A Study of Some Neutrosophic Differential Equations as a Direct Application of One-Dimensional Geometric AH-Isometry

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A Study of Some Neutrosophic Differential Equations as a Direct Application of One-Dimensional Geometric AH-Isometry

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Abstract

In this paper, we use the one dimensional AH-isometry to find the structure of the solutions of many neutrosophic differential equations. These equations will be handled by the algebraic direct image of the neutrosophic AH-isometry taken in one dimension.

Keywords: One-Dimensional Geometric AH-Isometry; Neutrosophic linear Differential Equation; Neutrosophicreal number.

1.Introduction

Neutrosophic logic. Neutrosophy, Neutrosophic set, Neutrosophic probability, are recently creations of Smarandache, being characterized by having the indeterminacy as component of their framework,and a notable feature of neutrosophic logic is that can be considered a generaliazationof fuzzy logics, encompassing the classical logic as well[1]. Also.F. Smarandache, has defined the concept of continuation of a neutrosophic function in year 2015 in[1],and neutrosophicmereo-limit[1], mereo-continuity.Moreover, in 2014, he has defined the concept of a neutro-oscillator differentialin [3], and mereo-derivative. Finallyin 2013 he introduced neutrosophic integration in [2], and mereo-integral, besides the classical defintions of limit, continuity, deverative, and integral respectively.
Among the recent applications there are: neutrosophic crisp set theory in image processing,neutrosophic setsmedical field [6-10], in information geographic systems and possible applications to database. Also,neutrosophic triplet group application to physics.Morever Several researches have made multiple contributions to neutrosophic topology and algebra [14-20, 34-50], Also More researches have made multiple contributions to neutrosophic analysis[21 − 33]. Finally the neutrosophic integration may have application in calculus the areas between two neutrosophic functions.

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2. Preliminaries

Definition :Neutrosophic Real Number

Suppose that w is a neutrosophic number, then it takes the following standard form: $w = a + bI$ where a, b are real coefficients, and I represents the indeterminacy, where $0.I = 0$ and $I^n = I$ for all positive integers n.

For example:

$w = 1 + 2I, w = 3 = 3 + 0I.$

Definition :Division of neutrosophic real numbers

Suppose that w_1, w_2 are two neutrosophic number,where

$$w_1 = a_1 + b_1 I, w_2 = a_2 + b_2 I$$

Then:

$$\frac{w_1}{w_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$$

Definition

Let $R(I) = \{a + bI\ ; a, b \in R\}$ where $I^2 = I$be the neutrosophic field of reals. The one-dimensional isometry (AH-Isometry) is defined as follows:

$$T: R(I) \to R \times R$$
$$T(a + bI) = (a, a + b)$$

Remark

T is an algebraic isomorphism between two rings, it has the following properties:

1) T is bijective.
2) T preserves addition and multiplication, i.e.:

$$T[(a + bI) + (c + dI)] = T(a + bI) + T(c + dI)$$
And
$$T[(a + bI) \cdot (c + dI)] = T(a + bI) \cdot T(c + dI)$$

3) Since T is bijective, then it is invertible by:

$$T^{-1}: R \times R \to R(I)$$
$$T^{-1}(a, b) = a + (b - a)I$$

4) T preserves distances, i.e.:

The distance on $R(I)$ can be defined as follows:

Let $A = a + bI, B = c + dI$ be two neutrosophic real numbers, then $L = \|\overrightarrow{AB}\| = d[(a + bI, c + dI)] = |a + bI - (c + dI)| = |(a - c) + I(b - d)| = |a - c| + I[|a + b - c - d| - |a - c|].$

On the other hand, we have:

$T(\|\overrightarrow{AB}\|) = (|a - c|, |(a + b) - (c + d)|) = (d(a, c), d(a + b, c + d)) = d[(a, a + b), (c, c + d)]$=$d(T(a + bI), T(c + dI))$

=$\|T(\overrightarrow{AB})\|$.

This implies that the distance is preserved up to isometry. i.e.$\|T(AB)\| = T(\|AB\|)$

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Neutrosophic linear differential equation.

Inthis section is defined a linear differential equation by Using the One-Dimensional Geometric AH-Isometry and solutions are found for this equation.

Neutrosophic identical linear differential equation.

Definition

Let $Y = y_1 + y_2 I$, $X = x_1 + x_2 I$We define the Neutrosophic identical linear differential equation by Using the One-Dimensional Geometric AH-Isometry as form:

$$\acute{Y} + f(x)Y = 0$$

This equation can be written as follow:

$$(y_1{}' + I[(y_1 + y_2)' - y_1{}'])(f(x_1) + I[f(x_1 + x_2) - f(x_1)])(y_1 + y_2 I) = 0$$

$$\Rightarrow y_1' + f(x_1)y_1 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)y_1)] = 0$$

Method of solution.

1. Take AH-Isometry for the differential equation, we have.

$$T(y_1' + f(x_1)y_1 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)y_1)]) = T(0)$$

$$\Rightarrow \big(y_1' + f(x_1)y_1, (y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2)\big) = (0,0)$$

Then.

$$\begin{cases} y_1' + f(x_1)y_1 = 0 \dots\dots (1) \\ (y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) = 0 \dots\dots (2) \end{cases}$$

The equations $(1)$and $(2)$are two differential equation classical.

2. We find the solution to the equations classical(1) and (2), we have.

y_1 the solution to the equation (1).

$(y_1 + y_2)$ the solution to the equation (2).

3. We Take invertibleAH-Isometry, then, we have the solution of aNeutrosophic identical linear differential equation.

$$Y = y_1 + y_2 I = T^{-1}(y_1, y_1 + y_2) = y_1 + \big((y_1 + y_2) - y_1\big)I$$

Example. Find a solution to the equation:

$$\acute{Y} + \frac{1}{X}Y = 0$$

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Solution.

Let $Y = y_1 + y_2 I, X = x_1 + x_2 I$. Then.

$$y_1' + \frac{1}{x_1}y_1 + I\left[(y_1 + y_2)' - \frac{1}{(x_1 + x_2)}(y_1 + y_2) - \left(y_1' + \frac{1}{x_1}y_1\right)\right] = (0)$$

Now, Take AH-Isometry for the differential equation, we have.

$$\begin{cases} y_1' + \frac{1}{x_1}y_1 = 0 \dots\dots (3) \\ (y_1 + y_2)' - \frac{1}{x_1 + x_2}(y_1 + y_2) = 0 \dots\dots (4) \end{cases}$$

The solution of equation (3) written as follow:

$y_1 = e^{-\int \frac{1}{x_1} dx_1} + a = e^{-\ln x_1} + a = \frac{1}{x_1} + a$, where $a \in R$.

The solution of equation (4) written as follow:

$(y_1 + y_2) = e^{-\int \frac{1}{x_1 + x_2} d(x_1 + x_2)} + b = e^{-\ln(x_1 + x_2)} + b = \frac{1}{x_1 + x_2} + b$, where $b \in R$

Now, Take invertibleAH-Isometry, then, we have the solution of aNeutrosophic identical linear differential equation .

$$Y = y_1 + y_2 I = T^{-1}\left(\frac{1}{x_1} + a, \frac{1}{x_1 + x_2} + b\right) = \frac{1}{x_1} + a + \left(\frac{1}{x_1 + x_2} + b - \frac{1}{x_1} - a\right) I$$

$$Y = y_1 + y_2 I = \frac{1}{x_1} + \left(\frac{1}{x_1 + x_2} - \frac{1}{x_1}\right) I + a + (b - a) I$$

we have.

$$\frac{1}{x_1} + \left(\frac{1}{x_1 + x_2} - \frac{1}{x_1}\right) I = \frac{1}{x_1} + \left(\frac{-x_2}{x_1(x_1 + x_2)}\right) I = \frac{1}{x_1 + x_2 I} = \frac{1}{X}$$

So that,

$$Y = y_1 + y_2 I = \frac{1}{X} + (a + bI)$$

where $a + bI \in R(I)$.

Example Find a solution to the equation:

$$\acute{Y} + 2XY = 0$$

Solution.

Let $Y = y_1 + y_2 I, X = x_1 + x_2 I$. Then.

$$\left(y_1' + 2x_1 y_1, (y_1 + y_2)' - 2(x_1 + x_2)(y_1 + y_2)\right) = (0)$$

Now, Take AH-Isometry for the differential equation, we have.

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$$\begin{cases} y_1' + 2x_1y_1 = 0 \ldots \ldots (5) \\ (y_1+y_2)' - 2(x_1+x_2)(y_1+y_2) = 0 \ldots \ldots (6) \end{cases}$$

The solution of equation (5) written as follow:

$y_1 = e^{-\int 2x_1\, dx_1} + a = e^{-(x_1)^2} + a$, where $a \in R$.

The solution of equation (6) written as follow:

$(y_1+y_2) = e^{-\int 2(x_1+x_2)\, d(x_1+x_2)} + b = e^{-(x_1+x_2)^2} + b$, where $b \in R$

Now, Take invertibleAH-Isometry, then, we have the solution of aNeutrosophic identical linear differential equation .

$$Y = y_1 + y_2 I = T^{-1}\left(e^{-(x_1)^2} + a, e^{-(x_1+x_2)^2} + b\right) = \frac{1}{x_1} + a + \left(e^{-(x_1+x_2)^2} + b - e^{-(x_1)^2} - a\right)I$$

$$Y = y_1 + y_2 I = e^{-(x_1)^2} + \left(e^{-(x_1+x_2)^2} - e^{-(x_1)^2}\right)I + a + (b-a)I$$

we have.

$$e^{-(x_1)^2} + \left(e^{-(x_1+x_2)^2} - e^{-(x_1)^2}\right)I = e^{-X^2}$$

So that,

$$Y = y_1 + y_2 I = e^{-X^2} + (a + bI)$$

where $a + bI \in R(I)$.

Neutrosophic non-homogeneous linear differential equation.

Definition

We define the Neutrosophic non-homogeneous linear differential equation is taken the following forms:

$$\acute{Y} + f(X)Y = g(X)$$

This equation can be written as follow:

$$(y_1{}' + I[(y_1+y_2)' - y_1{}'])(f(x_1) + I[f(x_1+x_2) - f(x_1)])(y_1 + y_2 I) = g(x_1) + I[g(x_1+x_2) - g(x_1)]$$

$$\Rightarrow y_1' + f(x_1)y_1 + I[(y_1+y_2)' - f(x_1+x_2)(y_1+y_2) - (y_1' + f(x_1)y_1)] = g(x_1) + I[g(x_1+x_2) - g(x_1)]$$

Method of solution.

1. Take AH-Isometry for the differential equation, we have.

$$T(y_1' + f(x_1)y_1 + I[(y_1+y_2)' - f(x_1+x_2)(y_1+y_2) - (y_1' + f(x_1)y_1)]) = T(g(x_1) + I[g(x_1+x_2) - g(x_1)])$$

$$\Rightarrow \left(y_1' + f(x_1)y_1, (y_1+y_2)' - f(x_1+x_2)(y_1+y_2)\right) = \left(g(x_1), g(x_1+x_2)\right)$$

Then.

$$\begin{cases} y_1' + f(x_1)y_1 = g(x_1) \ldots \ldots (7) \\ (y_1+y_2)' - f(x_1+x_2)(y_1+y_2) = g(x_1+x_2) \ldots \ldots (8) \end{cases}$$

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The equations (7) and (8)are two differential equation classical.

2. We find the solution to the equations classical (7) and (8), we have.

y_1 the solution to the equation (7).

$(y_1 + y_2)$ the solution to the equation (8).

3. We Take invertibleAH-Isometry, then, we have the solution of aNeutrosophic identical linear differential equation .

$$Y = y_1 + y_2 I = T^{-1}(y_1, y_1 + y_2) = y_1 + \big((y_1 + y_2) - y_1\big)I$$

Example Find the general solution for the following neutrosophic non- homogeneous linear differential equation:

$$\acute{Y} + 2XY = X$$

Solution.

Let $Y = y_1 + y_2 I, X = x_1 + x_2 I$. Then.

$$\big(y_1' + 2x_1 y_1, (y_1 + y_2)' - 2(x_1 + x_2)(y_1 + y_2)\big) = (x_1, x_1 + x_2)$$

Now, Take AH-Isometry for the differential equation, we have.

$$\begin{cases} y_1' + 2x_1 y_1 = x_1 \dots \dots (9) \\ (y_1 + y_2)' - 2(x_1 + x_2)(y_1 + y_2) = x_1 + x_2 \dots \dots (10) \end{cases}$$

The solution of equation (10) written as follow:

$y_1 = \frac{1}{\mu(x_1)}\left(a + \int x_1 \mu(x_1) dx_1\right) = \frac{1}{e^{(x_1)^2}}\left(a + \int x_1 e^{(x_1)^2} dx\right) = \frac{1}{e^{(x_1)^2}}\left(a + \frac{1}{2}e^{(x_1)^2}\right)$,where $a \in R$.

By the method same, The solution of equation (10) written as follow:

$(y_1 + y_2) = \frac{1}{e^{(x_1+x_2)^2}}\left(b + \frac{1}{2}e^{(x_1+x_2)^2}\right)$, where $b \in R$

Now, Take invertibleAH-Isometry, then, we have the solution of aNeutrosophic identical linear differential equation .

$$\begin{aligned} Y = y_1 + y_2 I &= T^{-1}\left(\frac{1}{e^{(x_1)^2}}\left(a + \frac{1}{2}e^{(x_1)^2}\right), \frac{1}{e^{(x_1+x_2)^2}}\left(b + \frac{1}{2}e^{(x_1+x_2)^2}\right)\right) \\ &= T^{-1}\left(\frac{1}{e^{(x_1)^2}}, \frac{1}{e^{(x_1+x_2)^2}}\right).\left[T^{-1}(a,b) + T^{-1}\left(\frac{1}{2}e^{(x_1)^2}, \frac{1}{2}e^{(x_1+x_2)^2}\right)\right] \end{aligned}$$

$$Y = y_1 + y_2 I = \left[\frac{1}{e^{(x_1)^2}} + \left(\frac{1}{e^{(x_1+x_2)^2}} - \frac{1}{e^{(x_1)^2}}\right)I\right].\left[a + (b-a)I + \frac{1}{2}e^{(x_1)^2} + \left(\frac{1}{2}e^{(x_1+x_2)^2} - \frac{1}{2}e^{(x_1)^2}\right)I\right]$$

By Definition 2.5, we have.

$$\frac{1}{e^{(x_1)^2}} + \left(\frac{1}{e^{(x_1+x_2)^2}} - \frac{1}{e^{(x_1)^2}}\right)I = \frac{1}{e^{(x_1+x_2 I)^2}} = \frac{1}{e^{X^2}}$$

And$\frac{1}{2}e^{(x_1)^2} + \left(\frac{1}{2}e^{(x_1+x_2)^2} - \frac{1}{2}e^{(x_1)^2}\right)I = \frac{1}{2}\left[e^{(x_1)^2} + \left(e^{(x_1+x_2)^2} - e^{(x_1)^2}\right)I\right] = \frac{1}{2}e^{(x_1+x_2 I)^2} = \frac{1}{2}e^{X^2}$

So that,

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$$Y = y_1 + y_2 I = \frac{1}{e^{X^2}}\left[a + bI + \frac{1}{2}e^{X^2}\right]$$

where $a + bI \in R(I)$.

Example Find the general solution for the following neutrosophic non- homogeneous linear differential equation:

$$\acute{Y} + cotg(X)Y = sin(X)$$

Solution.

Let $Y = y_1 + y_2 I, X = x_1 + x_2 I$. Then.

$$\left(y_1' + cotg(x_1)y_1, (y_1 + y_2)' + cotg(x_1 + x_2)(y_1 + y_2)\right) = \left(sin(x_1), sin(x_1 + x_2)\right)$$

Now, Take AH-Isometry for the differential equation, we have.

$$\begin{cases} y_1' + cotg(x_1)y_1 = sin(x_1) \dots\dots (11) \\ (y_1 + y_2)' + cotg(x_1 + x_2)(y_1 + y_2) = sin(x_1 + x_2) \dots\dots (12) \end{cases}$$

The solution of equation (11) written as follow:

$$y_1 = \frac{1}{\mu(x_1)}\left(a + \int x_1 \mu(x_1) dx_1\right) = \frac{1}{sin(x_1)}\left(a + \int sin(x_1).sin(x_1)dx\right) = \frac{1}{sin(x_1)}\left(a + \int sin^2(x_1)dx\right)$$

$y_1 = \frac{1}{sin(x_1)}\left(a + \frac{1}{2}x_1 + \frac{1}{4}sin(2x_1)\right)$,where $a \in R$.

By the method same, The solution of equation (12) written as follow:

$(y_1 + y_2) = \frac{1}{sin(x_1+x_2)}\left(b + \frac{1}{2}(x_1 + x_2) + \frac{1}{4}sin\big(2(x_1 + x_2)\big)\right)$, where $b \in R$

Now, Take invertibleAH-Isometry, then, we have the solution of aNeutrosophic identical linear differential equation .

$$Y = y_1 + y_2 I = T^{-1}\left(\frac{1}{sin(x_1)}\left(a + \frac{1}{2}x_1 + \frac{1}{4}sin(2x_1)\right), \frac{1}{sin(x_1 + x_2)}\left(b + \frac{1}{2}(x_1 + x_2) + \frac{1}{4}sin\big(2(x_1 + x_2)\big)\right)\right)$$

$$Y = y_1 + y_2 I = T^{-1}\left(\frac{1}{sin(x_1)}, \frac{1}{sin(x_1 + x_2)}\right).\Bigg[T^{-1}(a, b) + T^{-1}\left(\frac{1}{2}x_1, \frac{1}{2}(x_1 + x_2)\right) + T^{-1}\left(\frac{1}{4}sin(2x_1), \frac{1}{4}sin\big(2(x_1 + x_2)\big)\right)\Bigg]$$

$$Y = y_1 + y_2 I = \left[\frac{1}{sin(x_1)} + \left(\frac{1}{sin(x_1 + x_2)} - \frac{1}{sin(x_1)}\right)I\right].\Bigg[a + (b - a)I + \frac{1}{2}x_1 + \left(\frac{1}{2}(x_1 + x_2) - \frac{1}{2}x_1\right)I + \frac{1}{4}sin(2x_1) + \left(\frac{1}{4}sin\big(2(x_1 + x_2)\big) - \frac{1}{4}sin(2x_1)\right)I\Bigg]$$

On the other hand, we have.

$$\frac{1}{sin(x_1)} + \left(\frac{1}{sin(x_1 + x_2)} - \frac{1}{sin(x_1)}\right)I = \frac{1}{sin(x_1 + x_2 I)} = \frac{1}{sin(X)}$$

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And$\frac{1}{2}x_1 + \left(\frac{1}{2}(x_1 + x_2) - \frac{1}{2}x_1\right)I = \frac{1}{2}\left[x_1 + \left((x_1 + x_2) - x_1\right)I\right] = \frac{1}{2}(x_1 + x_2 I) = \frac{1}{2}X$

And $\frac{1}{4}sin(2x_1) + \left(\frac{1}{4}sin\left(2(x_1 + x_2)\right) - \frac{1}{4}sin(2x_1)\right)I = \frac{1}{4}\left[sin(2x_1) + \left(sin\left(2(x_1 + x_2)\right) - sin(2x_1)\right)\right] = \frac{1}{4}sin2(x_1 + x_2 I) = \frac{1}{4}sin2X$

So that,

$$Y = y_1 + y_2 I = \frac{1}{sin(X)}\left[a + bI + \frac{1}{2}X + \frac{1}{4}sin2X\right]$$

where $a + bI \in R(I)$.

Conclusion

In this work we have used the one dimensional AH-isometry to find the solutions of many neutrosophic diiferential equations. Also, we have illustrated many examples to clarify the validity of our work.

In the future, we aim to study refined neutrosophic differential equations.

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