

Research Article

A New Perspective for Solving the System of Differential Equations Describing Arm Race Model in Neutrosophic Environment

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The term “arms race” literally refers to competitions between nations for military superiority during times of peace. For a number of reasons, arms races have attracted a lot of attention. They are widely meant to have important repercussions for national security. The best course of action for a nation facing an aggressive foe is to engage in an armaments race. If one nation raises its arsenal, another will follow suit, in order the first nation responds by increasing its arsenal. Differential equations can be used to model this type of arms race. Both Engineering and Science rely only on differential equations. Modeling the problems in this field include some parameters that are uncertain or imprecise. To resolve this, neutrosophic differential equations are evolved. In this article, the arms race is modeled using a differential equation with a triangular neutrosophic number as the initial value. The system of neutrosophic differential equation is then solved using the MATLAB tool and the results are visualised graphically. The solution is compared by finding the value, ambiguity, and rank of neutrosophic numbers.

1. Introduction

The direct and instantaneous comprehension of reality is quite complicated. The comprehension of the real world predominantly benefits from the use of simplified reality-based models. These mathematical models are challenging, and it is difficult to find precise answers. Because of the data's inherent uncertainty, it is impossible to model problems in diverse disciplines using conventional methodologies. Given that there are uncertainties in natural contexts, the classical theory that relies on the crisp does not seem to be very appropriate for handling such uncertainty. An IFS, a generalisation of fuzzy set was developed by Atanassov [1] in 1986. There are some drawbacks to the sets stated above. Neutrosophic logic and sets, a new theory, was developed to address these drawbacks. The extensions of classical, fuzzy, and IFS are neutrosophic sets [2]. Degrees of truth, falsity, and ambiguity are all part of a neutrosophic set's membership function. When compared to fuzzy

sets and IFS, in this situation indeterminacy produces greater accuracy. Neutrosophy will therefore produce superior outcomes to fuzzy and intuitionistic fuzzy sets.

Both Engineering and Science predominantly rely on differential equations. Modeling the problems in this field include some parameters that are uncertain or imprecise. While indeterminacy does not occur, this uncertainty allows the formation of imprecise differential equations such as FDE [3] and IFDE. To resolve this, neutrosophic differential equation is evolved.

The term “arms race” literally refers to competitions between nations for military superiority during times of peace. For a number of reasons, arms races have attracted a lot of attention. They are widely meant to have important repercussions for national security. The best course of action for a nation facing an aggressive foe is to engage in an armaments race. Frederick William Lanchester emphasised the need of troop concentration in the contemporary combat in 1916. He

suggested differential equations, from which it would be possible to get the desired outcomes. Armed conflict deterministic equations are a straightforward advantage between two hostile forces. In particular, if one nation raises its arsenal, another will follow suit. In order, the first nation responds by increasing its arsenal. Differential equations can be used to model this type of arms race.

This research article uses the triangular neutrosophic number as the initial condition for the first-order system of differential equations which represents the arms race model. The equations are resolved, and the military performance of the two nations is calculated.

2. Literature Survey

Recently, the neutrosophic theory has received enormous support from several academicians. The quantity of articles on neutrosophic sets has significantly increased since 2015. Smarandache added neutrosophic sets to the literature in order to manage information that is insufficient, unclear, and inconsistent. In neutrosophic sets, an additional parameter is used to formally quantify indeterminacy. The concept of the neutrosophic number has been introduced by Chakraborty et al. [4] from many perspectives. They define many kinds of generalised triangular neutrosophic numbers, both linear and nonlinear, which are crucial for uncertainty theory. The de-neutrosophication notion for triangular neutrosophic numbers has also been introduced. This idea enables the transformation of neutrosophic numbers into crisp numbers.

Using fuzzy differential equations, this uncertainty has been modeled by Bede and Gal [5], Chalco-Cano and Roman-Flores [3], and Mondal et al. [6]. However, it merely takes the membership value into account. IFDE was further developed by Kumar et al. [7] and Wang et al. [8]. These two theories; however, do not contain the concept of indeterminacy. Neutrosophic differential equations were created as a result to model the indeterminacy. Researchers have developed a wide range of strategies for resolving fuzzy and intuitionistic fuzzy differential equations. Consequently, several approaches to the neutrosophic differential equation solution are a growing subject today. This article will be a valuable source for concepts, methods, and strategies for upcoming research on neutrosophic sets applications. Here is a list of some of the significant research that may aid in the development of the study on the neutrosophic differential equation. Using an application in bacteria culture models, Sumathi and MohanaPriya [9] suggested a method to solve differential equations utilizing neutrosophic numbers.

Fuzzy initial value problems were first described by Seikkala [10]. Bede and Gal [5] then proposed the generalised differentiability concepts. Using these concepts, they were able to solve fuzzy differential and partial differential equations.

A numerical method for the conventional Euler methodology for FDE was created by Ma et al. [11]. Then, Abbasbandy and Viranloo [12] introduced a special numerical approach for solving FDE based on the Taylor technique of order p . Khastan and Nieto [13] created a novel

method to deal with fuzzy boundary-value problems in which they investigated the problem to find solutions in various (n, m) -systems, where $n, m \in \{1, 2\}$.

The Milne–Simpson method of order five is used to solve FDE using an interval-valued fuzzy number which has recently been researched by Balakrishnan and Manigandan [14]. Numerous academics are still engaged in order to solve FDE analytically and numerically [15, 16]. All of the aforementioned works can now be adjusted and applied to a neutrosophic environment. Researchers have recently been focusing on neutrosophic differential equations.

2.1. Motivation. Every parameter does not need to have a fixed value when simulating a real-world scenario. Some parameters appear with ambiguous or approximate values. The idea of imprecise differential equations appears when representing uncertainty in a problem using differential equations. Neutrosophic set theory is created to address such circumstances. Moreover, it is a known fact that only a few publications are available on neutrosophic differential equations. Therefore, working in this field has a lot of potential.

2.2. Novelities. The uniqueness here is addressing the arms race problem in a neutrosophic environment with an initial condition as a triangular neutrosophic number, which is a novel approach despite some breakthroughs in the field of solving differential equations in a neutrosophic environment being worked on.

2.3. Paper Organization. The article is structured as follows: Introductions to fuzzy, intuitionistic fuzzy, and neutrosophic differential equations are listed in the first section. Literature review in the second section, and a discussion on initial impressions in the third section. The fourth section, which also transforms the arms race model into a system of differential equations with a base value of a TNN, discusses the implementation of the arms race model. The conclusion is presented in the fifth section.

3. Preliminaries

In this section, we present the necessary definitions and notations that will be used in this work as follows:

Definition 1 (See [9])

Let the universe of discourse be E . Neutrosophic single valued set MQ on E is defined as $MQ = \{ \langle T_{MQ}(x), I_{MQ}(x), F_{MQ}(x) \rangle : x \in E \}$ where $T_{MQ}(x), I_{MQ}(x), F_{MQ}(x) : E \rightarrow [0, 1]$ represents the membership degree, indeterministic degree, and nonmembership degree, respectively, of the element $x \in E$ with $0 \leq T_{MQ}(x) + I_{MQ}(x) + F_{MQ}(x) \leq 3$.

Definition 2 (See [9])

Neutrosophic set α, β, γ -cut is represented by $G(\alpha, \beta, \gamma)$, for, $\beta, \gamma \in [0, 1], \alpha + \beta + \gamma \leq 3$ is given by

$$G(\alpha, \beta, \gamma) = \{ \langle T_{MQ}(x), I_{MQ}(x), F_{MQ}(x) \rangle : x \in E, T_{MQ}(x) \geq \alpha, I_{MQ}(x) \leq \beta, F_{MQ}(x) \leq \gamma \}. \quad (1)$$

Definition 3 (See [9])

A neutrosophic set MQ over the set of real numbers R is said to be a neutrosophic number if it has the following properties.

- (i) MQ is normal if there exists $x_0 \in R$ such that $T_{MQ}(x_0) = 1, I_{MQ}(x_0) = 0, F_{MQ}(x_0) = 0$.
- (ii) MQ is convex set for the truth function $T_{MQ}(x_0)$ (i.e.)

$$T_{MQ}(\mu x_1 + (1 - \mu)x_2) \geq \min((T_{MQ}(x_1), T_{MQ}(x_2))) \text{ for every } x_1, x_2 \in R \text{ and } \mu \in [0, 1]. \quad (2)$$

- (iii) MQ is concave set for the indeterministic function $I_{MQ}(x)$ and falsehood function $F_{MQ}(x)$ where

$$\begin{aligned} I_{MQ}(\mu x_1 + (1 - \mu)x_2) &\geq \max((I_{MQ}(x_1), I_{MQ}(x_2))) \text{ for every } x_1, x_2 \in R \text{ and } \mu \in [0, 1], \\ F_{MQ}(\mu x_1 + (1 - \mu)x_2) &\geq \max((F_{MQ}(x_1), F_{MQ}(x_2))) \text{ for every } x_1, x_2 \in R \text{ and } \mu \in [0, 1]. \end{aligned} \quad (3)$$

Definition 4 (See [9])

A triangular neutrosophic number A_n is a subset of a neutrosophic number in R with the following truth function, indeterministic function, and falsity function given by

$$T_{A_n}(x_i) = \begin{cases} \left(\frac{x_i - a}{b - a}\right)g_A, & a \leq x_i \leq b, \\ g_A, & x_i = b, \\ \left(\frac{c - x_i}{c - b}\right)g_A, & b \leq x_i \leq c, \\ 0, & \text{elsewhere,} \end{cases}$$

$$I_{A_n}(x_i) = \begin{cases} \left(\frac{b - x_i}{b - a}\right)v_A, & a \leq x_i \leq b, \\ v_A, & x_i = b, \\ \left(\frac{x_i - c}{c - b}\right)v_A, & b \leq x_i \leq c, \\ 1, & \text{elsewhere,} \end{cases}$$

$$F_{A_n}(x_i) = \begin{cases} \left(\frac{b - x_i}{b - a}\right)k_A, & a \leq x_i \leq b, \\ k_A, & x_i = b, \\ \left(\frac{x_i - c}{c - b}\right)k_A, & b \leq x_i \leq c, \\ 1, & \text{elsewhere,} \end{cases} \quad (4)$$

where $a \leq b \leq c$ and TNN is denoted by $A_{TN} \langle (a, b, c); g_A, v_A, k_A \rangle$.

Definition 5 (See [9])

(α, β, γ) -cut of the neutrosophic number AB is given by

$$AB_{\alpha, \beta, \gamma} = \begin{cases} (AB_1(\alpha), AB_2(\alpha)) \text{ for } \alpha \in [0, 1], \\ (AB'_1(\beta), AB'_2(\beta)) \text{ for } \beta \in [0, 1], \\ (AB''_1(\gamma), AB''_2(\gamma)) \text{ for } \gamma \in [0, 1], \end{cases} \quad (5)$$

where $\alpha + \beta + \gamma \leq 3$

Here,

- (i) $dAB_1(\alpha)/d\alpha > 0, dAB_2(\alpha)/d\alpha < 0$, for all $\alpha \in [0, 1]$, $AB_1(1) \leq AB_2(1)$
- (ii) $dAB'_1(\beta)/d\beta < 0, dAB'_2(\beta)/d\beta > 0$, for all $\beta \in [0, 1]$, $AB'_1(0) \leq AB'_2(0)$
- (iii) $dAB''_1(\gamma)/d\gamma < 0, dAB''_2(\gamma)/d\gamma > 0$, for all $\gamma \in [0, 1]$, $AB''_1(0) \leq AB''_2(0)$

If $AB = (a, b, c; g_A, v_A)$ then (α, β, γ) - cut is given by

$$AB_{\alpha, \beta, \gamma} = \{[(a + \alpha(b - a))g_A, (c - \alpha(c - b))g_A], [(b - \beta(b - a))v_A, (b + \beta(c - b))v_A], [(b - \gamma(b - a))k_A, (b + \gamma(c - b))k_A]\}. \tag{6}$$

Definition 6. If $[x_1(t, \alpha), x_2(t, \alpha); x'_1(t, \beta), x'_2(t, \beta); x''_1(t, \gamma), x''_2(t, \gamma)]$ be the solution of the differential equation,

$$[x_1(t, \alpha = 0), x_1(t, \alpha = 1), x_2(t, \alpha = 0); x'_1(t, \beta = 1), x'_1(t, \beta = 0), x'_2(t, \beta = 1); x''_1(t, \gamma = 1), x''_1(t, \gamma = 0), x''_2(t, \gamma = 1)]. \tag{7}$$

Definition 7 (See [17])

Let $s = \langle (l, m, n); \mu, \gamma, k \rangle$ be an arbitrary SVTNN, then value and ambiguity of the SVTr-number is given by

$$\begin{aligned} V_\alpha(s) &= \left(\frac{l + 4m + n}{6}\right)\mu^2, \\ A_\alpha(s) &= \left(\frac{n - l}{6}\right)\mu^2, \\ V_\beta(s) &= \frac{(l + 4m + n)(1 - \gamma)^2}{6}, \\ A_\beta(s) &= \frac{(n - l)(1 - \gamma)^2}{6}, \\ V_\gamma(s) &= \left(\frac{l + 4m + n}{6}\right)(1 - k)^2, \\ A_\gamma(s) &= \left(\frac{n - l}{6}\right)(1 - k)^2. \end{aligned} \tag{8}$$

Definition 8 (See [17])

Let $s = \langle (l, m, n; \mu, \gamma, k) \rangle$ be a SVN, then for any $\delta \in [0, 1]$

(i) The δ -weighted value of the SVN “s” is defined as

$$V_\delta(s) = \delta V_\alpha(s) + (1 - \delta)V_\beta(s) + (1 - \delta)V_\gamma(s). \tag{9}$$

(ii) The δ -weighted ambiguity of the SVN “s” is defined as

$$A_\delta(s) = \delta A_\alpha(s) + (1 - \delta)A_\beta(s) + (1 - \delta)A_\gamma(s). \tag{10}$$

Definition 9 (See [17])

Let s and t be two SVN’s and for $\delta \in [0, 1]$ weighted values and ambiguities of the SVN’s, s and t the ranking order of s and t is defined as

then the solution is written as neutrosophic number as given in the following:

- (1) If $V_\delta(s) > V_\delta(t)$, then $s > t$
- (2) If $V_\delta(s) < V_\delta(t)$ then $s < t$
- (3) If $V_\delta(s) = V_\delta(t)$ * If $A_\delta(s) = A_\delta(t)$ then $s = t$ * If $A_\delta(s) > A_\delta(t)$ then $s > t$ and * If $A_\delta(s) < A_\delta(t)$ then $s < t$

4. Mathematical Modeling and Solution of Arm Race Model in Neutrosophic Environment

The study of the arms race is a fascinating application that results in a set of differential equations. Two nations are considered to be in conflict when there is a dispute between them and a war breaks out as a result. In this scenario, every nation will attempt to arm itself in order to protect itself from a potential assault by the opposing country in a fight. There begins the race for the gathering of arms. Here, we shall discuss Richardson’s model, a theoretical framework for understanding the arms race.

Assume that at time t , nation X ’s armaments are represented by $x(t)$ and nation Y ’s are represented by $y(t)$. How quickly one side’s armaments change is dependent on the other side’s arsenal, as if one side increases its arsenal, the other will do the same. The link between the rate of change of x with respect to t is proportional to y and vice versa. Further, proportionality constants are established to indicate the effectiveness of increased weaponry. The connection between two nations or alliances, each of which chooses to defend itself against a potential assault by the other, can be described by the provided set of equations $dp/dt = Aq(t)$ with $p(0) = p_0$ and $dq/dt = Bp(t)$ with $q(0) = q_0$ where $A > 0$ and $B > 0$ are constants.

Considering the arm race model in neutrosophic environment with initial condition as TNN, the model reduces to linear homogeneous system of equations $dx/dt = py, dy/dt = Qx$ with base condition $x(t_0) = x_0$ and $y(t_0) = y_0$ where x_0 and y_0 are TNN.

For the given system of equations, the solutions are $x(t)$ and $y(t)$, then its (α, β, γ) - cut is

$$\begin{aligned} x(t, \alpha, \beta, \gamma) &= [x_1(t, \alpha), x_2(t, \alpha); x'_1(t, \beta), x'_2(t, \beta); x''_1(t, \gamma), x''_2(t, \gamma)], \\ y(t, \alpha, \beta, \gamma) &= [y_1(t, \alpha), y_2(t, \alpha); y'_1(t, \beta), y'_2(t, \beta); y''_1(t, \gamma), y''_2(t, \gamma)]. \end{aligned} \tag{11}$$

The solution is strong if

- (i) $\frac{dx_1(t, \alpha)}{dt} > 0, \quad \frac{dx_2(t, \alpha)}{dt} < 0$
 $\frac{dy_1(t, \alpha)}{dt} > 0, \quad \frac{dy_2(t, \alpha)}{dt} < 0$
- (ii) $\frac{dx'_1(t, \alpha)}{dt} > 0, \quad \frac{dx'_2(t, \alpha)}{dt} < 0$
 $\frac{dy'_1(t, \alpha)}{dt} > 0, \quad \frac{dy'_2(t, \alpha)}{dt} < 0$
- (iii) $\frac{dx''_1(t, \alpha)}{dt} > 0, \quad \frac{dx''_2(t, \alpha)}{dt} < 0$
 $\frac{dy''_1(t, \alpha)}{dt} > 0, \quad \frac{dy''_2(t, \alpha)}{dt} < 0$

Otherwise the solution is feeble.

4.1. Determining the System of Differential Equations Solution. Take the differential equations system into consideration.

$$\frac{dx(t)}{dt} = py(t), \tag{12}$$

$$\frac{dy(t)}{dt} = Qx(t), \tag{13}$$

with $x(0) = x_0 = (a_1, a_2, a_3; \mu, \gamma, k)$ and $y(0) = y_0 = (b_1, b_2, b_3; \mu', \gamma', k')$

Case (i) If $P, Q > 0$

Taking (α, β, γ) - cut of (12), we obtain

$$\frac{d}{dt} [x_1(t, \alpha), x_2(t, \alpha); x'_1(t, \beta), x'_2(t, \beta); x''_1(t, \gamma), x''_2(t, \gamma)] = P [(y_1(t, \alpha), y_2(t, \alpha)); (y'_1(t, \beta), y'_2(t, \beta)); (y''_1(t, \gamma), y''_2(t, \gamma))]. \tag{14}$$

Taking (α, β, γ) - cut of (13), we have

$$\frac{d}{dt} [(y_1(t, \alpha), y_2(t, \alpha)); (y'_1(t, \beta), y'_2(t, \beta)); (y''_1(t, \gamma), y''_2(t, \gamma))] = Q [(x_1(t, \alpha), x_2(t, \alpha)); (x'_1(t, \beta), x'_2(t, \beta)); (x''_1(t, \gamma), x''_2(t, \gamma))], \tag{15}$$

with initial condition

$$\begin{aligned} x(t_0, \alpha, \beta, \gamma) &= ([AB_1(\alpha), AB_2(\alpha)]; [AB'_1(\beta), AB'_2(\beta)]; [AB''_1(\gamma), AB''_2(\gamma)]), \\ y(t_0, \alpha, \beta, \gamma) &= ([BC_1(\alpha), BC_2(\alpha)]; [BC'_1(\beta), BC'_2(\beta)]; [BC''_1(\gamma), BC''_2(\gamma)]), \end{aligned} \tag{16}$$

with $\alpha + \beta + \gamma \leq 3$ and $\alpha, \beta, \gamma \in [0, 1]$

$$\begin{aligned} \frac{dx_1(t, \alpha)}{dt} &= P y_1(t, \alpha), \\ \frac{dx_2(t, \alpha)}{dt} &= P y_2(t, \alpha), \\ \frac{dx'_1(t, \beta)}{dt} &= P y'_1(t, \beta), \\ \frac{dx'_2(t, \beta)}{dt} &= P y'_2(t, \beta), \\ \frac{dx''_1(t, \gamma)}{dt} &= P y''_1(t, \gamma), \\ \frac{dx''_2(t, \gamma)}{dt} &= P y''_2(t, \gamma). \end{aligned} \tag{17}$$

From (15), we obtain

$$\begin{aligned} \frac{dy_1(t, \alpha)}{dt} &= Qx_1(t, \alpha), \\ \frac{dy_2(t, \alpha)}{dt} &= Qx_2(t, \alpha), \\ \frac{dy'_1(t, \beta)}{dt} &= Qx'_1(t, \beta), \\ \frac{dy'_2(t, \beta)}{dt} &= Qx'_2(t, \beta), \\ \frac{dy''_1(t, \gamma)}{dt} &= Qx''_1(t, \gamma), \\ \frac{dy''_2(t, \gamma)}{dt} &= Qx''_2(t, \gamma), \end{aligned} \tag{18}$$

with base condition

$$\begin{aligned}
 x_1(t_0, \alpha) &= A_1(\alpha); \\
 x_2(t_0, \alpha) &= A_2(\alpha); \\
 x_1'(t_0, \beta) &= A_1'(\beta); \\
 x_2'(t_0, \beta) &= A_2'(\beta); \\
 x_1''(t_0, \gamma) &= A_1''(\gamma); \\
 x_2''(t_0, \gamma) &= A_2''(\gamma); \\
 y_1(t_0, \alpha) &= B_1(\alpha); \\
 y_2(t_0, \alpha) &= B_2(\alpha); \\
 y_1'(t_0, \beta) &= B_1'(\beta); \\
 y_2'(t_0, \beta) &= B_2'(\beta); \\
 y_1''(t_0, \gamma) &= B_1''(\gamma); \\
 y_2''(t_0, \gamma) &= B_2''(\gamma).
 \end{aligned} \tag{19}$$

From (17) to (18), we obtain

$$\begin{aligned}
 \frac{dx_1(t, \alpha)}{dt} &= P y_1(t, \alpha), \\
 \frac{dy_1(t, \alpha)}{dt} &= Q x_1(t, \alpha), \\
 \frac{d^2 x_1(t, \alpha)}{dt^2} &= P \frac{dy_1(t, \alpha)}{dt} \\
 &= PQ x_1(t, \alpha).
 \end{aligned} \tag{20}$$

The solution is $x_1(t, \alpha) = c_1 e^{\sqrt{PQ}t} + c_2 e^{-\sqrt{PQ}t}$ from (17), we obtain $c_1 e^{\sqrt{PQ}t} - c_2 e^{-\sqrt{PQ}t} = \sqrt{P/Q} y_1(t, \alpha)$

Using initial condition, we obtain

$$\begin{aligned}
 c_1 e^{\sqrt{PQ}t_0} + c_2 e^{-\sqrt{PQ}t_0} &= A_1(\alpha), \\
 c_1 e^{\sqrt{PQ}t_0} - c_2 e^{-\sqrt{PQ}t_0} &= \sqrt{\frac{P}{Q}} B_1(\alpha), \\
 c_1 &= \frac{1}{2} \left(A_1(\alpha) + \sqrt{\frac{P}{Q}} B_1(\alpha) \right) e^{-\sqrt{PQ}t_0}, \\
 c_2 &= \frac{1}{2} \left(A_1(\alpha) - \sqrt{\frac{P}{Q}} B_1(\alpha) \right) e^{\sqrt{PQ}t_0}.
 \end{aligned} \tag{21}$$

Therefore,

$$\begin{aligned}
 x_1(t, \alpha) &= \frac{1}{2} \left\{ A_1(\alpha) + \sqrt{\frac{P}{Q}} B_1(\alpha) \right\} e^{\sqrt{PQ}(t-t_0)} + \frac{1}{2} \left\{ A_1(\alpha) - \sqrt{\frac{P}{Q}} B_1(\alpha) \right\} e^{-\sqrt{PQ}(t-t_0)}, \\
 y_1(t, \alpha) &= \frac{1}{2} \sqrt{\frac{Q}{P}} \left\{ A_1(\alpha) + \sqrt{\frac{P}{Q}} B_1(\alpha) \right\} e^{\sqrt{PQ}(t-t_0)} - \frac{1}{2} \sqrt{\frac{Q}{P}} \left\{ A_1(\alpha) - \sqrt{\frac{P}{Q}} B_1(\alpha) \right\} e^{-\sqrt{PQ}(t-t_0)}.
 \end{aligned} \tag{22}$$

Similarly, we obtain

$$x_2(t, \alpha) = \frac{1}{2} \left\{ A_2(\alpha) + \sqrt{\frac{P}{Q}} B_2(\alpha) \right\} e^{\sqrt{PQ}(t-t_0)} + \frac{1}{2} \left\{ A_2(\alpha) - \sqrt{\frac{P}{Q}} B_2(\alpha) \right\} e^{-\sqrt{PQ}(t-t_0)}, \quad (23)$$

$$y_2(t, \alpha) = \frac{1}{2} \sqrt{\frac{Q}{P}} \left\{ A_2(\alpha) + \sqrt{\frac{P}{Q}} B_2(\alpha) \right\} e^{\sqrt{PQ}(t-t_0)} - \frac{1}{2} \sqrt{\frac{Q}{P}} \left\{ A_2(\alpha) - \sqrt{\frac{P}{Q}} B_2(\alpha) \right\} e^{-\sqrt{PQ}(t-t_0)},$$

$$x'_1(t, \beta) = \frac{1}{2} \left\{ A'_1(\beta) + \sqrt{\frac{P}{Q}} B'_1(\beta) \right\} e^{\sqrt{PQ}(t-t_0)} + \frac{1}{2} \left\{ A'_1(\beta) - \sqrt{\frac{P}{Q}} B'_1(\beta) \right\} e^{-\sqrt{PQ}(t-t_0)},$$

$$y'_1(t, \beta) = \frac{1}{2} \sqrt{\frac{Q}{P}} \left\{ A'_1(\beta) + \sqrt{\frac{P}{Q}} B'_1(\beta) \right\} e^{\sqrt{PQ}(t-t_0)} - \frac{1}{2} \sqrt{\frac{Q}{P}} \left\{ A'_1(\beta) + \sqrt{\frac{P}{Q}} B'_1(\beta) \right\} e^{-\sqrt{PQ}(t-t_0)}, \quad (24)$$

$$x'_2(t, \beta) = \frac{1}{2} \left\{ A'_2(\beta) + \sqrt{\frac{P}{Q}} B'_2(\beta) \right\} e^{\sqrt{PQ}(t-t_0)} + \frac{1}{2} \left\{ A'_2(\beta) - \sqrt{\frac{P}{Q}} B'_2(\beta) \right\} e^{-\sqrt{PQ}(t-t_0)},$$

$$y'_2(t, \beta) = \frac{1}{2} \sqrt{\frac{Q}{P}} \left\{ A'_2(\beta) + \sqrt{\frac{P}{Q}} B'_2(\beta) \right\} e^{\sqrt{PQ}(t-t_0)} - \frac{1}{2} \sqrt{\frac{Q}{P}} \left\{ A'_2(\beta) - \sqrt{\frac{P}{Q}} B'_2(\beta) \right\} e^{-\sqrt{PQ}(t-t_0)}.$$

Also,

$$x''_1(t, \gamma) = \frac{1}{2} \left\{ A''_1(\gamma) + \sqrt{\frac{P}{Q}} B''_1(\gamma) \right\} e^{\sqrt{PQ}(t-t_0)} + \frac{1}{2} \left\{ A''_1(\gamma) - \sqrt{\frac{P}{Q}} B''_1(\gamma) \right\} e^{-\sqrt{PQ}(t-t_0)},$$

$$y''_1(t, \gamma) = \frac{1}{2} \sqrt{\frac{Q}{P}} \left\{ A''_1(\gamma) + \sqrt{\frac{P}{Q}} B''_1(\gamma) \right\} e^{\sqrt{PQ}(t-t_0)} - \frac{1}{2} \sqrt{\frac{Q}{P}} \left\{ A''_1(\gamma) - \sqrt{\frac{P}{Q}} B''_1(\gamma) \right\} e^{-\sqrt{PQ}(t-t_0)}, \quad (25)$$

$$x''_2(t, \gamma) = \frac{1}{2} \left\{ A''_2(\gamma) + \sqrt{\frac{P}{Q}} B''_2(\gamma) \right\} e^{\sqrt{PQ}(t-t_0)} + \frac{1}{2} \left\{ A''_2(\gamma) - \sqrt{\frac{P}{Q}} B''_2(\gamma) \right\} e^{-\sqrt{PQ}(t-t_0)},$$

$$y''_2(t, \gamma) = \frac{1}{2} \sqrt{\frac{Q}{P}} \left\{ A''_2(\gamma) + \sqrt{\frac{P}{Q}} B''_2(\gamma) \right\} e^{\sqrt{PQ}(t-t_0)} - \frac{1}{2} \sqrt{\frac{Q}{P}} \left\{ A''_2(\gamma) - \sqrt{\frac{P}{Q}} B''_2(\gamma) \right\} e^{-\sqrt{PQ}(t-t_0)}.$$

4.2. Application. Consider the set of differential equations with base condition

$$\begin{aligned} \frac{dx(t)}{dt} &= 3y(t); \\ \frac{dy(t)}{dt} &= 4x(t), \end{aligned} \quad (26)$$

$$\begin{aligned} x(0) &= x_0 \\ &= (3, 4, 6; 0.7, 0.3, 0.5), \\ y(0) &= y_0 \\ &= (2, 5, 9; 0.6, 0.2, 0.4). \end{aligned} \quad (27)$$

The solution is

$$\begin{aligned}
 x_1(t, \alpha) &= \frac{1}{2} \left\{ (3 + \alpha)0.7 + \sqrt{\frac{3}{4}} (2 + 3\alpha)0.6 \right\} e^{\sqrt{12}t} + \frac{1}{2} \left\{ (3 + \alpha)0.7 - \sqrt{\frac{3}{4}} (2 + 3\alpha)0.6 \right\} e^{-\sqrt{12}t}, \\
 y_1(t, \alpha) &= \frac{1}{2} \sqrt{\frac{4}{3}} \left\{ (3 + \alpha)0.7 + \sqrt{\frac{3}{4}} (2 + 3\alpha)0.6 \right\} e^{\sqrt{12}t} - \frac{1}{2} \sqrt{\frac{4}{3}} \left\{ (3 + \alpha)0.7 - \sqrt{\frac{3}{4}} (2 + 3\alpha)0.6 \right\} e^{-\sqrt{12}t}, \\
 x_2(t, \alpha) &= \frac{1}{2} \left\{ (6 - 2\alpha)0.7 + \sqrt{\frac{3}{4}} (9 - 4\alpha)0.6 \right\} e^{\sqrt{12}t} + \frac{1}{2} \left\{ (6 - 2\alpha)0.7 - \sqrt{\frac{3}{4}} (9 - 4\alpha)0.6 \right\} e^{-\sqrt{12}t}, \\
 y_2(t, \alpha) &= \frac{1}{2} \sqrt{\frac{4}{3}} \left\{ (6 - 2\alpha)0.7 + \sqrt{\frac{3}{4}} (9 - 4\alpha)0.6 \right\} e^{\sqrt{12}t} - \frac{1}{2} \sqrt{\frac{4}{3}} \left\{ (6 - 2\alpha)0.7 - \sqrt{\frac{3}{4}} (9 - 4\alpha)0.6 \right\} e^{-\sqrt{12}t}, \\
 x'_1(t, \beta) &= \frac{1}{2} \left\{ (4 - \beta)0.3 + \sqrt{\frac{3}{4}} (5 - 3\beta)0.2 \right\} e^{\sqrt{12}t} + \frac{1}{2} \left\{ (4 - \beta)0.3 - \sqrt{\frac{3}{4}} (5 - 3\beta)0.2 \right\} e^{-\sqrt{12}t}, \\
 y'_1(t, \beta) &= \frac{1}{2} \sqrt{\frac{4}{3}} \left\{ (4 - \beta)0.3 + \sqrt{\frac{3}{4}} (5 - 3\beta)0.2 \right\} e^{\sqrt{12}t} - \frac{1}{2} \sqrt{\frac{4}{3}} \left\{ (4 - \beta)0.3 - \sqrt{\frac{3}{4}} (5 - 3\beta)0.2 \right\} e^{-\sqrt{12}t}, \\
 x'_2(t, \beta) &= \frac{1}{2} \left\{ (4 + 2\beta)0.3 + \sqrt{\frac{3}{4}} (5 + 4\beta)0.2 \right\} e^{\sqrt{12}t} + \frac{1}{2} \left\{ (4 + 2\beta)0.3 - \sqrt{\frac{3}{4}} (5 + 4\beta)0.2 \right\} e^{-\sqrt{12}t}, \\
 y'_2(t, \beta) &= \frac{1}{2} \sqrt{\frac{4}{3}} \left\{ (4 + 2\beta)0.3 + \sqrt{\frac{3}{4}} (5 + 4\beta)0.2 \right\} e^{\sqrt{12}t} - \frac{1}{2} \sqrt{\frac{4}{3}} \left\{ (4 + 2\beta)0.3 - \sqrt{\frac{3}{4}} (5 + 4\beta)0.2 \right\} e^{-\sqrt{12}t}, \\
 x''_1(t, \gamma) &= \frac{1}{2} \left\{ (4 - \gamma)0.5 + \sqrt{\frac{3}{4}} (5 - 3\gamma)0.4 \right\} e^{\sqrt{12}t} + \frac{1}{2} \left\{ (4 - \gamma)0.5 - \sqrt{\frac{3}{4}} (5 - 3\gamma)0.4 \right\} e^{-\sqrt{12}t}, \\
 y''_1(t, \gamma) &= \frac{1}{2} \sqrt{\frac{4}{3}} \left\{ (4 - \gamma)0.5 + \sqrt{\frac{3}{4}} (5 - 3\gamma)0.4 \right\} e^{\sqrt{12}t} - \frac{1}{2} \sqrt{\frac{4}{3}} \left\{ (4 - \gamma)0.5 - \sqrt{\frac{3}{4}} (5 - 3\gamma)0.4 \right\} e^{-\sqrt{12}t}, \\
 x''_2(t, \gamma) &= \frac{1}{2} \left\{ (4 + 2\gamma)0.5 + \sqrt{\frac{3}{4}} (5 + 4\gamma)0.4 \right\} e^{\sqrt{12}t} + \frac{1}{2} \left\{ (4 + 2\gamma)0.5 - \sqrt{\frac{3}{4}} (5 + 4\gamma)0.4 \right\} e^{-\sqrt{12}t}, \\
 y''_2(t, \gamma) &= \frac{1}{2} \sqrt{\frac{4}{3}} \left\{ (4 + 2\gamma)0.5 + \sqrt{\frac{3}{4}} (5 + 4\gamma)0.4 \right\} e^{\sqrt{12}t} - \frac{1}{2} \sqrt{\frac{4}{3}} \left\{ (4 + 2\gamma)0.5 - \sqrt{\frac{3}{4}} (5 + 4\gamma)0.4 \right\} e^{-\sqrt{12}t}.
 \end{aligned} \tag{28}$$

In Table 1, the solution of x is given for distinct values of (α, β, γ) when $t = 3$.

The graphical representation of the above table is shown in Figure 1.

In Table 2, the solution of y is given for distinct values of (α, β, γ) when $t = 3$.

The graphical representation of Table 2 is shown in Figure 2.

We can see from the preceding table and graphs that the prerequisites for a powerful solution are still true. As a result, the solution is solid.

4.3. Value, Ambiguity, and Ranking of a Solution. The system of differential equations solution may be written in neutrosophic number as given in the following:

$$\begin{aligned}
 \underline{x} &= \langle (5.0312, 8.6260, 14.1718; 3.3054, 1.9997, 5.3691; 4.6113, 5.9657, 9.7686; 0.7, 0.3, 0.5) \rangle, \\
 \underline{y} &= \langle (5.7711, 9.9532, 16.3813; 2.2890, 3.8048, 6.1910; 4.0300, 6.8789, 11.2909; 0.6, 0.2, 0.4) \rangle.
 \end{aligned} \tag{29}$$

TABLE 1: Interpretation of x -values when $t = 3$.

| (α, β, γ) | $x_1(t, \alpha)$ | $x_2(t, \alpha)$ | $x'_1(t, \beta)$ | $x'_2(t, \beta)$ | $x''_1(t, \gamma)$ | $x''_2(t, \gamma)$ |
|---------------------------|------------------|------------------|------------------|------------------|--------------------|--------------------|
| 0 | 51181.60 | 144721.89 | 33684.21 | 33684.21 | 60846.86 | 60846.86 |
| 0.1 | 54864.39 | 139050.65 | 32347.92 | 32347.92 | 58337.32 | 64736.38 |
| 0.2 | 58547.18 | 133379.42 | 31011.63 | 31011.63 | 55827.78 | 68625.90 |
| 0.3 | 62229.98 | 127708.18 | 29675.34 | 29675.34 | 53318.24 | 72515.42 |
| 0.4 | 65912.77 | 122036.94 | 28339.05 | 28339.05 | 50808.70 | 76404.93 |
| 0.5 | 69595.56 | 116365.71 | 27002.76 | 27002.76 | 48299.16 | 80294.45 |
| 0.6 | 73278.35 | 110694.47 | 25666.47 | 25666.47 | 45789.62 | 84183.97 |
| 0.7 | 76961.15 | 105023.23 | 24330.18 | 24330.18 | 43280.08 | 88073.48 |
| 0.8 | 80643.94 | 99352.00 | 22993.89 | 22993.89 | 40770.54 | 91963.00 |
| 0.9 | 84326.73 | 93680.76 | 21657.60 | 21657.60 | 38260.99 | 95852.52 |
| 1 | 88009.52 | 88009.52 | 20321.31 | 20321.31 | 35751.45 | 99742.03 |

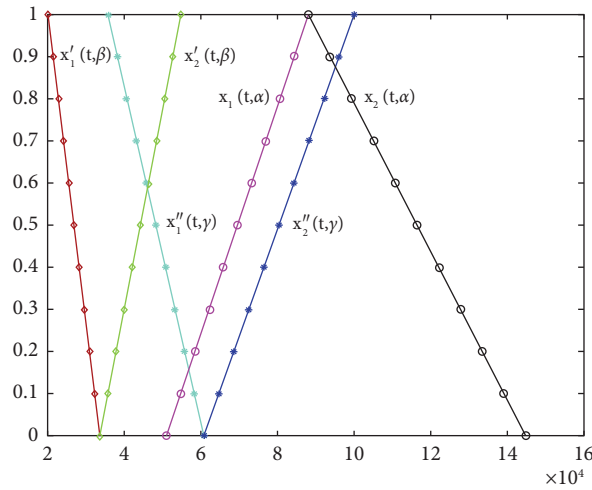


FIGURE 1: Interpretation of x values when $t = 3$.

TABLE 2: Interpretation of y values when $t = 3$.

| (α, β, γ) | $y_1(t, \alpha)$ | $y_2(t, \alpha)$ | $y'_1(t, \beta)$ | $y'_2(t, \beta)$ | $y''_1(t, \gamma)$ | $y''_2(t, \gamma)$ |
|---------------------------|------------------|------------------|------------------|------------------|--------------------|--------------------|
| 0 | 59099.42 | 167110.45 | 38895.17 | 38895.17 | 70259.91 | 70259.91 |
| 0.1 | 63351.94 | 160561.87 | 37352.16 | 41329.04 | 67362.14 | 74751.13 |
| 0.2 | 67604.47 | 154013.29 | 35809.14 | 43762.92 | 64464.37 | 79242.36 |
| 0.3 | 71856.99 | 147464.70 | 34266.13 | 46196.79 | 61566.60 | 83733.59 |
| 0.4 | 76109.51 | 140916.12 | 32723.11 | 48630.67 | 58668.83 | 88224.82 |
| 0.5 | 80362.03 | 134367.54 | 31180.10 | 51064.54 | 55771.06 | 92716.04 |
| 0.6 | 84614.55 | 127818.96 | 29637.08 | 53498.42 | 52873.30 | 97207.27 |
| 0.7 | 88867.08 | 121270.38 | 28094.07 | 55932.29 | 49975.53 | 101698.50 |
| 0.8 | 93119.60 | 114721.80 | 26551.05 | 58366.17 | 47077.76 | 106189.73 |
| 0.9 | 97372.12 | 108173.22 | 25008.04 | 60800.04 | 44179.99 | 110680.95 |
| 1 | 101624.64 | 101624.64 | 23465.02 | 63233.92 | 41282.22 | 115172.18 |

The value and ambiguity of SVTNN \underline{x} for truth, indeterminacy, and falsity membership are given by $V_\mu(\underline{x}) = 4.3861$ and $A_\mu(\underline{x}) = 0.7465$; $V_\gamma(\underline{x}) = 1.3617$ and $A_\gamma(\underline{x}) = 0.1685$; and $V_k(\underline{x}) = 1.5934$ and $A_k(\underline{x}) = 0.2149$.

Similarly, the value and ambiguity of SVTNN \underline{y} for truth, indeterminacy, and falsity membership are given by $V_\mu(\underline{y}) = 22.3075$ and $A_\mu(\underline{y}) = 0.6366$; $V_\gamma(\underline{y}) = 15.1675$ and $A_\gamma(\underline{y}) = 0.4162$; and $V_k(\underline{y}) = 15.4211$ and $A_k(\underline{y}) = 0.4357$.

The δ -weighted value of the SVTNN's \underline{x} and \underline{y} are given by $V_\delta(\underline{x}) = 1.4310\delta + 2.9551$ and $V_\delta(\underline{y}) = 30.5886 - 8.2811\delta$.

In the case of weighted values and ambiguities of the SVTNN's \underline{x} and \underline{y} and $\delta \in [0, 1]$, the order of ranking \underline{x} and \underline{y} is $\underline{x} < \underline{y}$.

Hence, the number of armaments of nation X is lesser than the number of armaments of nation Y.

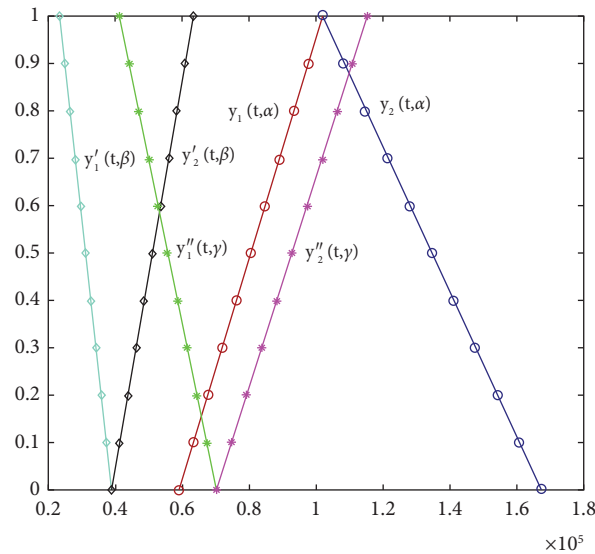


FIGURE 2: Interpretation of y values when $t = 3$.

5. Conclusion

In this research work, a set of neutrosophic differential equations were used to develop the arms race model and estimate the uncertain parameters, which is recognised as an important issue of research in international military planning.

By demonstrating the arm race using neutrosophic figures, it was possible to satisfy the indeterminate parameters as well, which was advantageously helpful for the planners of military activities in their analysis of armament expenditures. Additionally, it is found that ranking, ambiguity, and appraisal are necessary for contrasting these two alternatives. This strategy is also a promising one for resolving other comparable models in a neutrosophic setting. The findings of the indeterminacy function, truth function, and falsehood function were produced using MATLAB. Here, it is discovered that the suggested approach works well.

6. Future Scope

Neutrosophic fractional calculus and the system of nonlinear differential equations may both be the subject of future study. The area of solving neutrosophic differential equations using various numerical methods is one that is currently developing. In real-world applications, higher-order neutrosophic differential equations and partial differential equations utilizing neutrosophic numbers can be understood to resolve the issues for further research.

Abbreviations

| | |
|--------|---|
| IFS: | Intuitionistic fuzzy set |
| FDDE: | Fuzzy differential equation |
| IFDE: | Intuitionistic fuzzy differential equation |
| SVNN: | Single-valued neutrosophic number |
| TNN: | Triangular neutrosophic number |
| SVTNN: | Single-valued triangular neutrosophic number. |

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this study.

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