cup\theta_3|}m(\theta_1\cup\theta_2\cup\theta_3)=0.714$. It shows that $\frac{1}{|\theta_1\cup\theta_2\cup\theta_3|}m(\theta_1\cup\theta_2\cup\theta_3)<\frac{1}{|\theta_1|}m(\theta_1)<\frac{1}{|\theta_1\cap\theta_2|}m(\theta_1\cap\theta_2)$.The focal element θ_1, $\theta_1\cap\theta_2$ is remained but focal element $\theta_1\cup\theta_2\cup\theta_3$ whose bba is 0.5 is discarded. The result is not reasonable.

4. Focal element control rule based on improved energy function

The cardinal number is big in DSmT, so the focal element control rule based on energy function could not work well. We introduce the switch factor λ which reduces the cardinal number of composing focal element to improve the rule.

Define 3: The improved energy function is defined as:

$$E(A)=\frac{f(\lambda)}{1+\lambda|A|}m(A) \tag{3}$$

Where A is focal element, $|A|$ is cardinal number. λ is switch factor, $f(\lambda)$ is function of $f(\lambda)$, where $f(\lambda)=\begin{cases}1 & \lambda<1\\ \lambda & \lambda\geq 1\end{cases}$.

The switch factor λ is determined by two reasons: the first is value of cardinal number. For example, λ would value a less number (0.5) to make the value that bba divides cardinal number would not be variable (variable means the value that big bba divides big cardinal number is less than the value that small bba divides small cardinal number, as 0.7/7<0.2/1, which makes the result imprecise even absolute false) . The second is the frame of discernment. The range of cardinal number is largest in the free DSm model, the range is less in the hybrid DSm model and the range is least in the Shafer model when the quantity of single focal element is same. There is not an accordant formula for switch factor λ, so the range should be tested by experimentation.

Define 4: The improved average energy function in focal element control rule is defined as:

$$\overline{E}_i=\frac{\left(\sum_{j=1}^{N_j}\frac{f(\lambda)}{1+\lambda|A_j|}m_i(A_j)\right)}{N_j},\ A_j\subseteq\Theta, i=1,2,\ldots,n \tag{4}$$

Therefore we obtain improved energy function and improved average energy function in focal element control rule through formula (3) and (4). The focal element will be discarded when the energy of focal element is less than the average energy of focal element, contrariwise other focal element is remained to the process of fusion.

Example 3: Let consider $\Theta=\{\theta_1,\theta_2,\theta_3\}$ the frame of discernment of the fusion problem under free DSm model, there are three pieces of evidence:

$E_1: m_1(\theta_1)=0.6,\ m_1(\theta_2)=0.3,\ m_1(\theta_3)=0.1.$

$E_2: m_2(\theta_1)=0.6,\ m_2(\theta_1\cup\theta_2)=0.1,\ m_2(\theta_3)=0.3.$

$E_3: m_3(\theta_1)=0.3,\ m_3(\theta_1\cap\theta_2)=0.2,\ m_3(\theta_1\cup\theta_2\cup\theta_3)=0.5.$

The three evidence is used the improved focal element control rule t, $\lambda=0.5$. Evidence E_1 remain focal element θ_1 as $\frac{1}{1+0.5|\theta_1|}m(\theta_1)=0.2$, $\frac{1}{1+0.5|\theta_2|}m(\theta_2)=0.1$, $\frac{1}{1+0.5|\theta_3|}m(\theta_3)=0.033$ and average energy is 0.111 .Evidence E_2 remains focal element θ_1, as $\frac{1}{1+0.5|\theta_1|}m(\theta_1)=0.2$,

$\frac{1}{1+0.5|\theta_1 \cup \theta_2|} m(\theta_1 \cup \theta_2) = 0.024$, $\frac{1}{1+0.5|\theta_3|} m(\theta_3) = 0.1$ and average energy is 0.108. Evidence E_3 remains focal element $\theta_1 \cup \theta_2 \cup \theta_3$, $\frac{1}{1+0.5|\theta_1|} m(\theta_1) = 0.1$, $\frac{1}{1+0.5|\theta_1 \cap \theta_2|} m(\theta_1 \cap \theta_2) = 0.1$, $\frac{1}{1+0.5|\theta_1 \cup \theta_2 \cup \theta_3|} m(\theta_1 \cup \theta_2 \cup \theta_3) = 0.125$ and average energy is 0.108.

The example 3 shows that the focal element with big bba is remained to the further fusion, which illuminates the usefulness of the improved focal elements control rule.

5. Conclusion

DSmT is a new evidence reasoning theory to solve the problem of conflicting evidence. But DSmT introduces conflicting focal elements which makes the existing focal element control rule invalid. So an improved focal elements control rule is presented in this paper to weightiness of the focal element to obtain a reasonable result and a proper calculation complexity.

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