A new class of NeutroOpen, NeutroClosed, AntiOpen and AntiClosed sets in NeutroTopological and AntiTopological spaces

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A new class of NeutroOpen, NeutroClosed, AntiOpen and AntiClosed sets in NeutroTopological and AntiTopological spaces

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Abstract

A lot of research has been done on the types of open and closed sets in general topological space and also in general bitopological spaces. Types of sets like: pre-open sets and pre-closed sets, semi-open sets and semi-closed sets, α −open sets and α-closed sets, regular open sets and regular closed sets, g-open sets and gclosed sets and many more have been defined and studied. In the current study, attempt has been made to define and give examples of a new category of open and closed sets namely, NeutroOpen and NeutroClosed sets. Further, the concept of neutro-toplology is used to define NeutroPreOpen and NeutroPreClosed sets, NeutroSemiOpen and NeutroSemiClosed sets, NeutroAlphaOpen and NeutroAlphaClosed sets, NeutroRegularOpen and NeutroRegularClosed sets, NeutroBetaOpen and NeutroBetaClosed sets and several examples have been given to illustrate each of the new classes of sets. Also, the concept of AntiToplogy has been used to define another class of sets, namely, AntiOpen and AntiClosed sets of the above five classes of sets, namely, regular-open/closed; semi-open/closed, α −open/closed, β −open/closed pre-open/closed sets. Further, a new class of subsets are identified which are named as NeutroTauOpen and NeutroTauClosed sets. Similar subsets in anti-topological spaces are named as AntiTauOpen and AntiTauClosed sets.

Keywords: Neutro-Topological Space; NeutroAlphaOpen; Anti-Topological Space; AntiInterior, AntiTauOpen.
MSC Classification – 2010: 54F65, 54F99, 54H99, 54J05, 54J99, 94D05, 94D99, 97E20, 97E99

1. Introduction:

The notion of regular open sets was used extensively by M.H. Stone in his paper titled “Applications of the theory of Boolean rings to general topology”. He referred to regular open sets as the interior of closed sets [**9**]. The idea of $\alpha - set$ was first in 1965 by O. Njastad [**7**]. The concept of semi-open set was introduced by Levine [**5**]. Mashhour et al. [**6**] first coined the term pre-open sets from the notion of locally dense sets introduced by Corson and Michael [**4**]. The concept of neutro-topological space and anti-topological space was first discussed by MemetSahin et al. [**8**] in June, 2021. Basumatary et al. [**3**] first introduced the notion of NeutroInterior, NeutroClosure and NeutroBoundary and studied their properties in neutro-topological spaces, by introducing NeutroOpen and NeutroClosed sets. Basumataryet. al [**2**] first introduced the notions of AntiInterior, AntiClosure and Antiboundary and studied their properties in anti-topological spaces by introducing AntiOpen and AntiClosed sets. In this paper, we apply the concept of neutro-topology and anti-topology to four types of already defined and studied open sets, namely regular open/closed sets, α −open/closed sets, β −open/closed sets, semi-open/ closed sets and pre-open/closed sets by applying the concept of NeutroInterior, NeutroClosure, AntiInterior and AntiClosure. Further, we have introduced a new class of sets and named them towards the end of this paper.

2. Preliminaries:

Definition 2.1.1: [8**]** If X is a non-empty set and τ is a family of subsets of X, then τ will be called a NeutroTopology on X and the pair (X, τ) a NeutroTopological space, if at least one of the following conditions are satisfied:

(i) $(\emptyset_N \in \tau, X_N \notin \tau)$ or $(X_N \in \tau, \emptyset_N \notin \tau)$.

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(ii) For some n elements $a_1, a_2, \ldots, a_n \in \tau$, $\bigcap_{i=1}^{n} a_i \in \tau$ and for other n elements $b_1, b_2, \ldots, b_n \in \tau$, $p_1, p_2, \ldots, p_n \in \tau$; $[(\bigcap_{i=1}^{n} b_i \notin \tau)$

(iii) For some n elements $a_1, a_2, \ldots, a_n \in \tau$, $\bigcup_{i=1} a_i \in \tau$ [degree of truth T] and for other n elements $b_1, b_2, \ldots, b_n \in \tau$, $p_1, p_2, \ldots, p_n \in \tau$; $[(\bigcup_{i=I} b \notin \tau)$

Remark 2.1.1 [3]: In a neutro-topological space. The conditions (i), (ii), and (or) (iii) are satisfied.

(i) The empty set or the whole set is not open.
(ii) Union of at least two open set is open and union of at least two open set is not open.
(iii) Intersection of at least two open set is open and intersection of at least two open set is not open.

Remark 2.1.2 [3]: Open sets of a neutro-topological space are called NeutroOpen sets and their complements are called NeutroClosed sets.

Definition 2.1.2 [3]: If (X, τ) is a neutro-topological space on X and A a subset of X, then NeutroInterior of A is the union of all NeutroOpen subsets of A and is denoted by Neu$Int(A)$.

Definition 2.1.3 [3]: If (X, τ) is a neutro-topological space on X and A a subset of X, then NeutroClosure of A is the intersection of all NeutroClosed super sets of A and is denoted by NeuCl(A).

Definition 2.1.4 [8]: If X is a non-empty set, τ is a family of subsets of X and one of the following conditions {i, ii, iii} are satisfied then, τ is called an anti-topology and (X, τ) is called an anti-topological space.

(i) $\emptyset, X \notin \tau$
(ii) For all $q_1, q_2, q_3, \ldots, q_n \in \tau$, $\bigcap_{i=1}^{n} q_i \notin \tau$, where n is finite.
(iii) For all $q_1, q_2, q_3, \ldots, q_n \in \tau$, $\bigcup_{i \in I} q_i \notin \tau$, where I is an index set.

The open sets of an anti-topological space are called AntiOpen sets and their complements are called AntiClosed sets.

Remark 2.1.3 [2]: In an anti-topological space. The conditions (i), (ii), and (iii) are satisfied.

(i) Empty set and X are not AntiOpen.
(ii) Union of the AntiOpen sets is not AntiOpen.
(iii) Intersection of the AntiOpen sets is not AntiOpen.

Definition 2.1.5 [2]: If (X, τ) is an anti-topological space on X and A, a subset of X, then the AntiInterior of A is the union of all AntiOpen subsets of A.
That is, $AntiIn(A) = \cup \{B$, where B is AntiOpen and $B \subseteq A\}$

Definition 2.1.6 [2]: If (X, τ) is an anti-topological space on X and A, a subset of X, then the AntiClosure of A is the intersection of all AntiClosed super sets of A.
That is, $AntiC(A) = \cap \{G: G \supseteq A$ and $G$is AntiClosed$\}$

Definition 2.1.7: A subset A of a topological space (X, τ) is said to be **regular open** [**9**] if *A=Int(Cl(A));* **αopen** [**7**] if *A ⊆Int(Cl(Int(A)));* **semi-open** [**5**] if there exits an open set G such that *G⊆A⊆Cl(G)* equivalently *A ⊆Cl(Int(A),* **pre-open** [**6**] if *A ⊆Int(Cl(A))* andβ**-open [1]** if *A ⊆Cl(Int(Cl(A))).*

Definition 2.1.8: A subset A of a topological space (X, τ) is said to be **regular closed** if A^c is regular open in X, i.e. *A = Cl(Int(A));* **α-closed** if *Cl(Int(Cl(A)))⊆A;* **semi-closed** if *Int(Cl(A))⊆A,* **pre-closed** if *Cl(Int(A))⊆A* andβ**-closed** if *Int(Cl(Int(A)))⊆A.*

3. Main focus of the work

3.1: Neutro-Topology:

Definition 3.1.1: If X be a neutro-topological space and A is subset of the space (X, T), then A will be called NeutroPreOpen set if $A \subseteq$NeuIn(NeuCl(A)).

Example 3.1.1: Assume $X = \{1,2,3,4\}$, $T = \{\emptyset, \{1\}, \{2,3\}, \{3,4\}\}$ be a neutro-topology on X and say $A = \{2,3\}$ be a subset of X. Then the NeutroClosed subsets of T are: X, {2,3,4}, {1,4}, {1,2}.

Now, NeuInt(NeuCl(A)) = NeuInt(NeuCl({2,3})) = NeuInt({2,3,4}) = {2,3} ∪ {3,4}

= {2,3,4} ⊇ A. Hence, $A \subseteq$NeuIn(NeuCl(A), showing that A is a NeutroPreOpen set.

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Definition 3.1.2: If X be a neutro-topological space and A is subset of the space (X, T), then A will be called NeutroSemiOpen set if $A \subseteq$Neu$C$(NeuInt(A)).

Example 3.1.2: Assume $X = \{1,2,3,4\}$, $T = \{\emptyset, \{1\}, \{2,3\}, \{3,4\}\}$ be a neutro-topology for X and say $A = \{2,3\}$ be the subset of X. Then the NeutroClosed subsets of T are: X, $\{2,3,4\}$, $\{1,4\}$, $\{1,2\}$.

Now, Neu$C$(NeuInt(A)) = NeuCl(NeuInt($\{2,3\}$))
= Neu$C$($\{2,3\}$)=$\{2,3,4\} \supseteq$A.
Hence, $A \subseteq$Neu$C$(NeuInt(A), showing that A is a NeutroSemiOpen set.

Definition 3.1.3: If X be a neutro-topological space and $A \subseteq X$, then A will be called NeutroAlphaOpen if $A \subseteq$NeuIn(NeuCl(NeuInt(A))).

Example 3.1.3: Assume $X = \{1,2,3,4\}$, $T = \{\emptyset, \{1\}, \{2,3\}, \{3,4\}\}$ be a neutro-topology for X and say $A = \{2,3\}$ be a subset of X. Then the NeutroClosed subsets of T are: X, $\{2,3,4\}$, $\{1,4\}$, $\{1,2\}$.
Now, NeuInt(NeuCl(NeuInt(A))) = NeuInt(NeuCl(NeuInt($\{2,3\}$))) = NeuInt(NeuCl($\{2,3\}$)) =NeuInt($\{2,3,4\}$) = $\{2,3\}$ = A $\supseteq$ A.
Hence, $A \subseteq$NeuIn(NeuCl(NeuInt(A))), showing that A is a NeutroAlphaOpen set.

Definition 3.1.4: If X be a neutro-topological space and $A \subseteq X$, then A will be called NeutroRegularOpen if A = NeuIn(NeuCl(A)).

Example 3.1.4: Assume $X = \{1,2,3,4\}$, $T = \{\emptyset, \{1\}, \{2,3\}, \{3,4\}\}$ be a neutro-topology for X and say $A = \{2,3,4\}$ be a subset of X. Then the NeutroClosed subsets of T are: X, $\{2,3,4\}$, $\{1,4\}$, $\{1,2\}$.
Now, NeuInt(NeuCl(A)) = NeuInt(NeuCl($\{2,3,4\}$)) = NeuInt($\{2,3,4\}$) = $\{2,3\} \cup \{3,4\} = \{2,3,4\}$ = A. Hence, A = NeuIn(NeuCl(A), showing that A is a NeutroRegularOpen set.

Definition 3.1.5: If X be a neutro-topological space and $A \subseteq X$, then A will be called NeutroBetaOpen if $A \subseteq$Neu$C$(NeuInt(NeuCl(A))).

Example 3.1.5: Assume $X = \{1,2,3,4,5\}$, $T = \{\emptyset, \{1\}, \{1,3\}, \{2,4\}, \{3,4,5\}\}$. Then (X, T) is a neutro-topological space. The NeutroOpen sets are: $\emptyset$, $\{1\}$, $\{1,3\}$, $\{2,4\}$, $\{3,4,5\}$ and thus the NeutroClosed sets are: X, $\{2,3,4,5\}$, $\{2,4,5\}$, $\{1,3,5\}$, $\{1,2\}$.
Let $A = \{2,4\}$. Then NeuCl (NeuInt(NeuCl(2,4))) = NeuCl(NeuInt($\{2,4,5\}$)) = NeuCl($\{2,4\}$) = $\{2,4,5\}$ which shows that $A \subseteq$NeuCl(NeuInt(NeuCl(A))). Hence $A = \{2,4\}$ is NeutroBetaOpen.

Definition 3.1.6: A subset A of a neutro-topological space (X, T) will be called NeutroRegularClosedif A^c is NeutroRegularOpen in X or A = Neu*Cl(*Neu*Int(A))*.

Example 3.1.6: Assume $X = \{1,2,3,4\}$, $T = \{\emptyset, \{1\}, \{2,3\}, \{3,4\}, \{1,3,4\}\}$ be a Neutro-topology on X and say $A = \{1\}$ be the subset of X. Then the NeutroClosed subsets of T are: X, $\{2,3,4\}$, $\{1,4\}$, $\{1,2\}$, $\{2\}$. $A^c = \{2,3,4\}$.
Now, NeuInt(NeuCl(A^c)) = NeuInt(NeuCl($\{2,3,4\}$)) = NeuInt($\{2,3,4\}$) = $\{2,3\} \cup \{3,4\} = \{2,3,4\} = A^c$. Hence, A^c= NeuIn(NeuCl(A^c), showing that A^c is a NeutroRegularOpen set. Hence, A is a NeutroRegularClosed set.

Also, Neu*Cl(*Neu*Int(A))*=NeuCl(Neu$Int\{1\}$) = NeuCl($\{1\}$) = $\{1,4\} \cap \{1,2\} = \{1\} = A$.

Definition 3.1.7: A subset A of a neutro-topological space (X, τ) will be called NeutroAlphaClosedif Neu*Cl(*Neu*Int(*Neu*Cl(A)))⊆A.*

Example 3.1.7: Assume $X = \{1,2,3,4\}$, $T = \{\emptyset, \{1\}, \{2,3\}, \{3,4\}, \{1,3,4\}\}$ be a neutro-topology for X and say $A = \{2,3,4\}$ be the subset of X. Then the NeutroClosed subsets of T are: X, $\{2,3,4\}$, $\{1,4\}$, $\{1,2\}$, $\{2\}$.
Now, NeuCl(NeuInt(NeuCl(A))) = NeuCl(NeuInt(NeuCl($\{2,3,4\}$))) = NeuCl(NeuInt($\{2,3,4\}$)) = NeuCl($\{2,3\} \cup \{3,4\}$) = NeuCl($\{2,3,4\}$) = $\{2,3,4\}$ = A. Hence, Neu*Cl(*Neu*Int(*Neu*Cl(A)))⊆A*, showing that $A = \{2,3,4\}$ is NeutroAlphaClosed.

Definition 3.1.8: A subset A of a neutro-topological space (X, τ) will be called NeutroSemiClosedif NeuIn(NeuCl(A)) $\subseteq A$.

Example 3.1.8: Assume $X = \{1,2,3,4\}$, $T = \{\emptyset, \{1\}, \{2,3\}, \{3,4\}, \{1,3,4\}\}$ be a neutro-topology for X and say $A = \{1,4\}$ be the subset of X. Then the NeutroClosed subsets of T are: X, $\{2,3,4\}$, $\{1,4\}$, $\{1,2\}$, $\{2\}$.
Now, NeuIn(NeuCl(A)) = NeuInt(NeuCl($\{1,4\}$)) = NeuInt($\{1,4\}$) = $\{1\}$.
Hence, NeuIn(NeuCl(A)) $\subseteq A$, showing that $A = \{1,4\}$ is NeutroSemiClosed.

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Definition 3.1.9: A subset A of a neutro-topological space (X, τ) will be called NeutroPreClosedif NeuCl(Neu$Int(A)$)⊆A.

Example 3.1.9: Assume X = {1,2,3,4}, T = {∅, {1}, {2}, {1,3}, {2,4}} be a neutro-topology for X and say A = {1,3,4} be the subset of X. Then the NeutroClosed subsets of T are: X, {2,3,4}, {1,3,4}, {2,4}, {1,3}.

Now, NeuCl(NeuInt(A)) = NeuCl(NeuInt({1,3,4})) = NeuCl({1,3}) = {1,3,4} ∩ {1,3} = {1,3} ⊆A. Hence, Neu$C$(NeuInt(A)) ⊆A, showing that A = {1,3,4} is NeutroPreClosed.

Definition 3.1.10: If X be a neutro-topological space and A⊆X, then A will be called NeutroBetaClosed if NeuIn(NeuCl(NeuInt(A))) ⊆A.

Example 3.1.10: Assume X = {1,2,3,4,5}, T = {∅, {1}, {1,3}, {2,4}, {3,4,5}}. Then (X, T) is a neutrotopological space. The NeutroOpen sets are: ∅, {1}, {1,3}, {2,4}, {3,4,5} and thus the NeutroClosed sets are: X, {2,3,4,5}, {2,4,5}, {1,3,5}, {1,2}.

Let A = {2,4,5}. Then NeuInt (NeuCl(NeuInt(2,4,5))) = NeuInt(NeuCl({2,4})) = NeuInt({2,4,5}) = {2,4} which shows that NeuInt(NeuCl(NeuInt(A))) ⊆A. Hence A = {2,4,5} is NeutroBetaClosed.

3.2: Anti-Topology:

Definition 3.2.1: If (X, T) be an anti-topological space and A⊆X, then A will be called AntiPreOpen set if A⊆ $AntiIn$($AntiCl$(A)).

Example 3.2.1: Assume X = {1,2,3,4,5}, T = {{1}, {4}, {2,3}, {3,5}} be an anti-topology for X and say A = {2,3} be a subset of X. Then the AntiClosed subsets of T are: {2,3,4,5}, {1,2,3,5}, {1,4,5}, {1,2,4}.

Now, $AntiInt$($AntiCl$(A)) = $AntiInt$($AntiCl$({2,3})) = $AntiInt$({2,3,4,5 ∩ {1,2,3,5}}) = $AntiIn$({2,3,5}) = {2,3,5} ⊇ A. Hence, A⊆$AntiIn$($AntiCl$(A), showing that A is a AntiPreOpen set.

Definition 3.2.2: If (X, T) be an anti-topological space and A⊆X, then A will be called AntiSemiOpen set if A⊆$AntiC$($AntiInt$(A)).

Example 3.2.2: Assume X = {1,2,3,4,5}, T = {{1}, {4}, {2,3}, {3,5}} be an anti-topology for X and let A = {2,3,4} be the subset of X. Then the AntiClosed subsets of T are: {2,3,4,5}, {1,2,3,5}, {1,4,5}, {1,2,4}.

Now, $AntiC$($AntiInt$(A)) = $AntiCl$($AntiInt$({2,3,4}))

= $AntiC$({2,3,4}) = {2,3,4,5} ⊇ A. Hence, A⊆$AntiC$($AntiInt$(A),

showing that A is a AntiSemiOpen set.

Definition 3.2.3: If (X, T) be an anti-topological space and A⊆X, then A will be called AntiAlphaOpen if A⊆$AntiIn$($AntiCl$($AntiInt$(A))).

Example 3.2.3: Assume X = {1,2,3,4,5}, T = {{1}, {4}, {2,3}, {3,5}} be an anti-topology on X and say A = {2,3} be the subset of X. Then the AntiClosed subsets of T are: {2,3,4,5}, {1,2,3,5}, {1,4,5}, {1,2,4}.

Now,
$AntiIn$($AntiCl$($AntiInt$(A)))=$AntiInt$($AntiCl$($AntiInt$({2,3})))
= $AntiInt$($AntiCl$({2,3})) =$AntiInt$({2,3,4,5} ∩ {1,2,3,5}) = $AntiInt$({2,3,5}) = {2,3,5}.
Hence, A⊆$AntiIn$($AntiCl$($AntiInt$(A))), showing that A is a AntiAlphaOpen set.

Definition 3.2.4: If (X, T) be an anti-topological space and A⊆X, then A will be called AntiRegularOpen if A = $AntiIn$($AntiCl$(A)).

Example 3.2.4: Assume X = {1,2,3,4,5}, T = {{1}, {4}, {2,3}, {3,5}} be an anti-topology for X and say A = {1,4} be the subset of X. Then the AntiClosed subsets of T are: {2,3,4,5}, {1,2,3,5}, {1,4,5}, {1,2,4}.
Now, $AntiInt$($AntiCl$(A)) = $AntiInt$($AntiCl$({1,4})) = $AntiInt$({1,4,5} ∩ {1,2,4}) = $AntiInt$({1,4}) = {1} ∪ {4} = {1,4} = A. Hence, A = $AntiIn$($AntiCl$(A), showing that A is a AntiRegularOpen set.

Definition 3.2.5: If (X, T) be an anti-topological space and A⊆X, then A will be called AntiBetaOpen if A⊆$AntiC$($AntiInt$($AntiCl$(A))).

Example 3.2.5: Assume X = {1,2,3,4,5}, T = {{1}, {2,3}, {3,4,5}}. Then (X, T) is an anti-topological space. The AntiOpen sets are: {1}, {2,3}, {3,4,5} and thus the NeutroClosed sets are: {2,3,4,5}, {1,4,5}, {1,2}.

Let A = {3,4,5}, then $AntiCl$ ($AntiInt$($AntiCl$({3,4,5}))) = $AntiCl$($AntiInt$({2,3,4,5})) =

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$AntiCl(\{2,3,4,5\})$ = {2,3,4,5} which shows that $A \subseteq AntiCl(AntiInt(AntiCl(A)))$. Hence {3,4,5} is AntiBetaOpen.

Definition 3.2.6: A subset A of an anti-topological space (X, τ) will be called AntiRegularClosedif A^c is AntiRegularOpen in X or equivalently if $A = AntiCl(AntiInt(A))$.

Example 3.2.6: Assume $X = \{1,2,3,4,5\}$, $T = \{\{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}\}$ be an anti-topology for X and say $A = \{1,2,3\}$ be a subset of X. Then the AntiClosed subsets of T are: {3,4,5}, {1,4,5}, {1,2,5}, {1,2,3}.
Now, $AntiCl(AntiInt(A)) = AntiCl(AntiInt(\{1,2,3\})) = AntiCl(\{1,2,3\})$
$= \{1,2,3\} = A$.
Hence, the subset A is AntiRegularClosed.

Definition 3.2.7: A subset A of an anti-topological space (X, τ) will be called AntiAlphaClosedif $AntiCl(AntiInt(AntiCl(A))) \subseteq A$.

Example 3.2.7: Assume $X = \{1,2,3,4\}$, $T = \{\{1\}, \{2,3\}, \{3,4\}\}$ be an anti-topology for X and say $A = \{1,2\}$ be a subset of X. Then the AntiClosed subsets of T are: {2,3,4}, {1,4}, {1,2}.
Now, $AntiC(AntiInt(AntiCl(A))) = AntiCl(AntiInt(AntiCl(\{1,2\})))$
$= AntiC(AntiInt(\{1,2\})) = AntiCl(\{1\}) = \{1,4\} \cap \{1,2\} = \{1\} \subseteq A$.
Hence, $AntiCl(AntiInt(AntiCl(A))) \subseteq A$, showing that $A = \{1,2\}$ is AntiAlphaClosed.

Definition 3.2.8: A subset A of an anti-topological space (X, τ) will be called AntiSemiClosedif $AntiIn(AntiCl(A)) \subseteq A$.

Example 3.2.8: Assume $X = \{1,2,3,4,5\}$, $T = \{\{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}\}$ be an anti-topology for X and say $A = \{1,4,5\}$ be a subset of X. Then the AntiClosed subsets of T are: {3,4,5}, {1,4,5}, {1,2,5}, {1,2,3}.
Now, $AntiInt(AntiCl(A)) = AntiInt(AntiCl(\{1,4,5\})) = AntiInt(\{1,4,5\}) = \{4,5\}$.

Hence, $AntiIn(AntiCl(A)) \subseteq A$, showing that $A = \{1,4,5\}$ is AntiSemiClosed.

Definition 3.2.9: A subset A of an anti-topological space (X, τ) will be called AntiPreClosedif $AntiCl(AntiInt(A)) \subseteq A$.

Example 3.2.9: Assume $X = \{1,2,3,4,5\}$, $T = \{\{1\}, \{2\}, \{3\}, \{4,5\}\}$ be an anti-topology for X and say $A = \{1,2,4\}$ be a subset of X. Then the AntiClosed subsets of T are: {2,3,4,5}, {1,3,4,5}, {1,2,4,5}, {1,2,3}.
Now, $AntiCl(AntiInt(A)) = AntiCl(AntiInt(\{1,2,4\})) = AntiCl(\{1,2\}) = \{1,2,4,5\} \cap \{1,2,3\} = \{1,2\} \subseteq A$. Hence, $AntiC(AntiInt(A)) \subseteq A$, showing that $A = \{1,2,4\}$ is AntiPreClosed.

Definition 3.2.10: If X be an anti-topological space and $A \subseteq X$, then A will be called AntiBetaClosed if $AntiIn(AntiCl(AntiInt(A))) \subseteq A$.

Example 3.2.10: Assume $X = \{1,2,3,4,5\}$, $T = \{\{1\}, \{2,3\}, \{3,4,5\}\}$. Then (X, T) is an anti-topological space. The AntiOpen sets are: {1}, {2,3}, {3,4,5} and thus the NeutroClosed sets are: {2,3,4,5}, {1,4,5}, {1,2}.

Let $A = \{1,2\}$, then $AntiInt\ (AntiCl(AntiInt(\{1,2\}))) = AntiInt(AntiCl(\{1\})) = AntiInt(\{1\}) = \{1\}$ which shows that $AntiInt(AntiCl(AntiInt(A))) \subseteq A$. Hence {1,2} is AntiBetaClosed.

3.3: A new class of sets

Definition 3.3.1: If X be a neutro-topological space and $A \subseteq X$, then A will be called NeutroTauOpen if $A = \text{Neu}C(\text{Neu}Int(\text{Neu}Cl(A)))$.

Remark 3.3.1: In Example 3.4.1 the subsets $\{a\}$, $\{a, c, e\}$, $\{b, d, e\}$, $\{b, c, d, e\}$ are found to be NeutroTauOpen.

Definition 3.3.2: If X be a neutro-topological space and $A \subseteq X$, then A will be called NeutroTauClosed if $A = \text{Neu}In(\text{Neu}Cl(\text{Neu}Int(A)))$.

Remark 3.3.2: In Example 3.4.1 the subsets $\{a\}$, $\{a, c\}$, $\{b, d\}$, $\{b, c, d, e\}$ are found to be NeutroTauClosed.

Definition 3.3.3: If X be an anti-topological space and $A \subseteq X$, then A will be called AntiTauOpen if $A = AntiC(AntiInt(AntiCl(A)))$.

Remark 3.3.3: In Example 3.4.2 the subsets $\{a\}$ and $\{b, c, d, e\}$ are found to be AntiTauOpen.

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Definition 3.3.4: If X be an anti-topological space and $A \subseteq X$, then A will be called AntiTauClosed if A $= AntiIn(AntiCl(AntiInt$(A))).

Remark 3.3.4: In Example 3.4.2 the subsets $\{a\}$ and $\{b, c, d, e\}$ are found to be AntiTauClosed.

Remark 3.2.5: In the next examples, we take some neutro-topology and anti-topology and check the openness or closedness of the subsets in the neutro-topologies and anti-topologies in context to the definitions 3.1.1 to 3.1.10; 3.2.1 to 3.2.10 and 3.3.1 to 3.3.4.

3.4: Results and discussions:

Example 3.4.1: Assume $X = \{a, b, c, d, e\}$, $T = \{\emptyset, \{a\}, \{a, c\}, \{b, d\}, \{c, d, e\}\}$. Then (X, T) is a neutrotopological space. The NeutroOpen sets are: $\emptyset$, $\{a\}$, $\{a, c\}$, $\{b, d\}$, $\{c, d, e\}$ and thus the NeutroClosed sets are: X, $\{b, c, d, e\}$, $\{b, d, e\}$, $\{a, c, e\}$, $\{a, b\}$.

If $A$is NeutroOpen then NeuInt(A) = A and if A is NeutroClosedNeuCl(A) = A. Now, for the NeutroOpen sets:
Neu$C(\{a\}) = \{a, c, e\} \cap \{a, b\} = \{a\}$
Neu$C(\{a, c\}) = \{a, c, e\}$
Neu$C(\{b, d\}) = \{b, c, d, e\} \cap \{b, d, e\} = \{b, d, e\}$
Neu$C(\{c, d, e\}) = \{b, c, d, e\}$
Now, for the NeutroClosed sets, we find the NeutroInteriors:
Neu$In(X) = X$.
Neu$In(\{b, c, d, e\}) = \{b, d\} \cup \{c, d, e\} = \{b, c, d, e\}$
Neu$In(\{b, d, e\}) = \{b, d\}$
Neu$In(\{a, c, e\}) = \{a\} \cup \{a, c\} = \{a, c\}$
Neu$In(\{a, b\}) = \{a\}$
Now, NeuIn(Neu$Cl(\{a\})) = \{a\} \Rightarrow \{a\}$ is NeutroRegularOpen.
NeuIn(Neu$Cl(\{a, c\})) = \{a, c\} \Rightarrow \{a, c\}$ is NeutroRegularOpen.
NeuIn(Neu$Cl(\{b, d\})) = \{b, d\} \Rightarrow \{b, d\}$ is NeutroRegularOpen.
NeuIn(Neu$Cl(\{c, d, e\})) = \{b, c, d, e\} \Rightarrow \{c, d, e\}$ is NeutroPreOpen.
NeuIn(Neu$Cl(\{b, c, d, e\})) = \{b, c, d, e\} \Rightarrow \{b, c, d, e\}$ is NeutroRegularOpen.
NeuIn(Neu$Cl(\{b, d, e\})) = \{b, d\} \Rightarrow \{b, d, e\}$ is NeutroSemiClosed.
NeuIn(Neu$Cl(\{a, c, e\})) = \{a, c\} \Rightarrow \{a, c, e\}$ is NeutroSemiClosed.
NeuIn(Neu$Cl(\{a, b\})) = \{a\} \Rightarrow \{a, b\}$ is NeutroSemiClosed.
Next, Neu$C$(Neu$Int(\{a\})) = \{a\} \Rightarrow \{a\}$ is NeutroRegularClosed. Neu$C$(Neu$Int(\{a, c\})) = \{a, c, e\} \Rightarrow \{a, c\}$ is NeutroSemiOpen.
Neu$C$(Neu$Int(\{b, d\})) = \{b, d, e\} \Rightarrow \{b, d\}$ is NeutroSemiOpen
Neu$C$(Neu$Int(\{c, d, e\})) = \{b, c, d, e\} \Rightarrow \{c, d, e\}$ is NeutroSemiOpen.
Neu$C$(Neu$Int(\{b, c, d, e\})) = \{b, c, d, e\} \Rightarrow \{b, c, d, e\}$ is NeutroRegularClosed.
Neu$C$(Neu$Int(\{b, d, e\})) = \{b, d, e\} \Rightarrow \{b, d, e\}$ is NeutroRegularClosed.
Neu$C$(Neu$Int(\{a, c, e\})) = \{a, c, e\} \Rightarrow \{a, c, e\}$ is NeutroRegularClosed.
Neu$C$(Neu$Int(\{a, b\})) = \{a\} \Rightarrow \{a, b\}$ is NeutroPreClosed.
Next, NeuIn(NeuCl(Neu$Int(\{a\}))) = \{a\} \Rightarrow \{a\}$ is NeutroTauClosed.
NeuIn(NeuCl(Neu$Int(\{a, c\}))) = \{a, c\} \Rightarrow \{a, c\}$ is NeutroTauClosed.
NeuIn(NeuCl(Neu$Int(\{b, d\}))) = \{b, d\} \Rightarrow \{b, d\}$ is NeutroTauClosed
NeuIn(NeuCl(Neu$Int(\{c, d, e\}))) = \{b, c, d, e\} \Rightarrow \{c, d, e\}$ is NeutroAlphaOpen.
NeuIn(NeuCl(Neu$Int(\{b, c, d, e\}))) = \{b, c, d, e\} \Rightarrow \{b, c, d, e\}$ is NeutroTauClosed.
NeuIn(NeuCl(Neu$Int(\{b, d, e\}))) = \{b, d\} \Rightarrow \{b, d, e\}$ is NeutroBetaClosed.
NeuIn(NeuCl(Neu$Int(\{a, c, e\}))) = \{a, c\} \Rightarrow \{a, c, e\}$ is NeutroBetaClosed.
NeuIn(NeuCl(Neu$Int(\{a, b\}))) = \{a\} \Rightarrow \{a, b\}$ is NeutroBetaClosed.
Next, Neu$C$(NeuInt(Neu$Cl(\{a\}))) = \{a\} \Rightarrow \{a\}$ is NeutroTauOpen.
Neu$C$(NeuInt(Neu$Cl(\{a, c\}))) = \{a, c, e\} \Rightarrow \{a, c\}$ is NeutroBetaOpen.
Neu$C$(NeuInt(Neu$Cl(\{b, d\}))) = \{b, d, e\} \Rightarrow \{b, d\}$ is NeutroBetaOpen.
Neu$C$(NeuInt(Neu$Cl(\{c, d, e\}))) = \{b, c, d, e\} \Rightarrow \{c, d, e\}$ is NeutroBetaOpen.
Neu$C$(NeuInt(Neu$Cl(\{b, c, d, e\}))) = \{b, c, d, e\} \Rightarrow \{b, c, d, e\}$ is NeutroTauOpen.
Neu$C$(NeuInt(Neu$Cl(\{b, d, e\}))) = \{b, d, e\} \Rightarrow \{b, d, e\}$ is NeutroTauOpen.
Neu$C$(NeuInt(Neu$Cl(\{a, c, e\}))) = \{a, c, e\} \Rightarrow \{a, c, e\}$ is NeutroTauOpen. Neu$C$(NeuInt(Neu$Cl(\{a, b\}))) = \{a\} \Rightarrow \{a, b\}$ is NeutroAlphaClosed.

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Example 3.4.2: Assume $X = \{a, b, c, d, e\}$, $T = \{\{a\}, \{b, c\}, \{c, d, e\}\}$. Then (X, T) is an anti-topological space. The AntiOpen sets are: $\{a\}$, $\{b, c\}$, $\{c, d, e\}$ and thus the AntiClosed sets are: $\{b, c, d, e\}$, $\{a, d, e\}$, $\{a, b\}$.

If $A$is AntiOpen then $AntiInt$(A) = A and if A is AntiClosed$AntiCl$(A) = A. Now, for the AntiOpen sets we find the AntiClosures:
$AntiCl(\{a\}) = \{a, d, e\} \cap \{a, b\} = \{a\}$
$AntiC(\{b, c\}) = \{b, c, d, e\}$ $AntiCl(\{c, d, e\}) = \{b, c, d, e\}$
And, for the AntiClosed sets we find the AntiInteriors:
$AntiIn(\{b, c, d, e\}) = \{b, c, d, e\}$
$AntiIn(\{a, d, e\}) = \{a\}$
$AntiIn(\{a, b\}) = \{a\}$
Now, $AntiIn(AntiCl(\{a\})) = AntiInt(\{a\}) = \{a\} \Rightarrow \{a\}$ is AntiRegularOpen.
$AntiIn(AntiCl(\{b, c\})) = AntiInt(\{b, c, d, e\}) = \{b, c, d, e\} \Rightarrow \{b, c\}$ is AntiPreOpen.
$AntiIn(AntiCl(\{c, d, e\})) = AntiInt(\{b, c, d, e\}) = \{b, c, d, e\} \Rightarrow \{c, d, e\}$ is AntiPreOpen.
$AntiIn(AntiCl(\{b, c, d, e\})) = AntiInt(\{b, c, d, e\}) = \{b, c, d, e\} \Rightarrow \{b, c, d, e\}$ is AntiRegularOpen.
$AntiIn(AntiCl(\{a, d, e\})) = AntiInt(\{a, d, e\}) = \{a\} \Rightarrow \{a, d, e\}$ is AntiSemiClosed.
$AntiIn(AntiCl(\{a, b\})) = AntiInt(\{a, b\}) = \{a\} \Rightarrow \{a, b\}$ is AntiSemiClosed.
Next, $AntiC(AntiInt(\{a\}) = AntiCl(\{a\}) = \{a\} \Rightarrow \{a\}$ is AntiRegularClosed.
$AntiC(AntiInt(\{b, c\}) = AntiCl(\{b, c\}) = \{b, c, d, e\} \Rightarrow \{b, c\}$ is AntiSemiOpen.
$AntiC(AntiInt(\{c, d, e\}) = AntiCl(\{c, d, e\}) = \{b, c, d, e\} \Rightarrow \{c, d, e\}$ is AntiSemiOpen.
$AntiC(AntiInt(\{b, c, d, e\}) = \{b, c, d, e\} \Rightarrow \{b, c, d, e\}$ is AntiRegularClosed.
$AntiC(AntiInt(\{a, d, e\}) = AntiCl(\{a\}) = \{a\} \Rightarrow \{a\}$ is AntiRegularClosed.
$AntiC(AntiInt(\{a, b\}) = AntiCl(\{a\}) = \{a\} \Rightarrow \{a, b\}$ is AntiPreClosed.
Next, $AntiIn(AntiCl(AntiInt(\{a\})) = \{a\} \Rightarrow \{a\}$ is AntiTauClosed.
$AntiIn(AntiCl(AntiInt(\{b, c\})) = \{b, c, d, e\} \Rightarrow \{b, c\}$ is AntiAlphaOpen.
$AntiIn(AntiCl(AntiInt(\{c, d, e\})) = \{b, c, d, e\} \Rightarrow \{c, d, e\}$ is AntiAlphaOpen.
$AntiIn(AntiCl(AntiInt(\{b, c, d, e\})) = \{b, c, d, e\} \Rightarrow \{b, c, d, e\}$AntiTauClosed. $AntiIn(AntiCl(AntiInt(\{a, d, e\})) = \{a\} \Rightarrow \{a, d, e\}$ is AntiBetaClosed.
$AntiIn(AntiCl(AntiInt(\{a, b\})) = \{a\} \Rightarrow \{a, b\}$ is … AntiBetaClosed.
Next, $AntiC(AntiInt(AntiCl(\{a\})) = \{a\} \Rightarrow \{a\}$ is AntiTauOpen.
$AntiC(AntiInt(AntiCl(\{b, c\})) = \{b, c, d, e\} \Rightarrow \{b, c\}$ is AntiBetaOpen.
$AntiC(AntiInt(AntiCl(\{c, d, e\})) = \{b, c, d, e\} \Rightarrow \{c, d, e\}$ is …AntiBetaOpen.
$AntiC(AntiInt(AntiCl(\{b, c, d, e\})) = \{b, c, d, e\} \Rightarrow \{b, c, d, e\}$ is AntiTauOpen.
$AntiC(AntiInt(AntiCl(\{a, d, e\})) = \{a\} \Rightarrow \{a, d, e\}$ is AntiAlphaClosed.
$AntiC(AntiInt(AntiCl(\{a, b\})) = \{a\} \Rightarrow \{a, b\}$ is AntiAlphaClosed.

Proposition 3.4.1: Every NeutroTauClosed set is NeutroRegularOpen.

Proof: This comes from the fact: Neu$In(A) \subseteq A$**[3]**, and from the definition of the two class of sets.

Proposition 3.4.2: Every NeutroRegularClosed set is NeutroTauOpen.

Proof: This comes from the fact: $A \subseteq$Neu$C(A)$**[3]** and from the definition of the two class of sets.

Proposition 3.4.3: Every AntiTauClosed set is AntiRegularOpen.

Proof: This comes from the fact: $AntiIn(A) \subseteq A$**[2]** and from the definition of the two class of sets.

Proposition 3.4.4: Every AntiRegularClosed set is AntiTauOpen.

Proof: This comes from the fact: $A \subseteq AntiC(A)$**[2]** and from the definition of the two class of sets.

Definition 3.4.1: A subset of a neutro-topological space (X, T) is called NeutroClopen if it is both NeutroOpen and NeutroClosed.

Definition 3.4.2: A subset of an anti-topological space (X, T) is called AntiClopen if it is both AntiOpen and AntiClosed.

Proposition 3.4.5: Every NeutroClopen set is NeutroRegularOpen.

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Proposition 3.4.6: Every AntiClopen set is AntiRegularOpen.

Remark 3.4.1: Every NeutroRegularOpen set is not NeutroOpen.

Proof: Weprove it by giving a counter example.
Let X = {1,2,3,4}, T = {∅, {1}, {2,3}, {3,4}} be a neutro-topology on X and let A = {2,3,4} be the subset of X. Then the NeutroClosed subsets of T are: X, {2,3,4}, {1,4}, {1,2}.
Now, NeuInt(NeuCl(A)) = NeuInt(NeuCl({2,3,4})) = NeuInt({2,3,4}) = {2,3} ∪ {3,4} = {2,3,4} = A.
Hence, A = NeuIn(NeuCl(A), showing that A is a NeutroRegularOpen set. But {2,3,4} is not NeutroOpen.

Remark 3.4.2: Every AntiRegularOpen set is not AntiOpen.

Proof: Weprove it by giving a counter example.
Let X = {1,2,3,4,5}, T = {{1}, {4}, {2,3}, {3,5}} be an anti-topology on X and let A = {1,4} be the subset of X. Then the AntiClosed subsets of T are: {2,3,4,5}, {1,2,3,5}, {1,4,5}, {1,2,4}.
Now, $AntiInt$($AntiCl$(A)) = $AntiInt$($AntiCl$({1,4})) = $AntiInt$({1,4,5} ∩ {1,2,4}) = $AntiInt$({1,4}) = {1} ∪ {4} = {1,4} = A. Hence, A = $AntiIn$($AntiCl$(A), showing that A is a AntiRegularOpen set. But, A = {1,4} is not AntiOpen.

Remark 3.4.3: NeutroClopen sets are those which are both NeutroOpen and NeutroClosed in the neutrotopology in discussion. Similarly, for AntiClopen sets.

Remark 3.4.4:

(i) The union of two NeutroRegularOpen sets is not NeutroRegularOpen.
(ii) The intersection of two NeutroRegularOpen sets is not NeutroRegularOpen.
(iii) The union of two AntiRegularOpen sets is not AntiRegularOpen.
(iv) The intersection of two AntiRegularOpen sets is not AntiRegularOpen.

Remark 3.4.5: Finite intersection of NeutroRegularOpen sets is not NeutroRegularOpen.

A counter example will suffice. In Example 3.1.7 the sets {a, c} and {b, c, d, e} are both NeutroRegularOpen, and their intersection is {a, c} ∩ {b, c, d, e} = {c}.
Now, NeuIn(NeuCl({c})) = NeuInt({b, c, d, e} ∩ {a, c, e}) = NeuInt({c, e}) = ∅.

Remark 3.4.6: Finite union of AntiRegularClosed sets is not AntiRegularClosed.

A counter example will suffice. In Example 3.1.8 the sets {a} and {b, c, d, e} are both AntiRegularClosed, and their union is {a} ∪ {b, c, d, e} = {a, b, c, d, e}.
Now, $AntiC$($AntiInt$({a, b, c, d, e})) = $AntiCl$({a, b, c, d, e) does not exist in the anti-topology.

Remark 3.4.8: Every NeutroOpen subset of a neutro-topological space is not a NeutroPreOpen set.

A counter example will suffice. Let X = {a, b, c, d, e}, T = {∅, {a}, {a, c}, {b, d}, {c, d, e}}. Then (X, T) is a neutro-topological space. The NeutroOpen subsets are: ∅, {a}, {a, c}, {b, d}, {c, d, e} and the NeutroClosed subsets are: X, {b, c, d, e}, {b, d, e}, {a, c, e}, {a, b}.
For, the NeutroOpen subsets we have:
NeuIn(NeuCl({a})) = {a} ⇒ {a} is NeutroRegularOpen.
NeuIn(NeuCl({a, c})) = {a, c} ⇒ {a, c} is NeutroRegularOpen.
NeuIn(NeuCl({b, d})) = {b, d} ⇒ {b, d} is NeutroRegularOpen.
NeuIn(NeuCl({c, d, e})) = {b, c, d, e} ⇒ {c, d, e} is NeutroPreOpen.
Thus, it can be seen that only one of the NeutroOpen subsets is NeutroPreOpen.

Remark 3.4.9: Every NeutroClosed subset of a neutro-topological space is not a NeutroPreClosed set.

A counter example will suffice. Let X = {a, b, c, d, e}, T = {∅, {a}, {a, c}, {b, d}, {c, d, e}}. Then (X, T) is a neutro-topological space. The NeutroOpen subsets are: ∅, {a}, {a, c}, {b, d}, {c, d, e} and the NeutroClosed subsets are: X, {b, c, d, e}, {b, d, e}, {a, c, e}, {a, b}.

For, the NeutroClosed subsets we have:
Neu$C$(NeuInt({b, c, d, e})) = {b, c, d, e} ⇒ {b, c, d, e} is NeutroRegularClosed.
Neu$C$(NeuInt({b, d, e})) = {b, d, e} ⇒ {b, d, e} is NeutroRegularClosed.
Neu$C$(NeuInt({a, c, e})) = {a, c, e} ⇒ {a, c, e} is NeutroRegularClosed.

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Neu$C$(NeuInt($\{a, b\}$)) = $\{a\} \Rightarrow \{a, b\}$ is NeutroPreClosed.
Thus, it is observed that only one of the NeutroClosed subset is NeutroPreClosed.

4. Conclusion and future work:

It has been observed that many of the properties of the types of sets we have discussed, that holds good in general topological spaces, are not valid in neutro-topological spaces and anti-topological spaces. The obvious reason for this is that in a neutro-topological space the union or intersection of the subsets does not always belong to the space. And for an anti-topological space, neither the union nor the intersection of the subsets of the anti-topology belongs to the space. And, the exclusion of the whole set and the empty set from the anti-topology is also another reason for some of the dis-similarities in the properties. But, as can be observed in [9], the properties of AntiInterior, AntiClosure and AntiBoundary are basically more valid to those that are in general topological spaces as compared to those in [8], that is in case of the neutro-topological spaces. But it has been observed that all the types of sets we have discussed are existent in both the neutro-topological space and antitopological spaces.

Further study can be done on the other types of sets like $g - open/closed$ sets, $\partial - open/closed$ sets, $\pi - open/closed$ and the many other types of sets that has been so far defined, studied upon and put to application, if ever been.

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