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AN INTRODUCTION TO THE SMARANDACHE GEOMETRIES

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Abstract: In this paper we make a presentation of these exciting geometries and present a model for a particular one.

Introduction: An axiom is said *Smarandachely denied* if the axiom behaves in at least two different ways within the same space (i.e., validated and invalided, or only invalidated but in multiple distinct ways).

A *Smarandache Geometry* is a geometry which has at least one Smarandachely denied axiom (1969).

Notations: Let's note any point, line, plane, space, triangle, etc. in a smarandacheian geometry by s-point, s-line, s-plane, s-space, s-triangle respectively in order to distinguish them from other geometries.

Applications: Why these hybrid geometries? Because in reality there does not exist isolated homogeneous spaces, but a mixture of them, interconnected, and each having a different structure.

The Smarandache geometries (SG) are becoming very important now since they combine many spaces into one, because our world is not formed by perfect homogeneous spaces as in pure mathematics, but by non-homogeneous spaces. Also, SG introduce the degree of negation in geometry for the first time [for example an axiom is denied 40% and accepted 60% of the space] that's why they can become revolutionary in science and it thanks to the idea of partial denying/accepting of axioms/propositions in a space (making multi-spaces, i.e. a space formed by combination of many different other spaces), as in fuzzy logic the degree of truth (40% false and 60% true).

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Smarandache geometries are starting to have applications in physics and engineering because of dealing with non-homogeneous spaces.

In the Euclidean geometry, also called parabolic geometry, the fifth Euclidean postulate that there is only one parallel to a given line passing through an exterior point, is kept or validated.
In the Lobachevsky-Bolyai-Gauss geometry, called hyperbolic geometry, this fifth Euclidean postulate is invalidated in the following way: there are infinitely many lines parallels to a given line passing through an exterior point. While in the Riemannian geometry, called elliptic geometry, the fifth Euclidean postulate is also invalidated as follows: there is no parallel to a given line passing through an exterior point.

Thus, as a particular case, Euclidean, Lobachevsky-Bolyai-Gauss, and Riemannian geometries may be united altogether, in the same space, by some Smarandache geometries. These last geometries can be partially Euclidean and partially Non-Euclidean. Howard Iseri [3] constructed a model for this particular Smarandache geometry, where the Euclidean fifth postulate is replaced by different statements within the same space, i.e. one parallel, no parallel, infinitely many parallels but all lines passing through the given point, all lines passing through the given point are parallel.

Linfan Mao [4, 5] showed that SG are generalizations of Pseudo-Manifold Geometries, which in their turn are generalizations of Finsler Geometry, and which in its turn is a generalization of Riemann Geometry.

Let's consider Hilbert's 21 axioms of Euclidean geometry. If we Smarandachely deny one, two, three, and so on, up to 21 axioms respectively, then one gets:

$$C_{21}^{1} + C_{21}^{2} + ... + C_{21}^{21} = 2^{21} - 1 = 2097151$$

Smarandache geometries, however the number is much higher because one axiom can be Smarandachely denied in multiple ways.
Similarly, if one Smarandachely denies the axioms of Projective Geometry, etc.

It seems that Smarandache Geometries are connected with the Theory of Relativity (because they include the Riemannian geometry in a subspace) and with the Parallel Universes (because they combine separate spaces into one space only) too.

A *Smarandache manifold* is an *n*-D manifold that supports a smarandacheian geometry.

Examples:
As a particular case one mentions *Howard's Models* [3] where a *Smarandache manifold* is a 2-D manifold formed by equilateral triangles such that around a vertex there are 5 (for elliptic), 6 (for Euclidean), and 7 (for hyperbolic) triangles, two by two having in common a side. Or, more general, an n-D manifold constructed from n-D submanifolds (which have in common two by two at most one m-D frontier, where $m < n$) that supports a Smarandache geometry.

A Mode for a particular Smarandache Geometry:
Let's consider an Euclidean plane (α) and three non-collinear given points *A, B,* and *C* in it. We define as s-points all usual Euclidean points and s-lines any Euclidean line that passes through one and only one of the points *A, B,* or *C*. Thus the geometry formed is smarandacheian because two axioms are Smarandachely denied:

a) The axiom that through a point exterior to a given line there is only one parallel passing through it is now replaced by two statements: one parallel, and no parallel.

Examples:

Let's take the Euclidean line *AB* (which is not an s-line according to the definition because passes through two among the three given points *A, B, C*), and an s-line noted (*c*) that passes through s-point *C* and is parallel in the Euclidean sense to *AB*:
- through any s-point not lying on *AB* there is one s-parallel to (*c*).
- through any other s-point lying on the Euclidean line *AB*, there is no s-parallel to (*c*).

b) And the axiom that through any two distinct points there exist one line passing through them is now replaced by: one s-line, and no s-line.

Examples:
Using the same notations:
- through any two distinct s-points not lying on Euclidean lines *AB, BC, CA,* there is one s-line passing through them;
- through any two distinct s-points lying on *AB* there is no s-line passing through them.

Questions:
Is there a general model for all Smarandache Geometries in such a way that replacing some parameters one gets any of the desired particular SG?

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