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3 authors:

Bobin Albert
IFET College of Engineering

Thangaraja Paramasivam
Mahendra Engineering College

Thayalan Thaya
Annamalai University

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DECISION MAKING USING CUBIC HYPERSOFT TOPSIS METHOD

A. BOBIN*, P. THANGARAJA, H. PRATHAB AND S. THAYALAN

Abstract. In real-life scenarios, we may have to deal with real numbers or numbers in intervals or a combination of both to solve multi-criteria decision-making (MCDM) problems. Also, we may come across a situation where we must combine this interval and actual number membership values into a single real number. The most significant factor in combining these membership values into a single value is by using aggregation operators or scoring algorithms. To overcome such a situation, we suggest the cubic hypersoft set (CHSS) concept as a workaround. Ultimately, this makes it simple for the decision-maker to obtain information without misconceptions. The primary aim of this study is to establish some operational laws for the cubic hypersoft set, present the fundamental properties of aggregation operators and propose an algorithm by using the technique of order of preference by similarity to the ideal solution (TOPSIS) technique based on correlation coefficients to analyze the stress-coping skills of workers.

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1. Introduction

Zadeh [1] defined the logic of fuzzy set (FS), to handle uncertainty in science. This structure's generalization and several theoretical extensions enabled the researchers to use FS in various domains. It contains generalizations like interval-valued FS (IVFS)[2], intuitionistic FS (IFS) [3] and cubic set [4]. Molodsov's [5] introduction of the soft set (SS) resulted in parameterizing the universal set. To get around the limitations of SS, Smarandache [6] introduced the idea of a hypersoft set (HSS). Chinnadurai et al. [7] suggested a cubic soft matrix (CSM) for ranking the alternatives. Chinnadurai and Bobin [8] proposed reversing the ranking approach and employing max-min operations in CSM. In addition to using the TOPSIS approach to address multi-criteria decision-making (MCDM)

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issues, Zulqarnain et al. [9], [10] also proposed the ideas of intuitionistic hypersoft sets and Pythagorean hypersoft sets. The existing works of [11], [12], [13] and [14] explains the concept of TOPSIS technique in detail .

When the grades are a combination of interval and actual number grades for the provided qualities, CHSS proves to be a reliable method for predicting uncertainty. We present this study to show the significance of CHSS. To analyze the options based on cubic HSS (CHSS) data, we use aggregation operators and the TOPSIS technique based on correlation coefficient (CC). The whole field of study pertains to CHSS theory, associated development, and applications. Therefore, decision-makers assess the innovative technique proposed in this research and provide a practical solution. Furthermore, by applying the CHSS TOPSIS technique, we offer an appropriate workaround for analyzing workers' stress-coping abilities.

2. Preliminaries

Let $\mathcal{X}$ represent the universe, $x_i \in \mathcal{X}$, $P(\mathcal{X})$ be power set of $\mathcal{X}$, $\mathbb{N}$ denote natural numbers. Let the closed interval of real numbers be denoted as [0,1]. $C[0,1]$ represent closed sub intervals of [0,1] and $\mathcal{C}^U$ be cubic set (CS) over $\mathcal{X}$.

Definition 2.1. [4] A CS is represented as $\Phi = \{\langle \tilde{\mathcal{K}}_\Phi(x), \mathcal{K}_\Phi(x)\rangle, x \in \mathcal{X}\}$, where $\tilde{\mathcal{K}}_\Phi(x) : \mathcal{X} \to C[0,1]$, $\mathcal{K}_\Phi(x) : \mathcal{X} \to [0,1]$. $\tilde{\mathcal{K}}_\Phi(x)$ denote sub interval membership grades and $\mathcal{K}_\Phi(x)$ represent the membership grades of the element $x \in \mathcal{X}$. The lower and upper grades of $\tilde{\mathcal{K}}_\Phi(x)$ are given by $\underline{\mathcal{K}}_\Phi(x)$ and $\overline{\mathcal{K}}_\Phi(x)$.

Definition 2.2. [6] Let $\delta_1, \delta_2, ..., \delta_k$, be attribute sets. The sub-attributes are $\delta_1 = \{\Delta_{11}, \Delta_{12}, ..., \Delta_{1p}\}, \delta_2 = \{\Delta_{21}, \Delta_{22}, ..., \Delta_{2q}\}, ..., \delta_k = \{\Delta_{k1}, \Delta_{k2}, ..., \Delta_{kr}\}$, where $1 \le p \le x$, $1 \le q \le y$, $1 \le r \le z$ and x, y, $z \in \mathbb{N}$, $\delta_i \cap \delta_j = \emptyset$, $i, j \in \{1, 2, ..., k\}$ and $i \neq j$. Cartesian product of the attributes $\delta_1 \times \delta_2 \times ... \times \delta_k = \tilde{\delta} = \{\Delta_{1p} \times \Delta_{2q} \times ... \times \Delta_{kr}\}$, denote set of multi- attributes. A pair $(\Phi, \tilde{\delta})$ is called hypersoft set (HSS) over $\mathcal{V}$, if there exists a mapping $\Phi : \tilde{\delta} \to P(\mathcal{X})$ and denoted as $(\Phi, \tilde{\delta}) = \left\{(\tilde{\Delta}, \Phi(\tilde{\Delta}))|\tilde{\Delta} \in \tilde{\delta}, \Phi(\tilde{\Delta}) \in P(\mathcal{X})\right\}$.

3. Cubic hypersoft set

We provide the definition of cubic hypersoft set (CHSS) and few properties of CC and weighted CC (WCC) of CHSS.

Definition 3.1. A pair $(\Phi, \tilde{\delta})$ is called a CHSS over $\mathcal{X}$, if there exists a mapping $\Phi : \tilde{\delta} \to \mathcal{C}^U$. CHSS can be represented as $(\Phi, \tilde{\delta}) = \left\{(\tilde{\Delta}, \Phi(\tilde{\Delta}))|\tilde{\Delta} \in \tilde{\delta}, \Phi(\tilde{\Delta}) \in \mathcal{C}^U\right\}$, where $\Phi(\tilde{\Delta}) = \{\langle \tilde{\mathcal{K}}_{\Phi(\tilde{\Delta})}(x), \mathcal{K}_{\Phi(\tilde{\Delta})}(x)\rangle | x \in \mathcal{X}\}$, where $\tilde{\mathcal{K}}_{\Phi(\tilde{\Delta})}(x) : \mathcal{X} \to C[0,1]$, $\mathcal{K}_{\Phi(\tilde{\Delta})}(x) : \mathcal{X} \to [0,1]$. $\tilde{\mathcal{K}}_{\Phi(\tilde{\Delta})}(x)$ represent closed sub intervals of [0,1] and $\mathcal{K}_{\Phi(\tilde{\Delta})}(x)$ denote the membership grades of the element $x \in \mathcal{X}$. The lower and upper ends of $\tilde{\mathcal{K}}_{\Phi(\tilde{\Delta})}(x)$ are given as $\underline{\mathcal{K}}_{\Phi(\tilde{\Delta})}(x)$, $\overline{\mathcal{K}}_{\Phi(\tilde{\Delta})}(x)$.

3.1. Correlation coefficient of CHSS. Let the two CHSS over $\mathcal{X}$ be as below:

$$(\Phi_1, \tilde{\delta}_1) = \{(x_i, \langle[\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i), \overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i)], \mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_i)\rangle\} \textbf{and}$$
$$(\Phi_2, \tilde{\delta}_2) = \{(x_i, \langle[\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i), \overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i)], \mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i)\rangle\}.$$

Definition 3.2. The cubic informational energies of $(\Phi_1, \tilde{\delta}_1)$ and $(\Phi_2, \tilde{\delta}_2)$ are represented as

$$\Phi(\Phi_1, \tilde{\delta}_1) = \Sigma_{k=1}^{m} \Sigma_{i=1}^{n} \Big[(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 \Big], \tag{1}$$

$$\Phi(\Phi_2, \tilde{\delta}_2) = \Sigma_{k=1}^{m} \Sigma_{i=1}^{n} \Big[(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 + (\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 + (\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 \Big]. \tag{2}$$

Definition 3.3. The correlation measure between $(\Phi_1, \tilde{\delta}_1)$ and $(\Phi_2, \tilde{\delta}_2)$ can be represented as

$$\alpha_{\mathcal{K}}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) = \Sigma_{k=1}^{m} \Sigma_{i=1}^{n} \Big[(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i)) * (\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i)) + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i)) * (\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i)) + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_i)) * (\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i)) \Big]. \tag{3}$$

Proposition 3.4. *Let $(\Phi_1, \tilde{\delta}_1)$ and $(\Phi_2, \tilde{\delta}_2)$ be two CHSS. Then,*
(i) $\alpha_{\mathcal{K}}((\Phi_1, \tilde{\delta}_1), (\Phi_1, \tilde{\delta}_1)) = \Phi(\Phi_1, \tilde{\delta}_1)$
(ii) $\alpha_{\mathcal{K}}((\Phi_2, \tilde{\delta}_2), (\Phi_2, \tilde{\delta}_2)) = \Phi(\Phi_2, \tilde{\delta}_2)$.

Proof. Straight forward □

Definition 3.5. The CC between $(\Phi_1, \tilde{\delta}_1)$ and $(\Phi_2, \tilde{\delta}_2)$ is represented as

$$\alpha_C((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) = \frac{\alpha_{\mathcal{K}}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2))}{\sqrt{\Phi(\Phi_1, \tilde{\delta}_1)}\sqrt{\Phi(\Phi_2, \tilde{\delta}_2)}} \tag{4}$$

Proposition 3.6. *The following CC properties hold good for the CHSS $(\Phi_1, \tilde{\delta}_1)$ and $(\Phi_2, \tilde{\delta}_2)$.*
(i) $0 \leq \alpha_C((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) \leq 1$;
(ii) $\alpha_C((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) = \alpha_C((\Phi_2, \tilde{\delta}_2), (\Phi_1, \tilde{\delta}_1))$;
(iii) If $(\Phi_1, \tilde{\delta}_1) = (\Phi_2, \tilde{\delta}_2)$, then $\alpha_C((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) = 1$.

Proof. (i)We know that, $\alpha_C((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) \geq 0$.
So, let's show that $\alpha_C((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) \leq 1$.

$$\begin{aligned}
&\alpha_{\mathcal{K}}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) \\
&= \Sigma_{k=1}^{m} \Sigma_{i=1}^{n} \Big[(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i)) * (\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i)) + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i)) * (\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i)) \\
&\qquad + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_i)) * (\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i)) \Big]. \\
&= \Sigma_{k=1}^{m} \Big[\Big((\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_1)) * (\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_1)) + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_1)) * (\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_1)) \\
&\qquad + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_1)) * (\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_1)) \Big)
\end{aligned}$$

$$+\Big((\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_2))*(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_2))+(\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_2))*(\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_2))+(\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_2))*(\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_2))\Big)+\dots+\Big((\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_n))*(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_n))+(\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_n))*(\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_n))+(\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_n))*(\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_n))\Big].$$

By using Cauchy-Schwarz inequality

$$\alpha_K((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2))^2 \leq \Sigma_{k=1}^{m}\Big[\Big\{(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_1))^2+(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_2))^2+\dots+(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_n))^2\Big\}+\Big\{(\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_1))^2+(\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_2))^2+\dots+(\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_n))^2\Big\}+\Big\{(\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_1))^2+(\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_2))^2+\dots+(\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_n))^2\Big\}\Big]$$
$$\times\Sigma_{k=1}^{m}\Big[\Big\{(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_1))^2+(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_2))^2+\dots+(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_n))^2\Big\}+\Big\{(\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_1))^2+(\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_2))^2+\dots+(\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_n))^2\Big\}+\Big\{(\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_1))^2+(\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_2))^2+\dots+(\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_n))^2\Big\}\Big].$$

$$\alpha_K((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2))^2 \leq \Sigma_{k=1}^{m}\Sigma_{i=1}^{n}\Big[(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2+(\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2+(\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2\Big]\times\Sigma_{k=1}^{m}\Sigma_{i=1}^{n}\Big[(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2+(\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2+(\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2\Big].$$

$$\Rightarrow \alpha_K((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2))^2 \leq \Phi(\Phi_1,\tilde{\delta}_1)\times\Phi(\Phi_2,\tilde{\delta}_2).$$

$$\Rightarrow \alpha_K((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2)) \leq \sqrt{\Phi(\Phi_1,\tilde{\delta}_1)}\times\sqrt{\Phi(\Phi_2,\tilde{\delta}_2)}.$$

$$\Rightarrow \frac{\alpha_K((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2))}{\sqrt{\Phi(\Phi_1,\tilde{\delta}_1)}\times\sqrt{\Phi(\Phi_2,\tilde{\delta}_2)}} \leq 1.$$

By Definition 3.5, $\alpha_C((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2)) \leq 1$.
Hence, $0 \leq \alpha_C((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2)) \leq 1$. □

Proof. (ii) Straight forward. □

Proof. (iii) $\alpha_C((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2)) = \frac{\alpha_K((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2))}{\sqrt{\Phi(\Phi_1,\tilde{\delta}_1)}\times\sqrt{\Phi(\Phi_2,\tilde{\delta}_2)}}$.
Since, $(\Phi_1,\tilde{\delta}_1) = (\Phi_2,\tilde{\delta}_2)$.

$$\alpha_C((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2)) = \frac{\Sigma_{k=1}^{m}\Sigma_{i=1}^{n}\Big[(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2+(\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2+(\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2\Big]}{\sqrt{\Big\{\Sigma_{k=1}^{m}\Sigma_{i=1}^{n}\Big[(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2+(\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2+(\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2\Big]\Big\}}\times\sqrt{\Big\{\Sigma_{k=1}^{m}\Sigma_{i=1}^{n}\Big[(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2+(\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2+(\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2\Big]\Big\}}}$$

$\Rightarrow \alpha_C((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2)) = 1.$ □

Definition 3.7. Let $(\Phi_1,\tilde{\delta}_1)$ and $(\Phi_2,\tilde{\delta}_2)$ be two CPFHSS. Then, the CC between $(\Phi_1,\tilde{\delta}_1)$ and $(\Phi_2\tilde{\delta}_2)$ is defined as

$$\tilde{\alpha_C}((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2)) = \frac{\alpha_{\mathcal{K}}((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2))}{\max\left\{\Phi(\Phi_1,\tilde{\delta}_1),\Phi(\Phi_2,\tilde{\delta}_2)\right\}}. \tag{5}$$

$$\tilde{\alpha_C}((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2))$$

$$= \frac{\Sigma_{k=1}^{m}\Sigma_{i=1}^{n}\Big[(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))*(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i)) + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))*(\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i)) + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_i))*(\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i))\Big]}{\max\Big\{\Sigma_{k=1}^{m}\Sigma_{i=1}^{n}\Big[(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2\Big], \Sigma_{k=1}^{m}\Sigma_{i=1}^{n}\Big[(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 + (\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 + (\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2\Big]\Big\}}$$

Proposition 3.8. *The following CC properties hold good for the CHSS.*
(i) $0 \le \tilde{\alpha_C}((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2)) \le 1$;
(ii) $\tilde{\alpha_C}((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2)) = \tilde{\alpha_C}((\Phi_2,\tilde{\delta}_2),(\Phi_1,\tilde{\delta}_1))$;
(iii) If $(\Phi_1,\tilde{\delta}_1) = (\Phi_2,\tilde{\delta}_2)$, *then* $\tilde{\alpha_C}((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2)) = 1$.

Proof. (i) We know that, $\tilde{\alpha_C}((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2)) \ge 0$.
Let's show that $\tilde{\alpha_C}((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2)) \le 1$.

$$\begin{aligned}
&\alpha_{\mathcal{K}}((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2))\\
&=\Sigma_{k=1}^{m}\Sigma_{i=1}^{n}\Big[(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))*(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i)) + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))*(\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))\\
&\qquad + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_i))*(\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i))\Big].\\
&=\Sigma_{k=1}^{m}\Big[\Big((\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_1))*(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_1)) + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_1))*(\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_1))\\
&\qquad + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_1))*(\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_1))\Big)\\
&\quad + \Big((\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_2))*(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_2)) + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_2))*(\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_2))\\
&\qquad + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_2))*(\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_2))\Big) + \ldots\\
&\quad + \Big((\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_n))*(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_n)) + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_n))*(\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_n))\\
&\qquad + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_n))*(\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_n))\Big)\Big].
\end{aligned}$$

By Cauchy-Schwarz inequality,
$\alpha_K((\Phi_1,\tilde{\delta}_1),(\Phi_2,\tilde{\delta}_2))$

$$\leq \Big\{ \Sigma_{k=1}^{m} \Big[\Big\{ (\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_1))^2 + (\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_2))^2 + ... + (\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_n))^2 \Big\}$$
$$+ \Big\{ (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_1))^2 + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_2))^2 + ... + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_n))^2 \Big\}$$
$$+ \Big\{ (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_1))^2 + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_2))^2 + ... + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_n))^2 \Big\} \Big] \times$$
$$\Sigma_{k=1}^{m} \Big[\Big\{ (\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_1))^2 + (\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_2))^2 + ... + (\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_n))^2 \Big\}$$
$$+ \Big\{ (\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_1))^2 + (\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_2))^2 + ... + (\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_n))^2 \Big\}$$
$$+ \Big\{ (\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_1))^2 + (\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_2))^2 + ... + (\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_n))^2 \Big\} \Big] \Big\}^{\frac{1}{2}}.$$

$$\alpha_K((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2))$$
$$\leq \Big\{ \Sigma_{k=1}^{m} \Sigma_{i=1}^{n} \Big[(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 \Big]$$
$$\times \Sigma_{k=1}^{m} \Sigma_{i=1}^{n} \Big[(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 + (\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 + (\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 \Big] \Big\}^{\frac{1}{2}}.$$
$$\leq \Big\{ \Big(\max \Big\{ \Sigma_{k=1}^{m} \Sigma_{i=1}^{n} \Big[(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 \Big]$$
$$\times \Sigma_{k=1}^{m} \Sigma_{i=1}^{n} \Big[(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 + \overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 + (\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 \Big\}^{\frac{1}{2}}.$$
$$= \max \Big\{ \Sigma_{k=1}^{m} \Sigma_{i=1}^{n} \Big[(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 \Big]$$
$$\times \Sigma_{k=1}^{m} \Sigma_{i=1}^{n} \Big[(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 + (\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 + (\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 \Big] \Big\}.$$
$$\Rightarrow \alpha_K((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) \leq \max \Big\{ \Phi(\Phi_1, \tilde{\delta}_1) \times \Phi(\Phi_2, \tilde{\delta}_2) \Big\}.$$
$$\Rightarrow \frac{\alpha_K((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2))}{\max \Big\{ \Phi(\Phi_1, \tilde{\delta}_1) \times \Phi(\Phi_2, \tilde{\delta}_2) \Big\}} \leq 1.$$

By Definition 3.7, $\tilde{\alpha_C}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) \leq 1$.
Therefore, $0 \leq \tilde{\alpha_C}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) \leq 1$.
Refer Proposition 3.6 for Proofs of (ii) and (iii). □

3.2. Weighted correlation coefficient of CHSS. Weighted correlation coefficient (WCC) of CHSS are given in this section. Decision makers (DMs) use WCC to enable various weight values for criteria. Consider the weight vectors of alternatives and experts $\mathcal{P} = \{\mathcal{P}_1, \mathcal{P}_2, ..., \mathcal{P}_m\}$ and $\mathcal{W} = \{\mathcal{Q}_1, \mathcal{Q}_2, ..., \mathcal{Q}_n\}$, $\mathcal{P}_k, \mathcal{Q}_i > 0$ and $\Sigma_{k=1}^{m} \mathcal{P}_k = 1, \Sigma_{i=1}^{n} \mathcal{Q}_i = 1$, respectively.

Definition 3.9. Let $(\Phi_1, \tilde{\delta}_1)$ and $(\Phi_2, \tilde{\delta}_2)$ be two CHSS. Then, the WCC between $(\Phi_1, \tilde{\delta}_1)$ and $(\Phi_2, \tilde{\delta}_2)$ is defined as

$$\alpha_{C_{\mathcal{W}}}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) = \frac{\alpha_{\mathcal{K}}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2))}{\sqrt{\Phi(\Phi_1, \tilde{\delta}_1)}\sqrt{\Phi(\Phi_2, \tilde{\delta}_2)}} \qquad (6)$$

$$\alpha_{C_{\mathcal{W}}}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2))$$
$$= \frac{\Sigma_{k=1}^{m} \mathcal{P}_k \left(\Sigma_{i=1}^{n} \mathcal{Q}_i \left[\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i) * \underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i) + \overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i) * \overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i) + \mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_i) * \mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i) \right] \right)}{\sqrt{\left\{ \Sigma_{k=1}^{m} \mathcal{P}_k \left(\Sigma_{i=1}^{n} \mathcal{Q}_i \left[(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 \right] \right) \right\}} \times \sqrt{\left\{ \Sigma_{k=1}^{m} \mathcal{P}_k \left(\Sigma_{i=1}^{n} \mathcal{Q}_i \left[(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 + (\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 + (\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 \right] \right) \right\}}}.$$

Proposition 3.10. *WCC properties hold good for CHSS.*
(i) $0 \leq \alpha_{C_{\mathcal{W}}}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) \leq 1$;
(ii) $\alpha_{C_{\mathcal{W}}}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) = \alpha_{C_{\mathcal{W}}}((\Phi_2, \tilde{\delta}_2), (\Phi_1, \tilde{\delta}_1))$;
(iii) If $(\Phi_1, \tilde{\delta}_1) = (\Phi_2, \tilde{\delta}_2)$, *then* $\alpha_{C_{\mathcal{W}}}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) = 1$.

Proof. Refer Proposition 3.6. □

Definition 3.11. The WCC between $(\Phi_1, \tilde{\delta}_1)$ and $(\Phi_2 \tilde{\delta}_2)$ is defined as

$$\alpha_{\tilde{C}_{\mathcal{W}}}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) = \frac{\alpha_{\mathcal{K}}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2))}{\max \left\{ \Phi(\Phi_1, \tilde{\delta}_1), \Phi(\Phi_2, \tilde{\delta}_2) \right\}}. \tag{7}$$

$$\alpha_{\tilde{C}_{\mathcal{W}}}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2))$$
$$= \frac{\Sigma_{k=1}^{m} \mathcal{P}_k \left(\Sigma_{i=1}^{n} \mathcal{Q}_i \left[(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i)) * (\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i)) + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i)) * (\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i)) + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_i)) * (\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i)) \right] \right)}{\max \left\{ \Sigma_{k=1}^{m} \mathcal{P}_k \left(\Sigma_{i=1}^{n} \mathcal{Q}_i \left[(\underline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 + (\overline{\mathcal{K}}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 + (\mathcal{K}_{\Phi_1(\tilde{\Delta}_k)}(x_i))^2 \right] \right), \Sigma_{k=1}^{m} \mathcal{P}_k \left(\Sigma_{i=1}^{n} \mathcal{Q}_i \left[(\underline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 + (\overline{\mathcal{K}}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 + (\mathcal{K}_{\Phi_2(\tilde{\Delta}_k)}(x_i))^2 \right] \right) \right\}}.$$

Proposition 3.12. *The following WCC properties hold good for CHSS:*
(i) $0 \leq \alpha_{\tilde{C}_{\mathcal{W}}}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) \leq 1$;
(ii) $\alpha_{\tilde{C}_{\mathcal{W}}}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) = \alpha_{\tilde{C}_{\mathcal{W}}}((\Phi_2, \tilde{\delta}_2), (\Phi_1, \tilde{\delta}_1))$;
(iii) If $(\Phi_1, \tilde{\delta}_1) = (\Phi_2, \tilde{\delta}_2)$, *then* $\alpha_{\tilde{C}_{\mathcal{W}}}((\Phi_1, \tilde{\delta}_1), (\Phi_2, \tilde{\delta}_2)) = 1$.

Proof. Similiar to Proposition 3.6. □

4. Aggregation operators of CHSS

Cubic hypersoft weighted average (CHSWA) and cubic hypersoft weighted geometric (CHSWG) operators by using operational laws are presented. Let ρ denote cubic hypersoft numbers (CPFHSNs).

4.1. Operational laws of CHSS.

Definition 4.1. Let $\Phi_{e_{11}} = \left\langle [\underline{\mathcal{K}}_{11}, \overline{\mathcal{K}}_{11}], \mathcal{K}_{11} \right\rangle$ and $\Phi_{e_{12}} = \left\langle [\underline{\mathcal{K}}_{12}, \overline{\mathcal{K}}_{12}], \mathcal{K}_{12} \right\rangle)$ be CHSSs and δ represents an integer. Then,
(i) $\Phi_{e_{11}} \oplus \Phi_{e_{12}} = \left\langle [\underline{\mathcal{K}}_{11} + \underline{\mathcal{K}}_{12} - \underline{\mathcal{K}}_{11}\underline{\mathcal{K}}_{12}, \overline{\mathcal{K}}_{11} + \overline{\mathcal{K}}_{12} - \overline{\mathcal{K}}_{11}\overline{\mathcal{K}}_{12}], (\mathcal{K}_{11} + \mathcal{K}_{12} - \mathcal{K}_{11}\mathcal{K}_{12}) \right\rangle$;
(ii) $\Phi_{e_{11}} \otimes \Phi_{e_{12}} = \left\langle [\underline{\mathcal{K}}_{11}\underline{\mathcal{K}}_{12}, \overline{\mathcal{K}}_{11}\overline{\mathcal{K}}_{12}], (\mathcal{K}_{11}\mathcal{K}_{12}) \right\rangle$;

(iii) $\delta\Phi_{e_{11}} = \langle[(1-(1-\underline{\mathcal{K}}_{11})^{\delta},(1-(1-\overline{\mathcal{K}}_{11})^{\delta}],(1-(1-\mathcal{K}_{11})^{\delta}\rangle$;
(iv) $(\Phi_{e_{11}})^{\delta} = \langle[(\underline{\mathcal{K}}_{11})^{\delta},(\overline{\mathcal{K}}_{11})^{\delta}],(\mathcal{K}_{11})^{\delta}\rangle$.

4.2. Cubic hypersoft weighted average operator.

Definition 4.2. Let $\mathcal{P}_k$ and $\mathcal{Q}_i$ be weight vectors for alternatives and experts, with conditions that $\mathcal{P}_k, \mathcal{Q}_i > 0$ and $\Sigma_{k=1}^{m}\mathcal{P}_k = 1, \Sigma_{i=1}^{n}\mathcal{Q}_i = 1$ and $\Phi_{e_{ik}} = \langle[\underline{\mathcal{K}}_{ik},\overline{\mathcal{K}}_{ik}],\mathcal{K}_{ik}\rangle$ be a CHSN, where $i = \{1,2,...n\}$, $k = \{1,2,...m\}$. Then, $\mathcal{A} : \rho^n \to \rho$, CHSWAO is represented as

$$\mathcal{A}(\Phi_{e_{11}},\Phi_{e_{12}},...,\Phi_{e_{nm}}) = \bigoplus_{k=1}^{m}\mathcal{P}_k\left(\bigoplus_{i=1}^{n}\mathcal{Q}_i\Phi_{e_{ik}}\right).$$

Theorem 4.3. *Let $\Phi_{e_{ik}} = \langle[\underline{\mathcal{K}}_{ik},\overline{\mathcal{K}}_{ik}],\mathcal{K}_{ik}\rangle$ be a CHSN, where $i = \{1,2,...n\}$, $k = \{1,2,...m\}$. Then, the aggregated value of CHSWAO is also a CHSN, which is given by*

$$\mathcal{A}(\Phi_{e_{11}},\Phi_{e_{12}},...,\Phi_{e_{nm}}) = \left\langle\left[1-\Pi_{k=1}^{m}\left(\Pi_{i=1}^{n}\left(1-\underline{\mathcal{K}}_{ik}\right)^{\mathcal{Q}_i}\right)^{\mathcal{P}_k}, 1-\Pi_{k=1}^{m}\left(\Pi_{i=1}^{n}\left(1-\overline{\mathcal{K}}_{ik}\right)^{\mathcal{W}}_{i}\right)^{\mathcal{P}_k}\right],\right.$$
$$\left. 1-\Pi_{k=1}^{m}\left(\Pi_{i=1}^{n}\left(1-\mathcal{K}_{ik}\right)^{\mathcal{W}}_{i}\right)^{\mathcal{P}_k}\right\rangle.$$

Proof. Proof is similar to the Theorem 4.3 in [11] □

4.3. Cubic hypersoft weighted geometric operator.

Definition 4.4. Let $\mathcal{P}_k$ and $\mathcal{Q}_i$ be alternatives and experts weight vectors, with conditions that $\mathcal{P}_k, \mathcal{Q}_i > 0$ and $\Sigma_{k=1}^{m}\mathcal{P}_k = 1, \Sigma_{i=1}^{n}\mathcal{Q}_i = 1$ and $\Phi_{e_{ik}} = (\mathcal{K}_{ik}, \mathcal{E}_{ik})$ be a CHSN, where $i = \{1,2,...n\}$, $k = \{1,2,...m\}$. Then, $\mathcal{G} : \rho^n \to \rho$, CHSWGO is defined as $\mathcal{G}(\Phi_{e_{11}},\Phi_{e_{12}},...,\Phi_{e_{nm}}) = \bigotimes_{k=1}^{m}\left(\bigotimes_{i=1}^{n}\left(\Phi_{e_{ik}}\right)^{\mathcal{Q}_i}\right)^{\mathcal{P}_k}$.

Theorem 4.5. *Let $\Phi_{e_{ik}} = \langle[\underline{\mathcal{K}}_{ik},\overline{\mathcal{K}}_{ik}],\mathcal{K}_{ik}\rangle$ be a CHSN, where $i = \{1,2,...n\}$, $k = \{1,2,...m\}$. Then, the aggregated value of CHSWGO is also a CHSN, which is given by*

$$\mathcal{G}(\Phi_{e_{11}},\Phi_{e_{12}},...,\Phi_{e_{nm}}) = \left\langle\left[\Pi_{k=1}^{m}\left(\Pi_{i=1}^{n}\left(\underline{\mathcal{K}}_{ik}\right)^{\mathcal{Q}_i}\right)^{\mathcal{P}_k}, \Pi_{k=1}^{m}\left(\Pi_{i=1}^{n}\left(\overline{\mathcal{K}}_{ik}\right)^{\mathcal{Q}_i}\right)^{\mathcal{P}_k}\right], \Pi_{k=1}^{m}\left(\Pi_{i=1}^{n}\left(\mathcal{K}_{ik}\right)^{\mathcal{Q}_i}\right)^{\mathcal{P}_k}\right\rangle.$$

Proof. Proof is similar to the Theorem 4.3 in [11] □

5. MCDM problem by using TOPSIS method

Based on the minimum and maximum distances from the cubic positive ideal solution (CPIS) and cubic negative ideal solution, the TOPSIS technique assists in determining the optimum alternative (CNIS). Additionally, the TOPSIS technique accurately estimates the proximity coefficients when paired with CC rather than DMs. Finally, we present a case study to demonstrate CHSS TOPSIS method.

5.1. Algorithm to solve MCDM problems with CHSS data. Let $\mathcal{W} = \{\mathcal{W}^1, \mathcal{W}^2, ..., \mathcal{W}^x\}$ represent a set of workers. Let $\mathcal{X} = \{p_1, p_2, ..., p_n\}$ denote psychiatrists responsible to handle psychiatrist session with the workers. Let the weights be given as $\mathcal{Q}_i = (\mathcal{Q}_1, \mathcal{Q}_2, ..., \mathcal{Q}_n)$, $\mathcal{Q}_i > 0$ and $\Sigma_{i=1}^n \mathcal{Q}_i = 1$. Let $\tilde{\delta} = \left\{\tilde{\Delta}_1, \tilde{\Delta}_2, ..., \tilde{\Delta}_m\right\}$ be multi-valued sub-attributes with weights $\mathcal{P}_k = (\mathcal{P}_1, \mathcal{P}_2, ..., \mathcal{P}_m)$, $\mathcal{P}_k > 0$ and $\Sigma_{k=1}^m \mathcal{P}_k = 1$. The assessment of workers $\mathcal{A}^t$, $(t = 1, 2, ..., x)$ performed by the psychiatrists p_i, $(i = 1, 2, ..., n)$ based on the multi-valued sub-attributes $\tilde{\Delta}_k$, $(k = 1, 2, ..., m)$ are given in CHSS form, represented as $\mu_{ik}^t = \langle [\underline{\mathcal{K}}_{ik}^t, \overline{\mathcal{K}}_{ik}^t], \mathcal{K}_{ik}^t \rangle$.

Step 1. Create multi-valued sub-attribute in CHSS format:

$$[\mathcal{W}^t, \tilde{\delta}]_{n\times m} = [\mathcal{W}^t]_{n\times m} = \begin{matrix} & \tilde{\Delta}_1 & \tilde{\Delta}_2 & \cdots & \tilde{\Delta}_m \\ p_1 \\ p_2 \\ \vdots \\ p_n \end{matrix} \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1m} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{n1} & \mu_{n2} & \cdots & \mu_{nm} \end{bmatrix},$$

such that $[\mathcal{W}^t]_{n\times m} = \mu_{ik}^t = \langle [\underline{\mathcal{K}}_{ik}^t, \overline{\mathcal{K}}_{ik}^t], \mathcal{K}_{ik}^t \rangle, i = 1, 2, ..., n$ and $k = 1, 2, ..., m$.

Step 2. Compute the weighted decision matrix for each multi-valued sub-attribute, $[\tilde{W}_{ik}^t]_{n\times m}$

$$= \Bigg\langle \Bigg[1 - \Pi_{k=1}^m \Bigg(\Pi_{i=1}^n \Bigg(1 - \underline{\mathcal{P}}_{ik} \Bigg)^{\mathcal{Q}_i} \Bigg)^{\mathcal{P}_k}, 1 - \Pi_{k=1}^m \Bigg(\Pi_{i=1}^n \Bigg(1 - \overline{\mathcal{P}}_{ik} \Bigg)^{\mathcal{Q}_i} \Bigg)^{\mathcal{P}_k} \Bigg], 1 - \Pi_{k=1}^m \Bigg(\Pi_{i=1}^n \Bigg(1 - \mathcal{P}_{ik} \Bigg)^{\mathcal{Q}_i} \Bigg)^{\mathcal{P}_k} \Bigg\rangle$$

$$= \Bigg\langle \Bigg[\tilde{\underline{\mathcal{K}}}_{ik}, \tilde{\overline{\mathcal{K}}}_{ik} \Bigg], \tilde{\mathcal{K}}_{ik} \Bigg\rangle.$$

Step 3. Evaluate the CPIS and CNIS for weighted CHSS according to the instructions below:

$$\tilde{\mathcal{W}}^+ = \Bigg\langle \Bigg[\tilde{\underline{\mathcal{K}}}^+, \tilde{\overline{\mathcal{K}}}^+ \Bigg], \tilde{\mathcal{K}}^+ \Bigg\rangle_{n\times m} = \Bigg\langle \Bigg[\underline{\mathcal{K}}^{(\tilde{\vee}_{ij})}, \overline{\mathcal{K}}^{(\tilde{\vee}_{ij})} \Bigg], \mathcal{K}^{(\tilde{\vee}_{ij})} \Bigg\rangle,$$

$$\tilde{\mathcal{W}}^- = \Bigg\langle \Bigg[\tilde{\underline{\mathcal{K}}}^-, \tilde{\overline{\mathcal{K}}}^- \Bigg], \tilde{\mathcal{K}}^- \Bigg\rangle_{n\times m} = \Bigg\langle \Bigg[\underline{\mathcal{K}}^{(\tilde{\wedge}_{ij})}, \overline{\mathcal{K}}^{(\tilde{\wedge}_{ij})} \Bigg], \mathcal{K}^{(\tilde{\wedge}_{ij})} \Bigg\rangle,$$

where $\vee_{ij} = \arg\max_t \left\{\varphi_{ij}^t\right\}$ and $\wedge_{ij} = \arg\min_t \left\{\varphi_{ij}^t\right\}$.

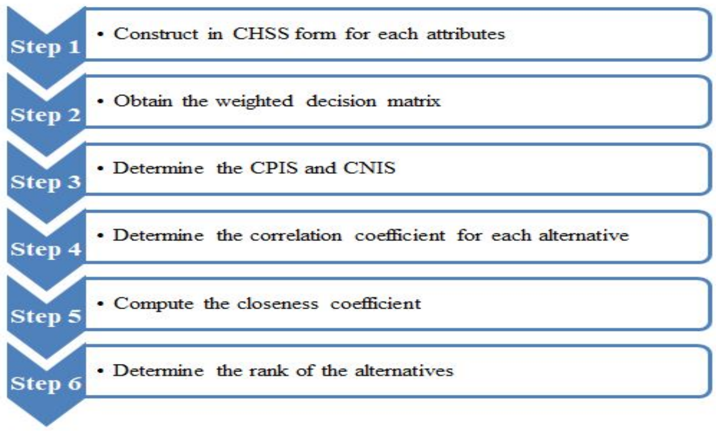
Step 4. Compute the CC for every possibility using CPIS and CNIS.

$$\chi^t = \alpha_C(\tilde{W}^t, \tilde{W}^+) = \frac{\alpha_{\mathcal{K}}(\tilde{W}^t, \tilde{W}^+)}{\sqrt{\Phi(\tilde{W}^t) * \sqrt{(\Phi\tilde{W}^+)}}} \text{ and}$$

$$\Delta^t = \alpha_C(\tilde{W}^t, \tilde{W}^-) = \frac{\alpha_{\mathcal{K}}(\tilde{W}^t, \tilde{W}^-)}{\sqrt{\Phi(\tilde{W}^t) * \sqrt{\Phi(\tilde{W}^-)}}}$$

Step 5. Compute the similarity coefficient of the cubic ideal solution using the

FIGURE 1. Flowchart of the proposed method



formula below:

$$\epsilon^t = \frac{1 - \Delta^t}{2 - \chi^t - \Delta^t}$$

Step 6. Compare the ϵ^t scores to the norms provided in Table 1 and assess the amount of stress coping for each alternative $\mathcal{A}^t$, $(t = 1, 2, ..., x)$. To manage stress, an individual with low stress coping abilities may need the assistance of a psychiatrist.

TABLE 1. Stress-coping norms.

Scores	Level
0-0.55	Low
0.56-0.70	Average
0.71-0.90	Good
0.91-1.00	Excellent

The graphical representation of the proposed method is given in Figure 1:

5.2. Application to analyze the stress-coping skills of workers using TOPSIS method. Let $\mathcal{W} = \{\mathcal{W}^1, \mathcal{W}^2, \mathcal{W}^3, \mathcal{W}^4\}$ be a set of workers. Let $\mathcal{X} = \{p_1, p_2, p_3, p_4\}$ represent psychiatrists for conducting sessions and the weights be $\mathcal{Q}_i = (0.16, 0.28, 0.34, 0.22)$ and $\mathcal{P}_k = (0.26, 0.32, 0.14, 0.28)$. Let δ_1, δ_2, δ_3 and δ_4 denote attribute sets. The respective sub-attributes are given below:

$\delta_1 =$ first stage $= \{\Delta_{11} =$ reactivity to stress$\}$,
$\delta_2 =$ second stage $= \{\Delta_{21} =$ ability to relax, $\Delta_{22} =$ self-reliance$\}$,
$\delta_3 =$ third stage $= \{\Delta_{31} =$ proactive attitude, $\Delta_{32} =$ adaptability and flexibility$\}$ and

δ_4 = fourth stage = $\{\varDelta_{41}$ = ability to access situations$\}$. Then $\tilde{\delta} = \delta_1 \times \delta_2 \times \delta_3 \times \delta_4$ is the distinct attribute set given by

$$\tilde{\delta} = \delta_1 \times \delta_2 \times \delta_3 \times \delta_4 = \{\varDelta_{11}\} \times \{\varDelta_{21}, \varDelta_{22}\} \times \{\varDelta_{31}, \varDelta_{32}\} \times \{\varDelta_{41}\}.$$
$$= \Big\{(\varDelta_{11}, \varDelta_{21}, \varDelta_{31}, \varDelta_{41}), (\varDelta_{11}, \varDelta_{21}, \varDelta_{32}, \varDelta_{41}), (\varDelta_{11}, \varDelta_{22}, \varDelta_{31}, \varDelta_{41}), (\varDelta_{11}, \varDelta_{22}, \varDelta_{32}, \varDelta_{41})\Big\}.$$
$$= \Big\{\tilde{\varDelta}_1, \tilde{\varDelta}_2, \tilde{\varDelta}_3, \tilde{\varDelta}_4\Big\}$$

We provide a workaround for a worker who may require the help of a psychiatrist to handle the stress effectively.

Step 1. Construct $\mathcal{W}^1$, $\mathcal{W}^2$, $\mathcal{W}^3$ and $\mathcal{W}^4$ for each multi-valued sub-attribute in CHSS format.

TABLE 2. $\mathcal{W}^1$ values expressed in CHSS format.

$\mathcal{W}^1$	$\tilde{\varDelta}_1$	$\tilde{\varDelta}_2$
p_1	$\langle[0.64, 0.66], 0.49\rangle$	$\langle[0.91, 0.92], 0.81\rangle$
p_2	$\langle[0.72, 0.75], 0.55\rangle$	$\langle[0.52, 0.56], 0.25\rangle$
p_3	$\langle[0.41, 0.45], 0.72\rangle$	$\langle[0.77, 0.79], 0.81\rangle$
p_4	$\langle[0.92, 0.93], 0.56\rangle$	$\langle[0.77, 0.79], 0.71\rangle$

$\mathcal{W}^1$	$\tilde{\varDelta}_3$	$\tilde{\varDelta}_4$
p_1	$\langle[0.36, 0.42], 0.57\rangle$	$\langle[0.48, 0.59], 0.69\rangle$
p_2	$\langle[0.73, 0.75], 0.55\rangle$	$\langle[0.81, 0.83], 0.23\rangle$
p_3	$\langle[0.42, 0.46], 0.56\rangle$	$\langle[0.34, 0.37], 0.41\rangle$
p_4	$\langle[0.58, 0.62], 0.49\rangle$	$\langle[0.54, 0.57], 0.64\rangle$

TABLE 3. $\mathcal{W}^2$ values expressed in CHSS format.

$\mathcal{W}^2$	$\tilde{\varDelta}_1$	$\tilde{\varDelta}_2$
p_1	$\langle[0.35, 0.41], 0.55\rangle$	$\langle[0.56, 0.61], 0.71\rangle$
p_2	$\langle[0.21, 0.25], 0.32\rangle$	$\langle[0.51, 0.53], 0.24\rangle$
p_3	$\langle[0.43, 0.48], 0.45\rangle$	$\langle[0.35, 0.39], 0.43\rangle$
p_4	$\langle[0.57, 0.64], 0.65\rangle$	$\langle[0.55, 0.59], 0.66\rangle$

$\mathcal{W}^2$	$\tilde{\varDelta}_3$	$\tilde{\varDelta}_4$
p_1	$\langle[0.84, 0.91], 0.83\rangle$	$\langle[0.91, 0.94], 0.52\rangle$
p_2	$\langle[0.48, 0.55], 0.24\rangle$	$\langle[0.71, 0.76], 0.58\rangle$
p_3	$\langle[0.72, 0.78], 0.36\rangle$	$\langle[0.21, 0.24], 0.74\rangle$
p_4	$\langle[0.71, 0.75], 0.58\rangle$	$\langle[0.74, 0.75], 0.58\rangle$

TABLE 4. $\mathcal{W}^3$ values expressed in CHSS format.

$\mathcal{W}^3$	$\tilde{\varDelta}_1$	$\tilde{\varDelta}_2$
p_1	$\langle[0.46, 0.61], 0.71\rangle$	$\langle[0.61, 0.62], 0.47\rangle$
p_2	$\langle[0.14, 0.17], 0.22\rangle$	$\langle[0.16, 0.17], 0.57\rangle$
p_3	$\langle[0.15, 0.21], 0.43\rangle$	$\langle[0.21, 0.27], 0.77\rangle$
p_4	$\langle[0.52, 0.59], 0.66\rangle$	$\langle[0.34, 0.48], 0.57\rangle$

$\mathcal{W}^3$	$\tilde{\Delta}_3$	$\tilde{\Delta}_4$
p_1	$\langle[0.39, 0.44], 0.54\rangle$	$\langle[0.51, 0.61], 0.68\rangle$
p_2	$\langle[0.24, 0.29], 0.34\rangle$	$\langle[0.16, 0.19], 0.29\rangle$
p_3	$\langle[0.46, 0.49], 0.44\rangle$	$\langle[0.38, 0.39], 0.44\rangle$
p_4	$\langle[0.62, 0.63], 0.64\rangle$	$\langle[0.57, 0.59], 0.67\rangle$

TABLE 5. $\mathcal{W}^4$ values expressed in CHSS format.

$\mathcal{W}^4$	$\tilde{\Delta}_1$	$\tilde{\Delta}_2$
p_1	$\langle[0.89, 0.92], 0.79\rangle$	$\langle[0.84, 0.92], 0.88\rangle$
p_2	$\langle[0.51, 0.56], 0.24\rangle$	$\langle[0.48, 0.56], 0.26\rangle$
p_3	$\langle[0.75, 0.79], 0.33\rangle$	$\langle[0.72, 0.77], 0.44\rangle$
p_4	$\langle[0.45, 0.46], 0.56\rangle$	$\langle[0.81, 0.82], 0.61\rangle$

$\mathcal{W}^4$	$\tilde{\Delta}_3$	$\tilde{\Delta}_4$
p_1	$\langle[0.56, 0.66], 0.77\rangle$	$\langle[0.31, 0.44], 0.58\rangle$
p_2	$\langle[0.89, 0.94], 0.25\rangle$	$\langle[0.35, 0.39], 0.38\rangle$
p_3	$\langle[0.35, 0.49], 0.44\rangle$	$\langle[0.41, 0.47], 0.48\rangle$
p_4	$\langle[0.55, 0.69], 0.67\rangle$	$\langle[0.51, 0.65], 0.68\rangle$

Step 2. Obtain $\tilde{\mathcal{W}}^1$, $\tilde{\mathcal{W}}^2$, $\tilde{\mathcal{W}}^3$ and $\tilde{\mathcal{W}}^4$, the weighted matrices for each multi-valued sub-attributes.

TABLE 6. Weighted value representation in CHSS for $\tilde{\mathcal{W}}^1$.

$\tilde{\mathcal{W}}^1$	$\tilde{\Delta}_1$	$\tilde{\Delta}_2$
p_1	$\langle[0.0416, 0.0439], 0.0276\rangle$	$\langle[0.1160, 0.1213], 0.0815\rangle$
p_2	$\langle[0.0885, 0.0960], 0.0565\rangle$	$\langle[0.0636, 0.0709], 0.0254\rangle$
p_3	$\langle[0.0456, 0.0515], 0.1064\rangle$	$\langle[0.1478, 0.1562], 0.1653\rangle$
p_4	$\langle[0.1345, 0.1411], 0.0459\rangle$	$\langle[0.0983, 0.1040], 0.0835\rangle$

$\tilde{\mathcal{W}}^1$	$\tilde{\Delta}_3$	$\tilde{\Delta}_4$
p_1	$\langle[0.0099, 0.0121], 0.0187\rangle$	$\langle[0.0289, 0.0392], 0.0511\rangle$
p_2	$\langle[0.0500, 0.0529], 0.0308\rangle$	$\langle[0.1221, 0.1297], 0.0203\rangle$
p_3	$\langle[0.0256, 0.0289], 0.0383\rangle$	$\langle[0.0388, 0.0430], 0.0490\rangle$
p_4	$\langle[0.0264, 0.0294], 0.0205\rangle$	$\langle[0.0467, 0.0507], 0.0610\rangle$

TABLE 7. Weighted value representation in CHSS for $\tilde{\mathcal{W}}^2$.

$\tilde{\mathcal{W}}^2$	$\tilde{\Delta}_1$	$\tilde{\Delta}_2$
p_1	$\langle[0.01780.0217], 0.0327\rangle$	$\langle[0.0412, 0.0471], 0.0614\rangle$
p_2	$\langle[0.01700.0207], 0.0277\rangle$	$\langle[0.0619, 0.0654], 0.0243\rangle$
p_3	$\langle[0.04850.0562], 0.0515\rangle$	$\langle[0.0458, 0.0524], 0.0593\rangle$
p_4	$\langle[0.04710.0568], 0.0583\rangle$	$\langle[0.0547, 0.0608], 0.0731\rangle$

$\tilde{\mathcal{W}}^2$	$\tilde{\Delta}_3$	$\tilde{\Delta}_4$
p_1	$\langle[0.0402, 0.0525], 0.0389\rangle$	$\langle[0.1023, 0.1184], 0.0323\rangle$
p_2	$\langle[0.0253, 0.0308], 0.0107\rangle$	$\langle[0.0925, 0.1059], 0.0658\rangle$
p_3	$\langle[0.0588, 0.0695], 0.0210\rangle$	$\langle[0.0222, 0.0258], 0.1204\rangle$
p_4	$\langle[0.0374, 0.0418], 0.0264\rangle$	$\langle[0.0796, 0.0819], 0.0520\rangle$

Table 8. Weighted value representation in CHSS format for $\tilde{\mathcal{W}}^3$.

$\tilde{\mathcal{W}}^3$	$\tilde{\Delta}_1$	$\tilde{\Delta}_2$
p_1	$\langle[0.0253, 0.0384], 0.0502\rangle$	$\langle[0.0471, 0.0483], 0.0320\rangle$
p_2	$\langle[0.0109, 0.0135], 0.0179\rangle$	$\langle[0.0155, 0.0166], 0.0728\rangle$
p_3	$\langle[0.0143, 0.0206], 0.0485\rangle$	$\langle[0.0253, 0.0337], 0.1478\rangle$
p_4	$\langle[0.0411, 0.0497], 0.0598\rangle$	$\langle[0.0288, 0.0450], 0.0577\rangle$

$\tilde{\mathcal{W}}^3$	$\tilde{\Delta}_3$	$\tilde{\Delta}_4$
p_1	$\langle[0.0110, 0.0129], 0.0172\rangle$	$\langle[0.0315, 0.0413], 0.0498\rangle$
p_2	$\langle[0.0107, 0.0133], 0.0162\rangle$	$\langle[0.0136, 0.0164], 0.0265\rangle$
p_3	$\langle[0.0289, 0.0315], 0.0272\rangle$	$\langle[0.0445, 0.0460], 0.0537\rangle$
p_4	$\langle[0.0294, 0.0302], 0.0310\rangle$	$\langle[0.0507, 0.0534], 0.0660\rangle$

Table 9. Weighted value representation in CHSS format for $\tilde{\mathcal{W}}^4$.

$\tilde{\mathcal{W}}^4$	$\tilde{\Delta}_1$	$\tilde{\Delta}_2$
p_1	$\langle[0.0877, 0.0997], 0.0629\rangle$	$\langle[0.0896, 0.1213], 0.1029\rangle$
p_2	$\langle[0.0506, 0.0580], 0.0198\rangle$	$\langle[0.0569, 0.0709], 0.0266\rangle$
p_3	$\langle[0.1153, 0.1289], 0.0348\rangle$	$\langle[0.1293, 0.1478], 0.0611\rangle$
p_4	$\langle[0.0336, 0.0346], 0.0459\rangle$	$\langle[0.1103, 0.1137], 0.0641\rangle$

$\tilde{\mathcal{W}}^4$	$\tilde{\Delta}_3$	$\tilde{\Delta}_4$
p_1	$\langle[0.0182, 0.0239], 0.0324\rangle$	$\langle[0.0165, 0.0256], 0.0381\rangle$
p_2	$\langle[0.0829, 0.1044], 0.0112\rangle$	$\langle[0.0332, 0.0380], 0.0368\rangle$
p_3	$\langle[0.0203, 0.0315], 0.0272\rangle$	$\langle[0.0490, 0.0587], 0.0604\rangle$
p_4	$\langle[0.0243, 0.0354], 0.0336\rangle$	$\langle[0.0430, 0.0626], 0.0678\rangle$

Step 3. Evaluate the CPIS and CNIS from $\tilde{\mathcal{W}}^1$, $\tilde{\mathcal{W}}^2$, $\tilde{\mathcal{W}}^3$ and $\tilde{\mathcal{W}}^4$.

Table 10. CPIS ($\tilde{\mathcal{W}}^+$) is represented by the weighted matrices.

$\tilde{\mathcal{W}}^+$	$\tilde{\Delta}_1$	$\tilde{\Delta}_2$
p_1	$\langle[0.0877, 0.0997], 0.0629\rangle$	$\langle[0.1160, 0.1213], 0.1029\rangle$
p_2	$\langle[0.0885, 0.0960], 0.0565\rangle$	$\langle[0.0636, 0.0709], 0.0728\rangle$
p_3	$\langle[0.1153, 0.1289], 0.1064\rangle$	$\langle[0.1478, 0.1562], 0.1653\rangle$
p_4	$\langle[0.1345, 0.1411], 0.0598\rangle$	$\langle[0.1103, 0.1137], 0.0835\rangle$

$\tilde{\mathcal{W}}^+$	$\tilde{\Delta}_3$	$\tilde{\Delta}_4$
p_1	$\langle[0.0402, 0.0525], 0.0389\rangle$	$\langle[0.1023, 0.1184], 0.0511\rangle$
p_2	$\langle[0.0829, 0.1044], 0.0308\rangle$	$\langle[0.1221, 0.1297], 0.0658\rangle$
p_3	$\langle[0.0588, 0.0695], 0.0383\rangle$	$\langle[0.0490, 0.0587], 0.1204\rangle$
p_4	$\langle[0.0374, 0.0418], 0.0336\rangle$	$\langle[0.0796, 0.0819], 0.0678\rangle$

Table 11. CPIS ($\tilde{\mathcal{W}}^{-}$) is represented by the weighted matrices.

$\tilde{\mathcal{W}}^{-}$	$\tilde{\Delta}_1$	$\tilde{\Delta}_2$
p_1	$\langle[0.0178, 0.0217], 0.0629\rangle$	$\langle[0.0412, 0.0471], 0.1029\rangle$
p_2	$\langle[0.0109, 0.0135], 0.0327\rangle$	$\langle[0.0155, 0.0166], 0.0320\rangle$
p_3	$\langle[0.0143, 0.0206], 0.0179\rangle$	$\langle[0.0253, 0.0337], 0.0243\rangle$
p_4	$\langle[0.0336, 0.0346], 0.0348\rangle$	$\langle[0.0288, 0.0450], 0.0593\rangle$

$\tilde{\mathcal{W}}^{-}$	$\tilde{\Delta}_3$	$\tilde{\Delta}_4$
p_1	$\langle[0.0099, 0.0121], 0.0389\rangle$	$\langle[0.0165, 0.0256], 0.0511\rangle$
p_2	$\langle[0.0107, 0.0133], 0.0172\rangle$	$\langle[0.0136, 0.0164], 0.0323\rangle$
p_3	$\langle[0.0203, 0.0289], 0.0107\rangle$	$\langle[0.0222, 0.0258], 0.0265\rangle$
p_4	$\langle[0.0243, 0.0294], 0.0210\rangle$	$\langle[0.0430, 0.0507], 0.0537\rangle$

Step 4. By using the values of CPIS and CNIS, compute the CC for the alternatives

$$\chi^1 = 0.9442, \chi^2 = 0.8953, \chi^3 = 0.8352 \text{ and } \chi^4 = 0.9045.$$

$$\lambda^1 = 0.7562, \lambda^2 = 0.7957, \lambda^3 = 0.8242 \text{ and } \lambda^4 = 0.7963.$$

Step 5. Derive the proximity coefficient of cubic ideal solution as below.

$$\epsilon^1 = 0.8139, \epsilon^2 = 0.6612, \epsilon^3 = 0.5162 \text{ and } \epsilon^4 = 0.6808.$$

Step 6. Compare the scores with the norms given in Table 1 and determine the stress-coping levels.

$$\epsilon^1 - \text{Good}, \epsilon^2 - \text{Average}, \epsilon^3 - \text{Low and } \epsilon^4 - \text{Average}.$$

$$\Rightarrow \mathcal{W}^1 - \text{Good}, \mathcal{W}^2 - \text{Average}, \mathcal{W}^3 - \text{Low and } \mathcal{W}^4 - \text{Average}$$

Hence, $\mathcal{W}^3$ may require the help of a psychiatrist to manage the stress effectively.

6. Conclusions

We have presented the notion of CPFHSS and established a few of its features in this study. We have presented aggregation operators and a TOPSIS-based application for analyzing the stress-coping abilities of telecommuters. In the suggested TOPSIS technique, we have utilized CC instead of DMs to examine the proximity coefficients. To demonstrate the validity of the proposed model, we conducted a comparison analysis by replacing CC with existing DMs in the proposed method. In the future, we can extend this structure to several aggregate operators, combine CHSS with N soft set, and in various decision-making problems.

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A. Bobin received his Ph.D., from Annamalai University. Currently, he is working as an assistant professor at IFET College of Engineering. His research interests are fuzzy and its hybrid sets.

Department of Mathematics, IFET College of Engineering, (Autonomous Institution), Villupuram, Tamilnadu, India.
e-mail: `bobinalbert@gmail.com`

P. Thangaraja received his Ph.D., from Annamalai University. Currently, he is working as an assistant professor at Mahendra Engineering College. His research interest is fuzzy topology.

Department of Mathematics, Mahendra Engineering College, (Autonomous Institution), Namakkal Tamilnadu, India.
e-mail: `thangarajap1991@gmail.com`

H. Prathab pursuing his Ph.D., at Pachaiyappa's college, University of Madras. Currently, he is working as an assistant professor at Saveetha Engineering College. His research interests are fuzzy and graph theory.

Department of Mathematics, Saveetha Engineering College, (Autonomous Institution), Chennai Tamilnadu, India.
e-mail: prathab@saveetha.ac.in

S. Thayalan received his Ph.D., from Annamalai University. Currently, he is working as a guest lecturer at Government Arts and Science College. His research interest is complex fuzzy set.

Department of Mathematics, Government Arts and Science College, Komarapalayam, Tamilnadu, India.
e-mail: ssthayalan@gmail.com