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RESEARCH ARTICLE

Extension of Interaction Aggregation Operators for the Analysis of Cryptocurrency Market Under q-Rung Orthopair Fuzzy Hypersoft Set

RANA MUHAMMAD ZULQARNAIN[1], IMRAN SIDDIQUE[2], SAYED M. ELDIN[3], AND SHAHID HUSSAIN GURMANI[1]

[1]College of Mathematics and Computer Science, Zhejiang Normal University, Jinhua, Zhejiang 321004, China
[2]Department of Mathematics, University of Management and Technology, Lahore 54770, Pakistan
[3]Center of Research, Faculty of Engineering, Future University in Egypt, New Cairo 11835, Egypt

Corresponding authors: Rana Muhammad Zulqarnain (ranazulqarnain7777@gmail.com) and Sayed M. Eldin (sayed.eldin22@fue.edu.eg)

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ABSTRACT One of the substantial innovations achieved through digitalization is cryptocurrencies, also known as simulated or digital currencies, which have been deliberated in the modern era as a new platform particularly suitable for financiers. Several cryptocurrencies, such as Bitcoin, Ethereum, Binance Coin, and Tether, do not trust a dominant expert. The classification and conduction of insecurity and the confirmation of digital currencies complicate decision-making. q-rung orthopair fuzzy hypersoft sets are an emerging arena of research intended to report the confidential restrictions of q-rung orthopair fuzzy soft sets on multiparameter indefinite functions. Such a function maps a tuple of sub-parameters to a power set of the universe. It emphasizes allocating attributes to their corresponding sub-attribute values in disjoint sets. These structures sort it an innovative systematic tool for addressing the obstacles of hesitancy. The q-rung orthopair fuzzy hypersoft set (q-ROFHSS) expertly compacts with tentative and ambagious facts equated to the existing q- rung orthopair fuzzy soft set and Pythagorean fuzzy hypersoft set (PFHSS). It is the most compelling mode for enlarging imprecise data in decision-making (DM). This investigation's ultimate impartiality is presenting interactional algebraic operational laws for q-ROFHSS. Furthermore, some interaction aggregation operators (AOs) have been anticipated via our proposed operational laws, such as q-rung orthopair fuzzy hypersoft interactive weighted average (q-ROFHSIWA) and q-rung orthopair fuzzy hypersoft interactive weighted geometric (q-ROFHSIWG) operators with their essential properties. In reality, a mathematical illustration of DM obstacles is pondered to substantiate the proven technique's dominance. Based on the projected interaction AOs, robust multi-criteria group decision-making (MCGDM) design has been offered, which carries the most practical consequences associated with predominant MCGDM methods. The significance spectacle is that the intentional methodology is more operative and steady in bearing weird facts based on q-ROFHSS.

INDEX TERMS q-rung orthopair fuzzy hypersoft set, q-rung orthopair fuzzy hypersoft interactive weighted average operator, q-rung orthopair fuzzy hypersoft interactive weighted geometric operator, MCGDM, cryptocurrency.

I. INTRODUCTION

The idea of cryptocurrencies has been broadly used in this era. Cryptocurrencies (CRCs) are also recognized as digital currencies, and several companies use crypto to authenticate dealings. Nakamoto [1] engendered CRCs peer-era automatic money transactions in 2009. CRCs are digital currencies used in the methodology and blockchain built on expertise [2]. With rapid fluctuations in the CRCs market, purchasing or retailing CRCs on online marketplaces is a daunting task. To solve this problem, we must analyze CRCs. The market's problems in decision-making the best option in a dataset

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is called an assessment. To mark the veracious judgment, numerous investigators gave one numeral of ideas. Decisions are made at the beginning of the era based on exact mathematical data, but this leads to inadequate and low conclusions pertinent to existing situations. Urquhart [3] trade activity and noteworthy capriciousness attracted the public's devotion to Bitcoin (BTC). However, he noted that no expectations of volatility could produce meaningful results by searching online. David et al. [4] operated on the opinion cycle among social gestures and EV signals in the BTC budget and a medium used to classify both suicides' optimistic opinion loops. Ramadani and Devianto [5] used the BTC price prediction model developed by the fuzzy time series Markov chain and Chen's logical technique. Carbonic acid time series can model different sorts of time series facts patterns because this technique is not affected by conventional assumptions.

MCGDM has been reflected as the most delicate technique for ruling the acceptable alternate since all credible adoptions, consequent criteria, or structures. Maximum assessments are made when real-life intents and boundaries are usually undefined or unreliable. Zadeh [6] originated the idea of a fuzzy set (FS) to astound these imprecise and uncertain details. It is essential for dealing with insignificant and hesitant decision-making (DM) problems. The current FS cannot deal with situations where experts regularly consider membership (MD) during the DM process. Experts primarily consider membership (MD) and non-membership (NMD) in DM plans where FS cannot specialize. Atanassov [7] overcame these confines and established the intuitionistic fuzzy set (IFS). Wang and Liu [8] presented numerous operations such as Einstein products, Einstein sums, and others, and AOs for IFS. Xu [9] protracted the concept of IFS and identified how to compare two intuitionistic fuzzy values through score and accuracy functions. Garg [10] enhanced the IFS's cosine similarity measures (SMs) and intended to adopt DM technology. Lin et al. [11] proposed an innovative multi-criteria decision-making (MCDM) method for IFS. De et al. [12] define basic IFS operations, such as concentration, normalization, and dilation. IFS can still not distinguish the inconsistent and ambiguous information as it imagines a linear asymmetry between MD and NMD. If the group of experts chooses the MD and NMD so that their sum exceeds 1, such as MD = 0.6 and NMD = 0.7, correspondingly, then the IFS, as stated formerly, cannot contract with it since $0.6 + 0.7 \geq 1$. Yager [13] presented the Pythagorean fuzzy set (PFS) to tenacity the scantiness stated above by revising the basic state $\mathfrak{T} + \mathfrak{J} \leq 1$ to $\mathfrak{T}^2 + \mathfrak{J}^2 \leq 1$ and proving some significance associated with score and accuracy function. Xiao and Ding [14] planned an advanced divergence measure for PFS, considering the Jensen-Shannon divergence. Thao and Smarandache [15] proposed entropy measures for PFS and demonstrated the MCDM method via the settled measures. Zhang et al. [16] became familiar with some of the new SMs used in PFS and confirmed that the fixed SMs is capable compared to existing SMs. Rahman et al. [17] intended Einstein weighted geometric operator on PFS and demonstrated the multi-attribute group decision-making (MAGDM) method through their scheduled operator. Zhang and Xu [18] lengthened the TOPSIS approach to deal with MCDM obstacles for PFS. Wei and Lu [19] provided the fuzzy power AO and basic properties of the Pythagorean fuzzy environment. Wang and Li [20] revealed the interaction law between the Pythagorean fuzzy number (PFN) and developed power Bonferroni mean AOs. Zhang [21] has proposed an advanced DM method based on SMs to resolve the MCGDM difficulties under the PFS setting. Yager [22] developed a q-rung orthopair fuzzy set (q-ROFS) with several elementary operations and their properties.

All of the above techniques have a wide range of applications, but the parametric attraction has some limitations due to the incompetence of these theories. Molodtsov [23] offered the soft set (SS) theory and explained specific rudimentary operations to deal with misunderstandings and ambiguities. Maji et al. [24] prolonged the SS theory and conventional several basic and binary operations. Cagman and Enginoglu [25] familiarized fuzzy parametric SS with obligatory operations. They also expanded the DM approach based on their anticipated theory and detached the uncertain complexities. Ali et al. [26] prolonged the perception of SS and described numerous basic operations of SS and their basic properties. Maji et al. [27] demonstrated a fuzzy soft set (FSS) by ideal belongings by captivating the two popular thoughts, FS and SS. Roy and Maji [28] identified a new DM method for FSS called imperfect multi-observational data identification. Cagman et al. [29] demarcated the AOs of FSS and proposed the DM technique via his anticipated operators. Feng et al. [30] presented amendable models in FSS and provided weighted FSS applications in DM. Maji et al. [31] settled the intuitionistic fuzzy soft set (IFSS), specific vital operations, and their core assets. Arora and Garg [32] solved the AOs for IFSS and provided the MCDM technique with the established operators. Çağman and Karataş [33] described certain of the operations of IFSS by their characteristics and developed DM techniques based on the operations they demonstrated. Muthukumar and Krishnan [34] acquainted SMs and weighted SMs for IFSS and deliberated the elementary operations of IFSS with their properties.

Peng et al. [35] combined two eminent concepts, PFS and SS, and settled the Pythagorean fuzzy soft set (PFSS). Zulqarnain et al. [36] proposed specific algebraic operational laws for PFSS and extended the AOs of PFSS. Athira et al. [37] offered entropy measures of PFSS, Hamming, and Euclid distances. Zulqarnain et al. [38] protracted the algebraic operational interaction laws of PFSS and extended the interaction AOs. Athira et al. [39] presented entropy measures of PFSS. Zulqarnain et al. [40, 41] extended Einstein's law of ordered operation for PFSS and introduced Einstein's ordered weighted average and geometric AOs into PFSS. They also formed DM technology to solve complex real-life problems. Zulqarnain et al. [42] settled the Einstein AOs for PFSS and established a MAGDM methodology to fortitude DM impediments. Hussain et al. [43] proposed the q-rung orthopair fuzzy soft sets (q-ROFSS) with their AOs. Zulqarnain et al. [44], [45] extended the AOs for q-ROFSS with their DM techniques and employed their presented DM methods for medical diagnoses.

Smarandache [46] anticipated the hypersoft set (HSS) theory. Smarandache HSS is the best proper model associated with SS and other standing concepts because it grips the multiple sub-attributes of the pondered constraints. There are various HSS extensions, and their DM systems are planned. Rahman et al. [47] premeditated the DM methods initiated on SM for the possibility IFHSS. Zulqarnain et al. [48], [49] prolonged the TOPSIS technique built on the CC. They also stretched the AOs for IFHSS. Zulqarnain et al. [50] lengthened the perception of IFHSS to PFHSS with fundamental operations. Zulqarnain et al. [51] settled a correlation-based TOPSIS methodology and used the technique they established to select the best anti-virus mask. Siddique et al. [52] provided the AOs of PFHSS and planned the MCDM technique through the operators they anticipated. Khan et al. [53] introduced the q-ROFHSS with some basic operations. q-ROFHSS is a combination rational formation of q-ROFSS. Gurmani et al. [54] extended the TOPSIS method using q-ROFHSS information to develop a MAGDM technique. Khan et al. [55] projected the AOs for q-ROFHSS and utilized their presented AOs to analyze the cryptocurrency market. An enhanced consolidating system fascinates detectives to imperfection incomprehensible and infrequent information to discourse these insufficiencies. Interpret the consideration significances, q-ROFHSS theatres an animated role in DM by amassing plentiful origins into a particular assessment.

A. MOTIVATION AND DRAWBACK OF EXISTING APPROACHES

The q-ROFHSS is a hybrid logical organization of HSS and q-ROFS, which is a dominant mathematical tool for processing uncertainty, coherence, and incomplete facts. It has been found that AOs are crucial in DM, so community assessment details from dissimilar sources can be written into specialized valuations. To our familiarity, the interaction of HSS hybridization with q-ROFS is not used in the literature. However, the above approaches do not yet have a relevant quantitative summary of q-rung orthopair fuzzy hypersoft numbers (q-ROFHSNs), nor be intentionally associated with MD and NMD of multi-sub-attributes of the parameters under consideration. Specifically, the impact of other degrees of MD or NMD on the resultant geometric or average AOs does not disturb the entire procedure. Also, the model states that the whole MD (NMD) functional level is autonomous of its NMD (MD) functional level. So, conferring to these AOs, the results are not beneficial, so the appropriate predilection for substitutes is not identified. Consequently, how to integrate these q-ROFHSNs into interactions is an attractive question. We'll push some interaction AOs q-ROFHSS, such as q-ROFHSIWA and q-ROFHSIWG operators, to solve these problems. As prevalent FS, IFS, SS, IFSS, IFSS, IFSS, QROFS, IFHSS, and PFHSS are exceptions to – q-ROFHSS, established interactive AOs have additional capabilities equated to prevailing FS hybrid configurations.

Consequently, the outcomes of the predominant replicas are undesirable and do not adequately order partiality for substitutes. Thus, integrating these q-ROFHSNs over interaction is a stimulating subject. Methods described in [55] are inadequate to test data in cavernous aptitude to achieve better concepts and precise results. For instance, $\mathfrak{U} = \{u_1, u_2\}$ be a set of two specialists with weights $\Omega_i = (0.7, 0.3)^T$ and d_1, d_2 be the nominated parameters with their conforming multi sub-attributes such as $d_1 = \{d_{11}, d_{12}\}$ and $d_2 = \{d_{21}\}$. Where $\mathfrak{L}'$ be a 2-tuple cartesian product of the deliberated parameters, can be stated as $\mathfrak{L}' = d_1 \times d_2 = \{d_{11}, d_{12}\} \times \{d_{21}\} = \{(d_{11}, d_{21}), (d_{12}, d_{21})\} = \left\{\check{d}_1, \check{d}_2\right\}$ whose weights are given as $\gamma_j = (0.4, .0.6)^T$. Let $\aleph$ be an alternative, then predilections of specialists can be précised as $\aleph = \begin{bmatrix} (0.7, 0.0) & (0.6, 0.7) \\ (0.8, 0.7) & (0.7, 0.2) \end{bmatrix}$ longsighted the multi-sub-attributes of the intentional factors in the form of q-ROFHSNs. Then, we attained the accumulated value by the q-ROFHSWA [55] operator is $\langle 0.6819, 0 \rangle$. Also, we employed the q-ROFHSWG [55] operator and attained an accumulated value $\langle 0.6667, 0 \rangle$. This displays that there is no influence on the communal outcome $\mathfrak{J}_{\check{d}_k}$. Since $\mathfrak{J}_{\check{d}_k} = \mathfrak{J}_{\check{d}_{11}} = 0.0$, $\mathfrak{J}_{\check{d}_{12}} = 0.7$, $\mathfrak{J}_{\check{d}_{21}} = 0.7$, and $\mathfrak{J}_{\check{d}_{22}} = 0.2$, which is unreasonable. An improved organizing methodology attracts investigators to crack baffling and insufficient facts. Interpreting the exploration results, q-ROFHSS executes a dynamic role in DM by assembling several sources into a single value.

B. CONTRIBUTION

Interaction AOs are identified as absorbing guesstimate AOs. It is observed that the prevalent AOs aspect is unresponsive to spotting the particular finding over the DM system in particular situations. To astound these exacting troubles, these AOs prerequisite to be reformed. So, to illuminate the present investigation on q-ROFHSS and the above boundaries, we will designate interaction AOs based on indeterminate information, and the core intentions of the study assumed below are as follows:

1. The interaction AOs for q-ROFHSS are familiar gorgeous assessment AOs. It is perceived that the predominant concept characteristic is unsympathetic to mark the specific outcome of the DM procedure in certain states. To stun these exacting impediments, we protracted the notion of q-ROFHSS and elongated the interaction AOs for q-ROFHSS.
2. The q-ROFHSS adroitly pacts the multidimensional apprehensions sighted the multi sub-attributes of the pondered factors in the DM system. To sanctuary this assistance in attention, we persist in the interaction AOs for q-ROFHSS.
3. q-ROFHSIWA and q-ROFHSIWG operators presented their crucial possessions by settled operational laws.

4. A unique algorithm with the intended interaction operators to insist the DM negligent is demonstrated to embrace MCGDM concerns under the q-ROFHSS setting and utilized for studying the cryptocurrency market.
5. A comparative analysis of the novel MCGDM method and prevailing methodologies is executed to ensure the efficacy and preeminence of the planned MCGDM model.

The association assumptions in this paper are as follows: The second section of this paper compacts with some of the elementary concepts that support our organization to grow follow-up research. Section 3 intends some interaction laws for the algebraic operation of q-ROFHSN. In addition, in the same section, the q-ROFHSIWA and q-ROFHSIWG operators are introduced according to the characteristics of the established operators. The MCGDM method in Section 4 is constructed using the expected AOs. Mathematical illustrations are discussed in this section to verify the pragmatism of the demonstrated scheme. Furthermore, an ephemeral comparative analysis is accompanied to recognize the merits of the established scheme in section 5.

II. PRELIMINARIES

This section remembers necessary concepts such as SS, HSS, PFHSS, and q-ROFHSS.

A. DEFINITION 1 [23]

Let $\mathcal{U}$ and E be the universe of discourse and attributes, respectively. Let $\mathcal{P}(\mathcal{U})$ be the power set of $\mathcal{U}$ and $\mathcal{A} \subseteq E$. A pair $(\mathcal{F}, \mathcal{A})$ is called a SS over $\mathcal{U}$; its mapping is expressed as follows:

$$\mathcal{F}: \mathcal{A} \to \mathcal{P}(\mathcal{U})$$

Also, it can be defined as follows:

$$(\mathcal{F}, \mathcal{A}) = \{\mathcal{F}(e) \in \mathcal{P}(\mathcal{U}) : e \in E, \mathcal{F}(e) = \emptyset \text{ if } e \notin \mathcal{A}\}$$

B. DEFINITION 2 [46]

Let $\mathcal{U}$ be a universe of discourse and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and $k = \{k_1, k_2, k_3, \ldots, k_n\}$, (n ≥ 1) and K_i signified the set of attributes and their consistent sub-attributes, such as $K_i \cap K_j = \varphi$, where $i \neq j$ for each $n \geq 1$ and $i, j \in \{1, 2, 3 \ldots n\}$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}} = \{d_{1h} \times d_{2k} \times \cdots \times d_{nl}\}$ is an assortment of sub-attributes, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and $\alpha, \beta, \gamma \in \mathbb{N}$. Then the pair $(\mathcal{F}, K_1 \times K_2 \times K_3 \times \ldots \times K_n = (\mathcal{F}, \ddot{\mathcal{A}})$ is known as HSS and is defined as follows:

$$\mathcal{F}: K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}} \to \mathcal{P}(\mathcal{U}).$$

It is also defined as

$$(\mathcal{F}, \ddot{\mathcal{A}}) = \left\{\check{d}, \mathcal{F}_{\ddot{\mathcal{A}}}\left(\check{d}\right) : \check{d} \in \ddot{\mathcal{A}}, \mathcal{F}_{\ddot{\mathcal{A}}}\left(\check{d}\right) \in \mathcal{P}(\mathcal{U})\right\}.$$

C. DEFINITION 3 [50]

Let $\mathcal{U}$ be a universe of discourse and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and $k = \{k_1, k_2, k_3, \ldots, k_n\}$, (n ≥ 1) and K_i signified the set of attributes and their consistent sub-attributes, such as $K_i \cap K_j = \varphi$, where $i \neq j$ for each $n \geq 1$ and $i, j \in \{1, 2, 3 \ldots n\}$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}} = \{d_{1h} \times d_{2k} \times \cdots \times d_{nl}\}$ is an assortment of sub-attributes, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and $\alpha, \beta, \gamma \in \mathbb{N}$. and $PFS^{\mathcal{U}}$ be an assembly of all Pythagorean fuzzy subsets over $\mathcal{U}$. Then the pair $(\mathcal{F}, K_1 \times K_2 \times K_3 \times \ldots \times K_n = (\mathcal{F}, \ddot{\mathcal{A}})$ is identified as PFHSS and defined as follows:

$$\mathcal{F}: K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}} \to PFS^{\mathcal{U}}.$$

It is also defined as $(\mathcal{F}, \ddot{\mathcal{A}}) = \left\{\left(\check{d}, \mathcal{F}_{\ddot{\mathcal{A}}}(\check{d})\right) : \check{d} \in \ddot{\mathcal{A}}, \mathcal{F}_{\ddot{\mathcal{A}}}(\check{d}) \in PFS^{\mathcal{U}} \in [0, 1]\right\}$, where $\mathcal{F}_{\ddot{\mathcal{A}}}(\check{d}) = \{\langle \delta, \mathcal{T}_{\mathcal{F}(\check{d})}(\delta), \mathcal{J}_{\mathcal{F}(\check{d})(\delta)}\rangle : \delta \in \mathcal{U}\}$, where $\mathcal{T}_{\mathcal{F}(\check{d})}(\delta)$ and $\mathcal{J}_{\mathcal{F}(\check{d})}(\delta)$ signifies the MD and NMD of the attributes:

$$\mathcal{T}_{\mathcal{F}\left(\check{d}\right)}(\delta), \mathcal{J}_{\mathcal{F}\left(\check{d}\right)}(\delta) \in [0, 1], \text{ and}$$
$$0 \leq \left(\mathcal{T}_{\mathcal{F}\left(\check{d}\right)}(\delta)\right)^2 + \left(\mathcal{J}_{\mathcal{F}\left(\check{d}\right)}(\delta)\right)^2 \leq 1.$$

D. DEFINITION 4 [53]

Let $\mathcal{U}$ be a universe of discourse and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and $k = \{k_1, k_2, k_3, \ldots, k_n\}$, (n ≥ 1) and K_i signified the set of attributes and their consistent sub-attributes, such as $K_i \cap K_j = \varphi$, where $i \neq j$ for each $n \geq 1$ and $i, j \in \{1, 2, 3 \ldots n\}$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}} = \{d_{1h} \times d_{2k} \times \cdots \times d_{nl}\}$ is an assortment of sub-attributes, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and $\alpha, \beta, \gamma \in \mathbb{N}$. and $q - ROFS^{\mathcal{U}}$ be a collection of all q-rung orthopair fuzzy subsets over $\mathcal{U}$. Then the pair $(\mathcal{F}, K_1 \times K_2 \times K_3 \times \ldots \times K_n = (\mathcal{F}, \ddot{\mathcal{A}})$ is identified as q-ROFHSS and is defined as follows:

$$\mathcal{F}: K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}} \to q - ROFS^{\mathcal{U}}.$$

It is also defined as $(\mathcal{F}, \ddot{\mathcal{A}}) = \left\{\left(\check{d}, \mathcal{F}_{\ddot{\mathcal{A}}}(\check{d})\right) : \check{d} \in \ddot{\mathcal{A}}, \mathcal{F}_{\ddot{\mathcal{A}}}(\check{d}) \in PFS^{\mathcal{U}} \in [0, 1]\right\}$, where $\mathcal{F}_{\ddot{\mathcal{A}}}\left(\check{d}\right) = \left\{\langle \delta, \mathcal{T}_{\mathcal{F}(\check{d})}(\delta), \mathcal{J}_{\mathcal{F}(\check{d})(\delta)}\rangle : \delta \in \mathcal{U}\right\}$, where $\mathcal{T}_{\mathcal{F}\left(\check{d}\right)}(\delta)$ and $\mathcal{J}_{\mathcal{F}\left(\check{d}\right)}(\delta)$ signifies the MD and NMD of the attributes:

$$\mathcal{T}_{\mathcal{F}\left(\check{d}\right)}(\delta), \mathcal{J}_{\mathcal{F}\left(\check{d}\right)}(\delta) \in [0, 1], \text{ and}$$
$$0 \leq \left(\mathcal{T}_{\mathcal{F}\left(\check{d}\right)}(\delta)\right)^q + \left(\mathcal{J}_{\mathcal{F}\left(\check{d}\right)}(\delta)\right)^q \leq 1.$$

A q-rung fuzzy hypersoft number (q-ROFHSN) can be specified as $\mathcal{F} = \left\{\left(\mathcal{T}_{\mathcal{F}\left(\check{d}\right)}(\delta), \mathcal{J}_{\mathcal{F}\left(\check{d}\right)}(\delta)\right)\right\}$, where $0 \leq \left(\mathcal{T}_{\mathcal{F}\left(\check{d}\right)}(\delta)\right)^q + \left(\mathcal{J}_{\mathcal{F}\left(\check{d}\right)}(\delta)\right)^q \leq 1$.

E. DEFINITION 5 [55]

Let $\mathfrak{I}_{\check{d}_k} = \left(\mathcal{T}_{\check{d}_k}, \mathcal{J}_{\check{d}_k}\right)$, $\mathfrak{I}_{\check{d}_{11}} = \left(\mathcal{T}_{\check{d}_{11}}, \mathcal{J}_{\check{d}_{11}}\right)$, and $\mathfrak{I}_{\check{d}_{12}} = \left(\mathcal{T}_{\check{d}_{12}}, \mathcal{J}_{\check{d}_{12}}\right)$ denotes the q-ROFHSNs, and $\alpha > 0$. Then, the operational laws for q-ROFHSNs can be defined as:

1. $\mathfrak{I}_{\check{d}_{11}} \oplus \mathfrak{I}_{\check{d}_{12}} = \left\langle \sqrt[q]{\mathcal{T}^q_{\check{d}_{11}} + \mathcal{T}^q_{\check{d}_{12}} - \mathcal{T}^q_{\check{d}_{11}}\mathcal{T}^q_{\check{d}_{12}}}, \mathcal{J}_{\check{d}_{11}}\mathcal{J}_{\check{d}_{12}} \right\rangle$

2. $\mathfrak{J}_{\check{d}_{11}} \otimes \mathfrak{J}_{\check{d}_{12}} = \left\langle \mathcal{T}_{\check{d}_{11}} \mathcal{T}_{\check{d}_{12}}, \sqrt[q]{\mathcal{I}^q_{\check{d}_{11}} + \mathcal{I}^q_{\check{d}_{12}} - \mathcal{I}^q_{\check{d}_{11}} \mathcal{I}^q_{\check{d}_{12}}} \right\rangle$
3. $\alpha \mathfrak{J}_{\check{d}_k} = \left\langle \sqrt[q]{1 - \left(1 - \mathcal{T}^q_{\check{d}_k}\right)^{\alpha}}, \mathcal{I}^{\alpha}_{\check{d}_k} \right\rangle$
4. $\mathfrak{J}^{\alpha}_{\check{d}_k} = \left\langle \mathcal{T}^{\alpha}_{\check{d}_k}, \sqrt[q]{1 - \left(1 - \mathcal{I}^q_{\check{d}_k}\right)^{\alpha}} \right\rangle$

For the assortment of q-rung orthopair fuzzy hypersoft numbers (q-ROFHSNs) $\mathfrak{J}_{\check{d}_k}$, where Ω_i and γ_j are weights for professionals and attributes, correspondingly, with certain circumstances $\Omega_i > 0, \sum_{i=1}^{n} \Omega_i = 1; \gamma_j > 0, \sum_{j=1}^{m} \gamma_j = 1$. Khan et al. [55] proposed the q-ROFHSWA and q-ROFHSWG operators given as follows:

$$q-ROFHSWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) = \left\langle \sqrt[q]{1 - \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(1 - \mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}}, \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(\mathcal{I}_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j} \right\rangle$$

$$q-ROFHSWG\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) = \left\langle \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(\mathcal{T}_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}, \sqrt[q]{1 - \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \left(1 - \mathcal{I}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}} \right\rangle$$

F. REMARK 1

1. If $\left(\mathcal{T}_{\mathcal{F}(\check{d})}(\delta)\right)^q + \left(\mathcal{I}_{\mathcal{F}(\check{d})}(\delta)\right)^q \le 1$ and $\left(\mathcal{T}_{\mathcal{F}(\check{d})}(\delta)\right)^2 + \left(\mathcal{I}_{\mathcal{F}(\check{d})}(\delta)\right)^2 \le 1$ both are holds. Then, q-ROFHSS is condensed to PFHSS [52].
2. If $\left(\mathcal{T}_{\mathcal{F}(\check{d})}(\delta)\right)^q + \left(\mathcal{I}_{\mathcal{F}(\check{d})}(\delta)\right)^q \le 1$ and $\mathcal{T}_{\mathcal{F}(\check{d})}(\delta) + \mathcal{I}_{\mathcal{F}(\check{d})}(\delta) \le 1$ both are holds. Then, q-ROFHSS is condensed to IFHSS [49].

For readers' aptness, the q-ROFHSN $\mathcal{F}_{\delta_i}(\check{d}_j) = \left\{\left(\mathcal{T}_{\mathcal{F}(\check{d}_j)}(\delta_i), \mathcal{I}_{\mathcal{F}(\check{d}_j)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ can be written as $\mathfrak{J}_{\check{d}_{ij}} = \left\langle \mathcal{T}_{\mathcal{F}(\check{d}_{ij})}, \mathcal{I}_{\mathcal{F}(\check{d}_{ij})} \right\rangle$. The score function for $\mathfrak{J}_{\check{d}_{ij}}$ is stated as follows:

Let $\mathfrak{J}_{\check{d}_{ij}} = \left\langle \mathcal{T}_{\mathcal{F}(\check{d}_{ij})}, \mathcal{I}_{\mathcal{F}(\check{d}_{ij})} \right\rangle$ be a q-ROFHSN. Then

$$\mathfrak{S}(\mathfrak{J}_{\check{d}_{ij}}) = \mathcal{T}^q_{\mathcal{F}(\check{d}_{ij})} - \mathcal{I}^q_{\mathcal{F}(\check{d}_{ij})} + \left(\frac{e^{\mathcal{T}^q_{\mathcal{F}(\check{d}_{ij})} - \mathcal{I}^q_{\mathcal{F}(\check{d}_{ij})}}}{e^{\mathcal{T}^q_{\mathcal{F}(\check{d}_{ij})} - \mathcal{I}^q_{\mathcal{F}(\check{d}_{ij})}} + 1} - \frac{1}{2} \right) \beth^q_{\mathfrak{J}_{\check{d}_{ij}}},$$

$$\text{for } q \ge 3 \text{ and } \mathfrak{S}(\mathfrak{J}_{\check{d}_{ij}}) \in [-1, 1]. \tag{1}$$

Let $\mathfrak{J}_{\check{d}_{11}} = \left(\mathcal{T}_{\mathcal{F}(\check{d}_{11})}, \mathcal{I}_{\mathcal{F}(\check{d}_{11})}\right)$ and $\mathfrak{J}_{\check{d}_{12}} = \left(\mathcal{T}_{\mathcal{F}(\check{d}_{12})}, \mathcal{I}_{\mathcal{F}(\check{d}_{12})}\right)$ be two q-ROFHSNs. Then

If $\mathfrak{S}(\mathfrak{J}_{\check{d}_{11}}) > \mathfrak{S}(\mathfrak{J}_{\check{d}_{12}})$, then $\mathfrak{J}_{\check{d}_{11}} \mathfrak{J}_{\check{d}_{12}}$.
If $\mathfrak{S}(\mathfrak{J}_{\check{d}_{11}}) < \mathfrak{S}(\mathfrak{J}_{\check{d}_{12}})$, then $\mathfrak{J}_{\check{d}_{11}} \mathfrak{J}_{\check{d}_{12}}$.
If $\mathfrak{S}\left(\mathfrak{J}_{\check{d}_{11}}\right) = \mathfrak{S}(\mathfrak{J}_{\check{d}_{12}})$, then
If $\beth_{\mathfrak{J}_{\check{d}_{11}}} > \beth_{\mathfrak{J}_{\check{d}_{12}}}$, then $\mathfrak{J}_{\check{d}_{11}} < \mathfrak{J}_{\check{d}_{12}}$
If $\beth^q_{\mathfrak{J}_{\check{d}_{11}}} = \beth^q_{\mathfrak{J}_{\check{d}_{12}}}$, then $\mathfrak{J}_{\check{d}_{11}} = \mathfrak{J}_{\check{d}_{12}}$

So, to equate two q-ROFHSNs $\mathfrak{J}_{\check{d}_{ij}}$ and $\mathfrak{J}_{\check{d}_{ij}}$. The subsequent comparison rules are demarcated

1. If $\mathfrak{S}(\mathfrak{J}_{\check{d}_{ij}}) > \mathfrak{S}(\mathfrak{J}_{\check{d}_{ij}})$, then $\mathfrak{J}_{\check{d}_{ij}} > \mathfrak{J}_{\check{d}_{ij}}$.
2. If $\mathfrak{S}(\mathfrak{J}_{\check{d}_{ij}}) = \mathfrak{S}(\mathfrak{J}_{\check{d}_{ij}})$, then
   - If $H\left(\mathfrak{J}_{\check{d}_{ij}}\right) > H\left(\mathfrak{J}_{\check{d}_{ij}}\right)$, then $\mathfrak{J}_{\check{d}_{ij}} > \mathfrak{J}_{\check{d}_{ij}}$
   - If $H(\mathfrak{J}_{\check{d}_{ij}}) = H(\mathfrak{J}_{\check{d}_{ij}})$, then $\mathfrak{J}_{\check{d}_{ij}} = \mathfrak{J}_{\check{d}_{ij}}$.

From the q-ROFHSWA and q-ROFHSWG operators, it is noticed that, in definite circumstances, these AOs deliver some repellant consequences. To overcome such scenarios, we introduce the interaction AOs for q-ROFHSS.

III. INTERACTION AGGREGATION OPERATORS FOR Q-RUNG ORTHOPAIR FUZZY HYPERSOFT NUMBERS

In this section, we will define operational laws under q-ROFHSNs. We shall also present q-ROFHSIWA and q-ROFHSIWG operators based on these operational laws.

1) DEFINITION

Let $\mathfrak{J}_{\check{d}_k} = \left(\mathcal{T}_{\check{d}_k}, \mathcal{I}_{\check{d}_k}\right)$, $\mathfrak{J}_{\check{d}_{11}} = \left(\mathcal{T}_{\check{d}_{11}}, \mathcal{I}_{\check{d}_{11}}\right)$, and $\mathfrak{J}_{\check{d}_{12}} = \left(\mathcal{T}_{\check{d}_{12}}, \mathcal{I}_{\check{d}_{12}}\right)$ denotes the q-ROFHSNs, and $\alpha > 0$. Then, the operational laws for q-ROFHSNs can be defined as:

$$\mathfrak{J}_{\check{d}_{11}} \oplus \mathfrak{J}_{\check{d}_{12}} = \left\langle \sqrt[q]{\mathcal{T}^q_{\check{d}_{11}} + \mathcal{T}^q_{\check{d}_{12}} - \mathcal{T}^q_{\check{d}_{11}} \mathcal{T}^q_{\check{d}_{12}}}, \sqrt[q]{\mathcal{T}^q_{\check{d}_{11}} + \mathcal{T}^q_{\check{d}_{12}} - \mathcal{T}^q_{\check{d}_{11}} \mathcal{T}^q_{\check{d}_{12}} - \mathcal{I}^q_{\check{d}_{11}} \mathcal{T}^q_{\check{d}_{12}} - \mathcal{T}^q_{\check{d}_{11}} \mathcal{I}^q_{\check{d}_{12}}} \right\rangle$$

$$\mathfrak{J}_{\check{d}_{11}} \otimes \mathfrak{J}_{\check{d}_{12}} = \left\langle \sqrt[q]{\mathcal{T}^q_{11} + \mathcal{T}^q_{12} - \mathcal{T}^q_{11}\mathcal{T}^q_{12} - \mathcal{T}^q_{11}\mathcal{T}^q_{12} - \mathcal{I}^q_{11}\mathcal{T}^q_{12}}, \sqrt[q]{\mathcal{I}^q_{11} + \mathcal{I}^q_{12} - \mathcal{I}^q_{11}\mathcal{I}^q_{12}} \right\rangle$$

$$\alpha \mathfrak{J}_{\check{d}_k} = \left\langle \sqrt{1 - \left(1 - \mathcal{T}^q_{\check{d}_k}\right)^{\alpha}}, \sqrt{\left(1 - \mathcal{T}^q_{\check{d}_k}\right)^{\alpha} - \left[1 - \left(\mathcal{T}^q_{\check{d}_k} + \mathcal{I}^q_{\check{d}_k}\right)\right]^{\alpha}} \right\rangle$$

$$\mathfrak{J}^{\alpha}_{\check{d}_k} = \left\langle \sqrt{\left(1 - \mathcal{I}^q_{\check{d}_k}\right)^{\alpha} - \left[1 - \left(\mathcal{T}^q_{\check{d}_k} + \mathcal{I}^q_{\check{d}_k}\right)\right]^{\alpha}}, \sqrt{1 - \left(1 - \mathcal{I}^q_{\check{d}_k}\right)^{\alpha}} \right\rangle$$

In light of the above-presented interactional operational laws, we will propose the interaction AOs for the q-ROFHSS.

2) DEFINITION

Let $\mathfrak{J}_{\check{d}_{ij}} = \left(\mathcal{T}_{\check{d}_{ij}}, \mathcal{I}_{\check{d}_{ij}}\right)$ be a q-ROFHSN, Ω_i and γ_j are weights for experts and multi sub-attributes of the deliberated attributes consistently along with identified environments

$\Omega_i > 0$, $\sum_{i=1}^{n} \Omega_i = 1$, $\gamma_j > 0$, $\sum_{j=1}^{m} \gamma_j = 1$. Then the q-ROFHSIWA operator can be defined as

q-ROFHSIWA: $\Delta^n \to \Delta$ defined as follows

$$q-ROFHSIWA\left(\mathfrak{I}_{\check{d}_{11}}, \mathfrak{I}_{\check{d}_{12}}, \ldots, \mathfrak{I}_{\check{d}_{nm}}\right) = \oplus_{j=1}^{m}\gamma_j\left(\oplus_{i=1}^{n}\Omega_i\mathfrak{I}_{\check{d}_{ij}}\right) \quad (2)$$

3) THEOREM

Let $\mathfrak{I}_{\check{d}_{ij}} = \left(\mathfrak{T}_{\check{d}_{ij}}, \mathfrak{J}_{\check{d}_{ij}}\right)$ be a q-ROFHSN. Then, the aggregated value using Equation 2 is also a q-ROFHSN and (3), as shown at the bottom of the page, where Ω_i and γ_j are weight vectors for experts and sub-attributes of the parameters, respectively, with given conditions $\Omega_i > 0$, $\sum_{i=1}^{n} \Omega_i = 1$, $\gamma_j > 0$, $\sum_{j=1}^{m} \gamma_j = 1$.

Proof: Using the mathematical induction

For $n = 1$, we get $\Omega_1 = 1$. Then, we have, as shown in the equation at the bottom of the page.

For $m = 1$, we get $\gamma_1 = 1$. Then, we have, as shown in the equation at the bottom of the page, for $n = 2$ and $m = 2$. Then, we have, as shown in the equation at the bottom of the next page.

So, for $n = 2$ and $m = 2$, Equation 3 fulfills. Suppose Equation (3) holds for $n = \beta_1$ and $m = \beta_2$, as shown in the equation at the bottom of page 8.

For $m = \beta_1 + 1$ and $n = \beta_2 + 1$, we have, as shown in the equation at the bottom of page 8.

So, it is true for $m = \beta_1 + 1$ and $n = \beta_2 + 1$.

Which shows that it holds $\forall n, m \geq 1$.

The most remarkable object is that the attained accumulated values with the q-ROFHSIWA operator are also q-ROFHSN. To demonstrate this, contemplate $\mathfrak{I}_{\check{d}_{ij}} = \left(\mathfrak{T}_{\check{d}_{ij}}, \mathfrak{J}_{\check{d}_{ij}}\right)$, where $0 \leq \mathfrak{T}_{\check{d}_{ij}}, \mathfrak{J}_{\check{d}_{ij}} \leq 1$ with condition $0 \leq \mathfrak{T}_{\check{d}_{ij}}^q + \mathfrak{J}_{\check{d}_{ij}}^q \leq 1$ and Ω_i and γ_j indicates the weights of experts and sub-attributes separately, such as $\Omega_i, \gamma_j \in [0, 1]$, $\Omega_i > 0$, $\sum_{i=1}^{n} \Omega_i = 1$, $\gamma_j > 0$, $\sum_{j=1}^{m} \gamma_j = 1$.

We know that

$$0 \leq \mathfrak{T}_{\check{d}_{ij}} \leq 1 \Rightarrow 0 \leq 1 - \mathfrak{T}_{\check{d}_{ij}} \leq 1 \Rightarrow 0 \leq \left(1 - \mathfrak{T}_{\check{d}_{ij}}^q\right)^{\Omega_i} \leq 1$$

$$\Rightarrow 0 \leq \prod_{i=1}^{n}\left(1 - \mathfrak{T}_{\check{d}_{ij}}^q\right)^{\Omega_i} \leq 1$$

$$q-ROFHSIWA\left(\mathfrak{I}_{\check{d}_{11}}, \mathfrak{I}_{\check{d}_{12}}, \ldots, \mathfrak{I}_{\check{d}_{nm}}\right) = \oplus_{j=1}^{m}\gamma_j\left(\oplus_{i=1}^{n}\Omega_i\mathfrak{I}_{\check{d}_{ij}}\right)$$
$$= \left\langle \sqrt[q]{1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \mathfrak{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \mathfrak{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1 - \left(\mathfrak{T}_{\check{d}_{ij}}^q + \mathfrak{J}_{\check{d}_{ij}}^q\right)\right]^{\Omega_i}\right)^{\gamma_j}} \right\rangle \quad (3)$$

$$q-ROFHSIWA\left(\mathfrak{I}_{\check{d}_{11}}, \mathfrak{I}_{\check{d}_{12}}, \ldots, \mathfrak{I}_{\check{d}_{nm}}\right) = \oplus_{j=1}^{m}\gamma_j\mathfrak{I}_{\check{d}_{ij}}$$

$$q-ROFHSIWA\left(\mathfrak{I}_{\check{d}_{11}}, \mathfrak{I}_{\check{d}_{12}}, \ldots, \mathfrak{I}_{\check{d}_{nm}}\right)$$
$$= \left\langle \sqrt[q]{1 - \prod_{j=1}^{m}\left(\left(1 - \mathfrak{T}_{\check{d}_{1j}}^q\right)^{\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{\prod_{j=1}^{m}\left(\left(1 - \mathfrak{T}_{\check{d}_{1j}}^q\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(\left[1 - \left(\mathfrak{T}_{\check{d}_{1j}}^q + \mathfrak{J}_{\check{d}_{1j}}^q\right)\right]^{\Omega_i}\right)^{\gamma_j}} \right\rangle$$
$$= \left\langle \sqrt[q]{1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1 - \mathfrak{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{\prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1 - \mathfrak{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left[1 - \left(\mathfrak{T}_{\check{d}_{ij}}^q + \mathfrak{J}_{\check{d}_{ij}}^q\right)\right]^{\Omega_i}\right)^{\gamma_j}} \right\rangle$$

$$q-ROFHSIWA\left(\mathfrak{I}_{\check{d}_{11}}, \mathfrak{I}_{\check{d}_{12}}, \ldots, \mathfrak{I}_{\check{d}_{nm}}\right) = \oplus_{i=1}^{n}\Omega_i\mathfrak{I}_{\check{d}_{ij}}$$
$$= \left\langle \sqrt[q]{1 - \prod_{i=1}^{n}\left(1 - \mathfrak{T}_{\check{d}_{i1}}^q\right)^{\Omega_i}}, \sqrt[q]{\prod_{i=1}^{n}\left(1 - \mathfrak{T}_{\check{d}_{i1}}^q\right)^{\Omega_i} - \prod_{i=1}^{n}\left[1 - \left(\mathfrak{T}_{\check{d}_{i1}}^q + \mathfrak{J}_{\check{d}_{i1}}^q\right)\right]^{\Omega_i}} \right\rangle$$
$$= \left\langle \sqrt[q]{1 - \prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(1 - \mathfrak{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(1 - \mathfrak{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left[1 - \left(\mathfrak{T}_{\check{d}_{ij}}^q + \mathfrak{J}_{\check{d}_{ij}}^q\right)\right]^{\Omega_i}\right)^{\gamma_j}} \right\rangle$$

$$\Rightarrow 0 \le \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}^{q}_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j} \le 1$$

$$\Rightarrow 0 \le \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}^{q}_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}} \le 1$$

Similarly,

$$0 \le \mathcal{T}^{q}_{\check{d}_{ij}}+\mathcal{J}^{q}_{\check{d}_{ij}} \le 1 \Rightarrow 0 \le 1-\left(\mathcal{T}^{q}_{\check{d}_{ij}}+\mathcal{J}^{q}_{\check{d}_{ij}}\right) \le 1$$

$$\Rightarrow 0 \le \prod_{i=1}^{n}\left[1-\left(\mathcal{T}^{q}_{\check{d}_{ij}}+\mathcal{J}^{q}_{\check{d}_{ij}}\right)\right]^{\Omega_i} \le 1$$

$$\Rightarrow 0 \le \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathcal{T}^{q}_{\check{d}_{ij}}+\mathcal{J}^{q}_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j} \le 1$$

As we know that

$$0 \le \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}^{q}_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j} \le 1$$

So, as shown in the equation at the bottom of the next page.

Therefore, as shown in the equation at the bottom of the next page.

So, it is verified that the attained consequence over the q-ROFHSIWA operator is also a q-ROFHSN.

4) EXAMPLE

Let $\mathfrak{U} = \{u_1, u_2, u_3\}$ represents the set of experts with weights $\Omega_i = (0.143, 0.514, 0.343)^T$. Professionals precise the attractiveness of a community under a defined

$$q-ROFHSIWA\left(\Im_{\check{d}_{11}},\Im_{\check{d}_{12}},\ldots,\Im_{\check{d}_{nm}}\right)$$

$$= \oplus_{j=1}^{m}\gamma_j\left(\oplus_{i=1}^{n}\Omega_i\Im_{\check{d}_{ij}}\right)$$

$$= \oplus_{j=1}^{2}\gamma_j\left(\oplus_{i=1}^{2}\Omega_i\Im_{\check{d}_{ij}}\right)$$

$$= \gamma_1\left(\oplus_{i=1}^{2}\Omega_i\Im_{\check{d}_{i1}}\right)\oplus\gamma_2\left(\oplus_{i=1}^{2}\Omega_i\Im_{\check{d}_{i2}}\right)$$

$$= \gamma_1\left(\Omega_1\Im_{\check{d}_{11}}\oplus\Omega_2\Im_{\check{d}_{21}}\right)\oplus\gamma_2\left(\Omega_1\Im_{\check{d}_{12}}\oplus\Omega_2\Im_{\check{d}_{22}}\right)$$

$$= \gamma_1\left\{\begin{array}{l}\sqrt[q]{1-\left(1-\mathcal{T}^{q}_{\check{d}_{11}}\right)^{\Omega_1}},\sqrt[q]{\left(1-\mathcal{T}^{q}_{\check{d}_{11}}\right)^{\Omega_1}-\left[1-\left(\mathcal{T}^{q}_{\check{d}_{11}}+\mathcal{J}^{q}_{\check{d}_{11}}\right)\right]^{\Omega_1}}\oplus\\ \sqrt[q]{1-\left(1-\mathcal{T}^{q}_{\check{d}_{21}}\right)^{\Omega_2}},\sqrt[q]{\left(1-\mathcal{T}^{q}_{\check{d}_{21}}\right)^{\Omega_2}-\left[1-(\mathcal{T}^{q}_{\check{d}_{21}}+\mathcal{J}^{q}_{\check{d}_{21}})\right]^{\Omega_2}}\end{array}\right\}$$

$$\oplus\gamma_2\left\{\begin{array}{l}\sqrt[q]{1-\left(1-\mathcal{T}^{q}_{\check{d}_{12}}\right)^{\Omega_1}},\sqrt[q]{\left(1-\mathcal{T}^{q}_{\check{d}_{12}}\right)^{\Omega_1}-\left[1-\left(\mathcal{T}^{q}_{\check{d}_{12}}+\mathcal{J}^{q}_{\check{d}_{12}}\right)\right]^{\Omega_1}}\oplus\\ \sqrt[q]{1-\left(1-\mathcal{T}^{q}_{\check{d}_{22}}\right)^{\Omega_2}},\sqrt[q]{\left(1-\mathcal{T}^{q}_{\check{d}_{22}}\right)^{\Omega_2}-\left[1-(\mathcal{T}^{q}_{\check{d}_{22}}+\mathcal{J}^{q}_{\check{d}_{22}})\right]^{\Omega_2}}\end{array}\right\}$$

$$= \gamma_1\left\langle\sqrt[q]{1-\prod_{i=1}^{2}\left(1-\mathcal{T}^{q}_{\check{d}_{i1}}\right)^{\Omega_i}},\sqrt[q]{\prod_{i=1}^{2}\left(1-\mathcal{T}^{q}_{\check{d}_{i1}}\right)^{\Omega_i}-\prod_{i=1}^{2}\left[1-\left(\mathcal{T}^{q}_{\check{d}_{i1}}+\mathcal{J}^{q}_{\check{d}_{i1}}\right)\right]^{\Omega_i}}\right\rangle$$

$$\oplus\gamma_2\left\langle\sqrt[q]{1-\prod_{i=1}^{2}\left(1-\mathcal{T}^{q}_{\check{d}_{i2}}\right)^{\Omega_i}},\sqrt[q]{\prod_{i=1}^{2}\left(1-\mathcal{T}^{q}_{\check{d}_{i2}}\right)^{\Omega_i}-\prod_{i=1}^{2}\left[1-\left(\mathcal{T}^{q}_{\check{d}_{i2}}+\mathcal{J}^{q}_{\check{d}_{i2}}\right)\right]^{\Omega_i}}\right\rangle$$

$$= \left\langle\sqrt[q]{1-\left(\prod_{i=1}^{2}\left(1-\mathcal{T}^{q}_{\check{d}_{i1}}\right)^{\Omega_i}\right)^{\gamma_1}},\sqrt[q]{\left(\prod_{i=1}^{2}\left(1-\mathcal{T}^{q}_{\check{d}_{i1}}\right)^{\Omega_i}\right)^{\gamma_1}-\left(\prod_{i=1}^{2}\left[1-\left(\mathcal{T}^{q}_{\check{d}_{i1}}+\mathcal{J}^{q}_{\check{d}_{i1}}\right)\right]^{\Omega_i}\right)^{\gamma_1}}\right\rangle$$

$$\oplus\left\langle\sqrt[q]{1-\left(\prod_{i=1}^{2}\left(1-\mathcal{T}^{q}_{\check{d}_{i2}}\right)^{\Omega_i}\right)^{\gamma_2}},\sqrt[q]{\left(\prod_{i=1}^{2}\left(1-\mathcal{T}^{q}_{\check{d}_{i2}}\right)^{\Omega_i}\right)^{\gamma_2}-\left(\prod_{i=1}^{2}\left[1-\left(\mathcal{T}^{q}_{\check{d}_{i2}}+\mathcal{J}^{q}_{\check{d}_{i2}}\right)\right]^{\Omega_i}\right)^{\gamma_2}}\right\rangle$$

$$= \left\langle\sqrt[q]{1-\prod_{j=1}^{2}\left(\prod_{i=1}^{2}\left(1-\mathcal{T}^{q}_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_1}},\sqrt[q]{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}^{q}_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathcal{T}^{q}_{\check{d}_{ij}}+\mathcal{J}^{q}_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j}}\right\rangle$$

$$= \left\langle\sqrt[q]{1-\prod_{j=1}^{2}\left(\prod_{i=1}^{2}\left(1-\mathcal{T}^{q}_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}},\sqrt[q]{\prod_{j=1}^{2}\left(\prod_{i=1}^{2}\left(1-\mathcal{T}^{q}_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}-\prod_{j=1}^{2}\left(\prod_{i=1}^{2}\left[1-\left(\mathcal{T}^{q}_{\check{d}_{ij}}+\mathcal{J}^{q}_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j}}\right\rangle$$

set of attributes $\mathfrak{L}' = \{d_1 = lawn, d_2 = security\ system\}$ with their conforming sub-attributes, Lawn $= d_1 = \{d_{11} = with\ grass, d_{12} = without\ graas\}$. Security system $= d_2 = \{d_{21} = guards, d_{22} = cameras\}$. Let $\mathfrak{L}' = d_1 \times d_2$ be a set of multi sub-attributes $\mathfrak{L}' = d_1 \times d_2 = \{d_{11}, d_{12}\} \times \{d_{21}, d_{22}\} = \{(d_{11}, d_{21}), (d_{11}, d_{22}), (d_{12}, d_{21}), (d_{12}, d_{22})\}$ $\mathfrak{L}' = \{\check{d}_1, \check{d}_2, \check{d}_3, \check{d}_4\}$ designates the set of multi sub-attributes with their weights $\gamma_j = (0.35, 0.15, 0.2, 0.3)^T$. Expert's judgment for each multi-sub-attribute in the form of q-ROFHSNs $(\mathfrak{I}, \mathfrak{L}') = \langle \mathcal{T}_{\check{d}_{ij}}, \mathcal{J}_{\check{d}_{ij}} \rangle_{3\times 4}$ given as follows:

$$(\mathfrak{I}, \mathfrak{L}') = \begin{bmatrix} (.3, .8) & (.4, .6) & (.3, .6) & (.5, .6) \\ (.8, .3) & (.7, .4) & (.7, .3) & (.7, .8) \\ (.3, .6) & (.5, .7) & (.6, .5) & (.5, .4) \end{bmatrix}$$

Using Equation 3, as shown in the equation at the bottom of the page.

For, as shown in the equation shown at the bottom of the next page.

A. PROPERTIES OF Q-ROFHSIWA OPERATOR

1) IDEMPOTENCY

If $\mathfrak{I}_{\check{d}_{ij}} = \mathfrak{I}_{\check{d}} = \left(\mathcal{T}_{\check{d}_{ij}}, \mathcal{J}_{\check{d}_{ij}}\right) \forall i, j$, then,

$$q-ROFHSIWA\left(\mathfrak{I}_{\check{d}_{11}}, \mathfrak{I}_{\check{d}_{12}}, \ldots, \mathfrak{I}_{\check{d}_{nm}}\right) = \mathfrak{I}_{\check{d}}.$$

Proof: As we know that all $\mathfrak{I}_{\check{d}_{ij}} = \mathfrak{I}_{\check{d}} = \left(\mathcal{T}_{\check{d}_{ij}}, \mathcal{J}_{\check{d}_{ij}}\right)$, then by Equation 4, as shown in the equation at the bottom of the next page.

$$\begin{aligned} &q-ROFHSIWA\left(\mathfrak{I}_{\check{d}_{11}}, \mathfrak{I}_{\check{d}_{12}}, \ldots, \mathfrak{I}_{\check{d}_{nm}}\right) \\ &= \oplus_{j=1}^{\beta_2}\gamma_j\left(\oplus_{i=1}^{\beta_1}\Omega_i\mathfrak{I}_{\check{d}_{ij}}\right) \\ &= \left\langle \sqrt[q]{1-\prod_{j=1}^{\beta_2}\left(\prod_{i=1}^{\beta_1}\left(1-\mathcal{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{\prod_{j=1}^{\beta_2}\left(\prod_{i=1}^{\beta_1}\left(1-\mathcal{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{\beta_2}\left(\prod_{i=1}^{\beta_1}\left[1-\left(\mathcal{T}_{\check{d}_{ij}}^q + \mathcal{J}_{\check{d}_{ij}}^q\right)\right]^{\Omega_i}\right)^{\gamma_j}} \right\rangle \end{aligned}$$

$$\begin{aligned} &\oplus_{j=1}^{\beta_2+1}\gamma_j\left(\oplus_{i=1}^{\beta_1+1}\Omega_i\mathfrak{I}_{\check{d}_{ij}}\right) \\ &= \oplus_{j=1}^{\beta_2+1}\gamma_j\left(\oplus_{i=1}^{\beta_1}\Omega_i\mathfrak{I}_{\check{d}_{ij}} \oplus \Omega_{\beta_1+1}\mathfrak{I}_{\check{d}_{(\beta_1+1)j}}\right) \\ &= \oplus_{j=1}^{\beta_2+1}\oplus_{i=1}^{\beta_1}\gamma_j\Omega_i\mathfrak{I}_{\check{d}_{ij}} \oplus_{j=1}^{\beta_2+1}\gamma_j\Omega_{\beta_1+1}\mathfrak{I}_{\check{d}_{(\beta_1+1)j}} \\ &= \left\langle \begin{matrix} \sqrt[q]{1-\prod_{j=1}^{\beta_2+1}\left(\prod_{i=1}^{\beta_1}\left(1-\mathcal{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j}} \oplus \sqrt[q]{1-\prod_{j=1}^{\beta_2+1}\left(\left(1-\mathcal{T}_{\check{d}_{(\beta_1+1)j}}^q\right)^{\Omega_{\beta_1+1}}\right)^{\gamma_j}}, \\ \sqrt[q]{\prod_{j=1}^{\beta_2+1}\left(\prod_{i=1}^{\beta_1}\left(1-\mathcal{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{\beta_2+1}\left(\prod_{i=1}^{\beta_1}\left[1-\left(\mathcal{T}_{\check{d}_{ij}}^q+\mathcal{J}_{\check{d}_{ij}}^q\right)\right]^{\Omega_i}\right)^{\gamma_j}} \oplus \sqrt[q]{\prod_{j=1}^{\beta_2+1}\left(\left(1-\mathcal{T}_{\check{d}_{(\beta_1+1)j}}^q\right)^{\Omega_{\beta_1+1}}\right)^{\gamma_j} - \prod_{j=1}^{\beta_2+1}\left(\left[1-\left(\mathcal{T}_{\check{d}_{(\beta_1+1)j}}^q+\mathcal{J}_{\check{d}_{(\beta_1+1)j}}^q\right)\right]^{\Omega_{\beta_1+1}}\right)^{\gamma_j}} \end{matrix} \right\rangle \\ &= \left\langle \sqrt[q]{1-\prod_{j=1}^{\beta_2+1}\left(\prod_{i=1}^{\beta_1+1}\left(1-\mathcal{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{\prod_{j=1}^{\beta_2+1}\left(\prod_{i=1}^{\beta_1+1}\left(1-\mathcal{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{\beta_2+1}\left(\prod_{i=1}^{\beta_1+1}\left[1-\left(\mathcal{T}_{\check{d}_{ij}}^q+\mathcal{J}_{\check{d}_{ij}}^q\right)\right]^{\Omega_i}\right)^{\gamma_j}} \right\rangle \end{aligned}$$

$$\begin{aligned} &0 \le \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathcal{T}_{\check{d}_{ij}}^q+\mathcal{J}_{\check{d}_{ij}}^q\right)\right]^{\Omega_i}\right)^{\gamma_j} \le 1 \\ &\Rightarrow 0 \le \sqrt[q]{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathcal{T}_{\check{d}_{ij}}^q+\mathcal{J}_{\check{d}_{ij}}^q\right)\right]^{\Omega_i}\right)^{\gamma_j}} \le 1 \end{aligned}$$

$$0 \le \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j}} + \sqrt[q]{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathcal{T}_{\check{d}_{ij}}^q+\mathcal{J}_{\check{d}_{ij}}^q\right)\right]^{\Omega_i}\right)^{\gamma_j}} \le 1.$$

$$\begin{aligned} &q-ROFHSIWA\left(\mathfrak{I}_{\check{d}_{11}}, \mathfrak{I}_{\check{d}_{12}}, \ldots, \mathfrak{I}_{\check{d}_{nm}}\right) \\ &= \left\langle \sqrt[q]{1-\prod_{j=1}^{4}\left(\prod_{i=1}^{3}\left(1-\mathcal{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{\prod_{j=1}^{4}\left(\prod_{i=1}^{3}\left(1-\mathcal{T}_{\check{d}_{ij}}^q\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{4}\left(\prod_{i=1}^{3}\left[1-\left(\mathcal{T}_{\check{d}_{ij}}^q+\mathcal{J}_{\check{d}_{ij}}^q\right)\right]^{\Omega_i}\right)^{\gamma_j}} \right\rangle \end{aligned}$$

2) BOUNDEDNESS

Let $\mathfrak{I}_{\check{d}_{ij}}$ be a collection of q-ROFHSNs and $\mathfrak{I}^{-}_{\check{d}_{ij}} = \left\langle \underset{j}{min}\,\underset{i}{min}\left\{\mathfrak{T}_{\check{d}_{ij}}\right\}, \underset{j}{max}\,\underset{i}{max}\left\{\mathfrak{J}_{\check{d}_{ij}}\right\}\right\rangle$ and $\mathfrak{I}^{+}_{\check{d}_{ij}} = \left\langle \underset{j}{max}\,\underset{i}{max}\left\{\mathfrak{T}_{\check{d}_{ij}}\right\}, \underset{j}{min}\,\underset{i}{min}\left\{\mathfrak{J}_{\check{d}_{ij}}\right\}\right\rangle$, then $\mathfrak{I}^{-}_{\check{d}_{ij}} \leq q-ROFHSIWA\left(\mathfrak{I}_{\check{d}_{11}}, \mathfrak{I}_{\check{d}_{12}}, \ldots, \mathfrak{I}_{\check{d}_{nm}}\right) \leq \mathfrak{I}^{+}_{\check{d}_{ij}}$.

Proof: As we know that $\mathfrak{I}_{\check{d}_{ij}} = \left(\mathfrak{T}_{\check{d}_{ij}}, \mathfrak{J}_{\check{d}_{ij}}\right)$ be a q-ROFHSN, then

For each $i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$, we have, (4), as shown at the bottom of the next page.

Similarly, (5), as shown at the bottom of the next page.

Let $\check{d}_k = q - ROFHSIWA\left(\mathfrak{I}_{\check{d}_{11}}, \mathfrak{I}_{\check{d}_{12}}, \ldots, \mathfrak{I}_{\check{d}_{nm}}\right) = \left(\mathfrak{T}_{\check{d}_{ij}}, \mathfrak{J}_{\check{d}_{ij}}\right) = \mathfrak{I}_{\check{d}_k}$, then utilizing Equation 1.

$$\begin{aligned}
&\mathfrak{S}(\check{d}_k)\\
&= \mathfrak{T}^q_{\check{d}_k} - \mathfrak{J}^q_{\check{d}_k} + \left(\frac{e^{\mathfrak{T}^q_{\check{d}_k} - \mathfrak{J}^q_{\check{d}_k}}}{e^{\mathfrak{T}^q_{\check{d}_k} - \mathfrak{J}^q_{\check{d}_k}} + 1} - \frac{1}{2}\right)\pi^q_{\check{d}_k}\\
&\leq \left(\underset{j}{max}\,\underset{i}{max}\left\{\mathfrak{T}_{\check{d}_{ij}}\right\}\right)^q - \left(\underset{j}{min}\,\underset{i}{min}\left\{\mathfrak{J}_{\check{d}_{ij}}\right\}\right)^q\\
&\quad + \left(\frac{e^{\left(\underset{j}{max}\,\underset{i}{max}\left\{\mathfrak{T}_{\check{d}_{ij}}\right\}\right)^q - \left(\underset{j}{min}\,\underset{i}{min}\left\{\mathfrak{J}_{\check{d}_{ij}}\right\}\right)^q}}{e^{\left(\underset{j}{max}\,\underset{i}{max}\left\{\mathfrak{T}_{\check{d}_{ij}}\right\}\right)^q - \left(\underset{j}{min}\,\underset{i}{min}\left\{\mathfrak{J}_{\check{d}_{ij}}\right\}\right)^q} + 1} - \frac{1}{2}\right)\pi^q_{\mathfrak{I}^{+}_{\check{d}_{ij}}}\\
&= \mathfrak{S}(\mathfrak{I}^{+}_{\check{d}_{ij}})\\
&\Rightarrow \mathfrak{S}(\check{d}_k) \leq \mathfrak{S}(\mathfrak{I}^{+}_{\check{d}_{ij}})
\end{aligned}$$

and

$$\begin{aligned}
&\mathfrak{S}(\check{d}_k)\\
&= \mathfrak{T}^q_{\check{d}_k} - \mathfrak{J}^q_{\check{d}_k} + \left(\frac{e^{\mathfrak{T}^q_{\check{d}_k} - \mathfrak{J}^q_{\check{d}_k}}}{e^{\mathfrak{T}^q_{\check{d}_k} - \mathfrak{J}^q_{\check{d}_k}} + 1} - \frac{1}{2}\right)\pi^q_{\check{d}_k}\\
&\geq \left(\underset{j}{min}\,\underset{i}{min}\left\{\mathfrak{T}_{\check{d}_{ij}}\right\}\right)^q - \left(\underset{j}{max}\,\underset{i}{max}\left\{\mathfrak{J}_{\check{d}_{ij}}\right\}\right)^q\\
&\quad + \left(\frac{e^{\left(\underset{j}{min}\,\underset{i}{min}\left\{\mathfrak{T}_{\check{d}_{ij}}\right\}\right)^q - \left(\underset{j}{max}\,\underset{i}{max}\left\{\mathfrak{J}_{\check{d}_{ij}}\right\}\right)^q}}{e^{\left(\underset{j}{min}\,\underset{i}{min}\left\{\mathfrak{T}_{\check{d}_{ij}}\right\}\right)^q - \left(\underset{j}{max}\,\underset{i}{max}\left\{\mathfrak{J}_{\check{d}_{ij}}\right\}\right)^q} + 1} - \frac{1}{2}\right)\pi^q_{\mathfrak{I}^{-}_{\check{d}_{ij}}}\\
&= \mathfrak{S}(\mathfrak{I}^{-}_{\check{d}_{ij}}) \Rightarrow \mathfrak{S}(\check{d}_k) \geq \mathfrak{S}(\mathfrak{I}^{-}_{\check{d}_{ij}})
\end{aligned}$$

Seeing the above process, we have the consequent circumstances:

$$\begin{aligned}
q &= 3\\
&= \left\langle \begin{array}{l}\sqrt[3]{1 - \left(\begin{array}{l}\left\{(0.97)^{0.143}(0.49)^{0.514}(0.97)^{0.343}\right\}^{0.35}\left\{(0.94)^{0.143}(0.66)^{0.514}(0.88)^{0.343}\right\}^{0.15}\\ \left\{(0.97)^{0.143}(0.66)^{0.514}(0.78)^{0.343}\right\}^{0.2}\left\{(0.88)^{0.143}(0.66)^{0.514}(0.88)^{0.343}\right\}^{0.3}\end{array}\right)},\\ \sqrt[3]{\left\{\begin{array}{l}\left(\begin{array}{l}\left\{(0.97)^{0.143}(0.49)^{0.514}(0.97)^{0.343}\right\}^{0.35}\left\{(0.94)^{0.143}(0.66)^{0.514}(0.88)^{0.343}\right\}^{0.15}\\ \left\{(0.97)^{0.143}(0.66)^{0.514}(0.78)^{0.343}\right\}^{0.2}\left\{(0.88)^{0.143}(0.66)^{0.514}(0.88)^{0.343}\right\}^{0.3}\end{array}\right)\\ -\left(\begin{array}{l}\left\{(0.46)^{0.143}(0.46)^{0.514}(0.76)^{0.343}\right\}^{0.35}\left\{(0.72)^{0.143}(0.59)^{0.514}(0.53)^{0.343}\right\}^{0.15}\\ \left\{(0.76)^{0.143}(0.63)^{0.514}(0.66)^{0.343}\right\}^{0.2}\left\{(0.66)^{0.143}(0.15)^{0.514}(0.81)^{0.343}\right\}^{0.3}\end{array}\right)\end{array}\right\}}\end{array}\right\rangle\\
&= \left\langle \begin{array}{c}\sqrt[3]{1 - \{(0.8750)\,(0.9608)(0.7385)(0.7591)\}},\\ \sqrt[3]{\{(8750)\,(0.9608)(0.7385)(0.7591)\} - \{(0.5465)(0.5851)(0.9196)(0.3306)\}}\end{array}\right\rangle\\
&= \left\langle \sqrt[3]{1 - 0.4713}, \sqrt[3]{0.4713 - 0.0972}\right\rangle\\
&= \langle 0.8103, 0.7229\rangle .
\end{aligned}$$

$$\begin{aligned}
&q - ROFHSIWA\left(\mathfrak{I}_{\check{d}_{11}}, \mathfrak{I}_{\check{d}_{12}}, \ldots, \mathfrak{I}_{\check{d}_{nm}}\right)\\
&= \left\langle \sqrt[q]{1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \mathfrak{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \mathfrak{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1 - \left(\mathfrak{T}^q_{\check{d}_{ij}} + \mathfrak{J}^q_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j}}\right\rangle\\
&= \left\langle \sqrt[q]{1 - \left(\left(1 - \mathfrak{T}^q_{\check{d}_{ij}}\right)^{\sum_{i=1}^{n}\Omega_i}\right)^{\sum_{j=1}^{m}\gamma_j}}, \sqrt[q]{\left(\left(1 - \mathfrak{T}^q_{\check{d}_{ij}}\right)^{\sum_{i=1}^{n}\Omega_i}\right)^{\sum_{j=1}^{m}\gamma_j} - \left(\left[1 - \left(\mathfrak{T}^q_{\check{d}_{ij}} + \mathfrak{J}^q_{\check{d}_{ij}}\right)\right]^{\sum_{i=1}^{n}\Omega_i}\right)^{\sum_{j=1}^{m}\gamma_j}}\right\rangle\\
&= \left\langle \sqrt[q]{1 - \left(1 - \mathfrak{T}^q_{\check{d}_{ij}}\right)}, \sqrt[q]{\left(1 - \mathfrak{T}^q_{\check{d}_{ij}}\right) - \left[1 - \left(\mathfrak{T}^q_{\check{d}_{ij}} + \mathfrak{J}^q_{\check{d}_{ij}}\right)\right]}\right\rangle\\
&= \left\langle \sqrt[q]{\mathfrak{T}^q_{\check{d}_{ij}}}, \sqrt[q]{\mathfrak{J}^q_{\check{d}_{ij}}}\right\rangle = \left(\mathfrak{T}_{\check{d}_{ij}}, \mathfrak{J}_{\check{d}_{ij}}\right) = \mathfrak{I}_{\check{d}}.
\end{aligned}$$

If $\mathfrak{S}(\check{d}_k) < \mathfrak{S}(\mathfrak{J}^{+}_{\check{d}_{ij}})$ and $\mathfrak{S}(\check{d}_k) > \mathfrak{S}(\mathfrak{J}^{-}_{\check{d}_{ij}})$, then $\mathfrak{J}^{-}_{\check{d}_{ij}} < q-ROFHSIWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) < \mathfrak{J}^{+}_{\check{d}_{ij}}$.

If $\mathfrak{S}(\check{d}_k) = \mathfrak{S}(\mathfrak{J}^{+}_{\check{d}_{ij}})$, i.e.,

$$\mathcal{T}^q_{\check{d}_k} - \mathcal{J}^q_{\check{d}_k} + \left(\frac{e^{\mathcal{T}^q_{\check{d}_k} - \mathcal{J}^q_{\check{d}_k}}}{e^{\mathcal{T}^q_{\check{d}_k} - \mathcal{J}^q_{\check{d}_k}} + 1} - \frac{1}{2}\right)\pi^q_{\check{d}_k}$$
$$= \left(\max_j \max_i \left\{\mathcal{T}_{\check{d}_{ij}}\right\}\right)^q - \left(\min_j \min_i \left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right)^q + \left(\frac{e^{\left(\max_j \max_i \left\{\mathcal{T}_{\check{d}_{ij}}\right\}\right)^q - \left(\min_j \min_i \left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right)^q}}{e^{\left(\max_j \max_i \left\{\mathcal{T}_{\check{d}_{ij}}\right\}\right)^q - \left(\min_j \min_i \left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right)^q} + 1} - \frac{1}{2}\right)\pi^q_{\mathfrak{J}^{+}_{\check{d}_{ij}}},$$

utilizing the above inequalities, we obtain $\mathcal{T}_{\check{d}_k} = \max_j \max_i \left\{\mathcal{T}_{\check{d}_{ij}}\right\}$, and $\mathcal{J}_{\check{d}_k} = \min_j \min_i \left\{\mathcal{J}_{\check{d}_{ij}}\right\}$. Hence, $\pi^q_{\check{d}_k} = \pi^q_{\mathfrak{J}^{+}_{\check{d}_{ij}}}$. Then $q-ROFHSIWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) = \mathfrak{J}^{+}_{\check{d}_{ij}}$.

If $\mathfrak{S}(\check{d}_k) = \mathfrak{S}(\mathfrak{J}^{-}_{\check{d}_{ij}})$, i.e.,

$$\mathcal{T}^q_{\check{d}_k} - \mathcal{J}^q_{\check{d}_k} + \left(\frac{e^{\mathcal{T}^q_{\check{d}_k} - \mathcal{J}^q_{\check{d}_k}}}{e^{\mathcal{T}^q_{\check{d}_k} - \mathcal{J}^q_{\check{d}_k}} + 1} - \frac{1}{2}\right)\pi^q_{\check{d}_k}$$
$$= \left(\min_j \min_i \left\{\mathcal{T}_{\check{d}_{ij}}\right\}\right)^q - \left(\max_j \max_i \left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right)^q + \left(\frac{e^{\left(\min_j \min_i \left\{\mathcal{T}_{\check{d}_{ij}}\right\}\right)^q - \left(\max_j \max_i \left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right)^q}}{e^{\left(\min_j \min_i \left\{\mathcal{T}_{\check{d}_{ij}}\right\}\right)^q - \left(\max_j \max_i \left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right)^q} + 1} - \frac{1}{2}\right)\pi^q_{\mathfrak{J}^{+}_{\check{d}_{ij}}},$$

utilizing the above inequalities, we obtain $\mathcal{T}_{\check{d}_k} = \min_j \min_i \left\{\mathcal{T}_{\check{d}_{ij}}\right\}$, and $\mathcal{J}_{\check{d}_k} = \max_j \max_i \left\{\mathcal{J}_{\check{d}_{ij}}\right\}$. Hence, $\pi^q_{\check{d}_k} = \pi^q_{\mathfrak{J}^{-}_{\check{d}_{ij}}}$. Then by comparing two q-ROFHSNs, we obtain

$$q-ROFHSIWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) = \mathfrak{J}^{-}_{\check{d}_{ij}}.$$

So, it proved that

$$\mathfrak{J}^{-}_{\check{d}_{ij}} \leq q-ROFHSIWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) \leq \mathfrak{J}^{+}_{\check{d}_{ij}}.$$

3) SHIFT INVARIANCE

If $\mathfrak{J}_{\check{d}_k} = \left\langle \mathcal{T}_{\check{d}_k}, \mathcal{J}_{\check{d}_k} \right\rangle$ be a q-ROFHSN. Then,

$$q-ROFHSIWA\left(\mathfrak{J}_{\check{d}_{11}} \oplus \mathfrak{J}_{\check{d}_k}, \mathfrak{J}_{\check{d}_{12}} \oplus \mathfrak{J}_{\check{d}_k}, \ldots, \mathfrak{J}_{\check{d}_{nm}} \oplus \mathfrak{J}_{\check{d}_k}\right) = q-ROFHSIWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) \oplus \mathfrak{J}_{\check{d}_k}.$$

Proof: Consider $\mathfrak{J}_{\check{d}_k}$ and $\mathfrak{J}_{\check{d}_{ij}}$ be two q-ROFHSNs. Then by Definition 1 (1)

$$\mathfrak{J}_{\check{d}_k} \oplus \mathfrak{J}_{\check{d}_{ij}} = \left\langle \sqrt[q]{\mathcal{T}^q_{\check{d}_k} + \mathcal{T}^q_{\check{d}_{12}} - \mathcal{T}^q_{\check{d}_k}\mathcal{T}^q_{\check{d}_{12}}}, \sqrt[q]{\mathcal{T}^q_{\check{d}_k} + \mathcal{T}^q_{\check{d}_{12}} - \mathcal{T}^q_{\check{d}_k}\mathcal{T}^q_{\check{d}_{12}} - \mathcal{J}^q_{\check{d}_k}\mathcal{T}^q_{\check{d}_{12}} - \mathcal{T}^q_{\check{d}_k}\mathcal{J}^q_{\check{d}_{12}}} \right\rangle$$

$$\begin{aligned}
&\min_j \min_i \left\{\mathcal{T}^q_{\check{d}_{ij}}\right\} \leq \mathcal{T}^q_{\check{d}_{ij}} \leq \max_j \max_i \left\{\mathcal{T}^q_{\check{d}_{ij}}\right\} \\
&\Rightarrow 1 - \max_j \max_i \left\{\mathcal{T}^q_{\check{d}_{ij}}\right\} \leq 1 - \mathcal{T}^q_{\check{d}_{ij}} \leq 1 - \min_j \min_i \left\{\mathcal{T}^q_{\check{d}_{ij}}\right\} \\
&\Leftrightarrow \left(1 - \max_j \max_i \left\{\mathcal{T}^q_{\check{d}_{ij}}\right\}\right)^{\Omega_i} \leq \left(1 - \mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i} \leq \left(1 - \min_j \min_i \left\{\mathcal{T}^q_{\check{d}_{ij}}\right\}\right)^{\Omega_i} \\
&\Leftrightarrow \left(1 - \max_j \max_i \left\{\mathcal{T}^q_{\check{d}_{ij}}\right\}\right)^{\sum_{i=1}^n \Omega_i} \leq \prod_{i=1}^n \left(1 - \mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i} \leq \left(1 - \min_j \min_i \left\{\mathcal{T}^q_{\check{d}_{ij}}\right\}\right)^{\sum_{i=1}^n \Omega_i} \\
&\Leftrightarrow \left(1 - \max_j \max_i \left\{\mathcal{T}^q_{\check{d}_{ij}}\right\}\right)^{\sum_{j=1}^m \gamma_j} \leq \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j} \leq \left(1 - \min_j \min_i \left\{\mathcal{T}^q_{\check{d}_{ij}}\right\}\right)^{\sum_{j=1}^m \gamma_j} \\
&\Leftrightarrow 1 - \max_j \max_i \left\{\mathcal{T}^q_{\check{d}_{ij}}\right\} \leq \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j} \leq 1 - \min_j \min_i \left\{\mathcal{T}^q_{\check{d}_{ij}}\right\} \\
&\Leftrightarrow \min_j \min_i \left\{\mathcal{T}^q_{\check{d}_{ij}}\right\} \leq 1 - \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j} \leq \max_j \max_i \left\{\mathcal{T}^q_{\check{d}_{ij}}\right\} \\
&\Leftrightarrow \min_j \min_i \left\{\mathcal{T}_{\check{d}_{ij}}\right\} \leq \sqrt[q]{1 - \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}} \leq \max_j \max_i \left\{\mathcal{T}_{\check{d}_{ij}}\right\} \qquad (4)
\end{aligned}$$

$$\min_j \min_i \left\{\mathcal{J}_{\check{d}_{ij}}\right\} \leq \sqrt[q]{\prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^m \left(\prod_{i=1}^n \left[1 - \left(\mathcal{T}^q_{\check{d}_{ij}} + \mathcal{J}^q_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j}} \leq \max_j \max_i \left\{\mathcal{J}_{\check{d}_{ij}}\right\}. \qquad (5)$$

Therefore, as shown in the equation at the bottom of the page.

4) HOMOGENEITY

Prove that $q-ROFHSWA\left(\alpha\mathfrak{J}_{\check{d}_{11}}, \alpha\mathfrak{J}_{\check{d}_{12}}, \ldots, \alpha\mathfrak{J}_{\check{d}_{nm}}\right) = \alpha q-ROFHSIWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right)$ for any $\alpha > 0$.

Proof: Let $\mathfrak{J}_{\check{d}_{ij}}$ be a q-ROFHSN and $\alpha > 0$, then by using Definition 1 (3), we have

$$\alpha\mathfrak{J}_{\check{d}_k} = \left\langle \sqrt{1-\left(1-\mathcal{T}^q_{\check{d}_k}\right)^{\alpha}}, \sqrt{\left(1-\mathcal{T}^q_{\check{d}_k}\right)^{\alpha} - \left[1-\left(\mathcal{T}^q_{\check{d}_k}+\mathcal{I}^q_{\check{d}_k}\right)\right]^{\alpha}} \right\rangle.$$

So, as shown in the equation at the bottom of the page.

5) DEFINITION

Let $\mathfrak{J}_{\check{d}_{ij}} = \left(\mathcal{T}_{\check{d}_{ij}}, \mathcal{I}_{\check{d}_{ij}}\right)$ be a q-ROFHSN, Ω_i and γ_j are weights for specialists and multi sub-attributes of the deliberated aspects consistently along with stated environments $\Omega_i > 0$, $\sum_{i=1}^{n}\Omega_i = 1$, $\gamma_j > 0$, $\sum_{j=1}^{m}\gamma_j = 1$. Then the q-ROFHSIWG operator can be defined as follows:

q-ROFHSIWG: $\Delta^n \rightarrow \Delta$ defined as follows

$$q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) = \otimes_{j=1}^{m}\left(\otimes_{i=1}^{n}\mathfrak{J}_{\check{d}_{nm}}^{\Omega_i}\right)^{\gamma_j} \quad (6)$$

6) THEOREM

Let $\mathfrak{J}_{\check{d}_{ij}} = \left(\mathcal{T}_{\check{d}_{ij}}, \mathcal{I}_{\check{d}_{ij}}\right)$ be a q-ROFHSN. Then, attained collected values with Equation 6 is also a q-ROFHSN and (7), as shown at the bottom of the next page, Ω_i and γ_j are

$$\begin{aligned}
&q-ROFHSIWA\left(\mathfrak{J}_{\check{d}_{11}} \oplus \mathfrak{J}_{\check{d}_k}, \mathfrak{J}_{\check{d}_{12}} \oplus \mathfrak{J}_{\check{d}_k}, \ldots, \mathfrak{J}_{\check{d}_{nm}} \oplus \mathfrak{J}_{\check{d}_k}\right) \\
&= \oplus_{j=1}^{m}\gamma_j\left(\oplus_{i=1}^{n}\Omega_i\left(\mathfrak{J}_{\check{d}_{ij}} \oplus \mathfrak{J}_{\check{d}_k}\right)\right) \\
&= \left\langle \begin{array}{c} \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\left(1-\mathcal{T}^q_{\check{d}_k}\right)^{\Omega_i}\right)^{\gamma_j}}, \\ \sqrt[q]{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\left(1-\mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{I}^q_{\check{d}_{ij}}\right)\right]^{\Omega_i}\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{I}^q_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j}} \end{array} \right\rangle \\
&= \left\langle \begin{array}{c} \sqrt[q]{1-\left(1-\mathcal{T}^q_{\check{d}_k}\right)\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}}, \\ \sqrt[q]{\left(1-\mathcal{T}^q_{\check{d}_{ij}}\right)\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j} - \left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{I}^q_{\check{d}_{ij}}\right)\right]\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{I}^q_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j}} \end{array} \right\rangle \\
&= \left\langle \begin{array}{c} \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}}, \\ \sqrt[q]{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{I}^q_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j}} \end{array} \right\rangle \oplus \left\langle \mathcal{T}_{\check{d}_k}, \mathcal{I}_{\check{d}_k} \right\rangle \\
&= q-ROFHSIWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) \oplus \mathfrak{J}_{\check{d}_k}.
\end{aligned}$$

$$\begin{aligned}
&q-ROFHSIWA\left(\alpha\mathfrak{J}_{\check{d}_{11}}, \alpha\mathfrak{J}_{\check{d}_{12}}, \ldots, \alpha\mathfrak{J}_{\check{d}_{nm}}\right) \\
&= \left\langle \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}^q_{\check{d}_{ij}}\right)^{\alpha\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}^q_{\check{d}_{ij}}\right)^{\alpha\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{I}^q_{\check{d}_{ij}}\right)\right]^{\alpha\Omega_i}\right)^{\gamma_j}} \right\rangle \\
&\times \left\langle \sqrt[q]{1-\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}\right)^{\alpha}}, \right. \\
&\left. \sqrt[q]{\prod_{j-1}^{m}\left(\left(\prod_{i=1}^{n}\left(1-\mathcal{T}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}\right)^{\alpha} - \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{I}^q_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j}\right)^{\alpha}} \right\rangle \\
&= \alpha q-ROFHSIWA\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right).
\end{aligned}$$

weights for specialists and multi sub-attributes of the deliberated parameters consistently along with stated environments $\Omega_i > 0, \sum_{i=1}^{n} \Omega_i = 1, \gamma_j > 0, \sum_{j=1}^{m} \gamma_j = 1$.

Proof: By using mathematical induction.

For $n = 1$, we get $\Omega_1 = 1$. Then, we have, as shown in the equation at the bottom of the page.

For $m = 1$, we have $\gamma_1 = 1$. Then, as shown in the equation at the bottom of the page.

For $n = 1$ and $m = 1$, Equation 7 fulfills the q-ROFHSIWG operator. Let Equation 7 holds for $m = \beta_1 + 1$, $n = \beta_2$ and $m = \beta_1$, $n = \beta_2 + 1$, such as, shown in the equation at the bottom of the page.

For $m = \beta_1 + 1$ and $n = \beta_2 + 1$, we have, as shown in the equation at the bottom of the next page.

Hence, it is true for $m = \beta_1 + 1$ and $n = \beta_2 + 1$.

7) EXAMPLE

Let $\mathfrak{U} = \{u_1, u_2, u_3\}$ represents the set of experts with weights $\Omega_i = (0.143, 0.514, 0.343)^T$. Professionals precise the attractiveness of a community under a defined set of attributes $\mathfrak{L}' = \{d_1 = lawn, d_2 = security\ system\}$ with their conforming sub-attributes, Lawn $= d_1 = \{d_{11} = with\ grass, d_{12} = without\ grass\}$. Security system $= d_2 = \{d_{21} = guards, d_{22} = cameras\}$. Let $\mathfrak{L}' = d_1 \times d_2$ be a set of multi sub-attributes $\mathfrak{L}' = d_1 \times d_2 = \{d_{11}, d_{12}\} \times \{d_{21}, d_{22}\} = \left\{ (d_{11}, d_{21}), (d_{11}, d_{22}), (d_{12}, d_{21}), (d_{12}, d_{22}) \right\}$ $\mathfrak{L}' = \left\{ \check{d}_1, \check{d}_2, \check{d}_3, \check{d}_4 \right\}$ designates the set of multi sub-attributes with their weights $\gamma_j = (0.35, 0.15, 0.2, 0.3)^T$. Expert's judgment for each multi-sub-attribute in the form of q-ROFHSNs $(\mathfrak{J}, \mathfrak{L}') = \left\langle \mathfrak{T}_{\check{d}_{ij}}, \mathfrak{I}_{\check{d}_{ij}} \right\rangle_{3\times 4}$ given

$$q-ROFHSIWG\left(\aleph_{e_{11}}, \aleph_{e_{12}}, \ldots, \aleph_{e_{nm}}\right) = \otimes_{j=1}^{m}\left(\otimes_{i=1}^{n}\mathfrak{J}_{\check{d}_{nm}}^{\Omega_i}\right)^{\gamma_j}$$
$$= \left\langle \sqrt[q]{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathfrak{I}_{\check{d}_{ij}}^{q}\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathfrak{T}_{\check{d}_{ij}}^{q}+\mathfrak{I}_{\check{d}_{ij}}^{q}\right)\right]^{\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathfrak{I}_{\check{d}_{ij}}^{q}\right)^{\Omega_i}\right)^{\gamma_j}} \right\rangle \tag{7}$$

$$q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) = \otimes_{j=1}^{m}\mathfrak{J}_{\check{d}_{1j}}^{\gamma_j}$$
$$q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right)$$
$$= \left\langle \sqrt[q]{\prod_{j=1}^{m}\left(1-\mathfrak{I}_{\check{d}_{1j}}^{q}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(1-\left(\mathfrak{T}_{\check{d}_{1j}}^{q}+\mathfrak{I}_{\check{d}_{1j}}^{q}\right)\right)^{\gamma_j}}, \sqrt[q]{1-\prod_{j=1}^{m}\left(1-\mathfrak{I}_{\check{d}_{1j}}^{q}\right)^{\gamma_j}} \right\rangle$$
$$= \left\langle \sqrt[q]{\prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1-\mathfrak{I}_{\check{d}_{ij}}^{q}\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left[1-\left(\mathfrak{T}_{\check{d}_{ij}}^{q}+\mathfrak{I}_{\check{d}_{ij}}^{q}\right)\right]^{\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1-\mathfrak{I}_{\check{d}_{ij}}^{q}\right)^{\Omega_i}\right)^{\gamma_j}} \right\rangle$$

$$q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right)$$
$$= \otimes_{i=1}^{n}\left(\mathfrak{J}_{\check{d}_{i1}}\right)^{\Omega_i}$$
$$= \left\langle \sqrt[q]{\prod_{i=1}^{n}\left(1-\mathfrak{I}_{\check{d}_{i1}}^{q}\right)^{\Omega_i} - \prod_{i=1}^{n}\left[1-\left(\mathfrak{T}_{\check{d}_{i1}}^{q}+\mathfrak{I}_{\check{d}_{i1}}^{q}\right)\right]^{\Omega_i}}, \sqrt[q]{1-\prod_{i=1}^{n}\left(1-\mathfrak{I}_{\check{d}_{i1}}^{q}\right)^{\Omega_i}} \right\rangle$$
$$= \left\langle \sqrt[q]{\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(1-\mathfrak{I}_{\check{d}_{ij}}^{q}\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left[1-\left(\mathfrak{T}_{\check{d}_{ij}}^{q}+\mathfrak{I}_{\check{d}_{ij}}^{q}\right)\right]^{\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{1-\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(1-\mathfrak{I}_{\check{d}_{ij}}^{q}\right)^{\Omega_i}\right)^{\gamma_j}} \right\rangle$$

$$\otimes_{j=1}^{\beta_1+1}\left(\otimes_{i=1}^{\beta_2}\left(\mathfrak{J}_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}$$
$$= \left\langle \sqrt[q]{\prod_{j=1}^{\beta_1+1}\left(\prod_{i=1}^{\beta_2}\left(1-\mathfrak{I}_{\check{d}_{ij}}^{q}\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{\beta_1+1}\left(\prod_{i=1}^{\beta_2}\left[1-\left(\mathfrak{T}_{\check{d}_{ij}}^{q}+\mathfrak{I}_{\check{d}_{ij}}^{q}\right)\right]^{\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{1-\prod_{j=1}^{\beta_1+1}\left(\prod_{i=1}^{\beta_2}\left(1-\mathfrak{I}_{\check{d}_{ij}}^{q}\right)^{\Omega_i}\right)^{\gamma_j}} \right\rangle$$
$$\otimes_{j=1}^{\beta_1}\left(\otimes_{i=1}^{\beta_2}\left(\mathfrak{J}_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}$$
$$= \left\langle \sqrt[q]{\prod_{j=1}^{\beta_1}\left(\prod_{i=1}^{\beta_2+1}\left(1-\mathfrak{I}_{\check{d}_{ij}}^{q}\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{\beta_1}\left(\prod_{i=1}^{\beta_2+1}\left[1-\left(\mathfrak{T}_{\check{d}_{ij}}^{q}+\mathfrak{I}_{\check{d}_{ij}}^{q}\right)\right]^{\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{1-\prod_{j=1}^{\beta_1}\left(\prod_{i=1}^{\beta_2+1}\left(1-\mathfrak{I}_{\check{d}_{ij}}^{q}\right)^{\Omega_i}\right)^{\gamma_j}} \right\rangle$$

as follows:

$$(\mathfrak{J},\mathfrak{L}')=\begin{bmatrix}(.3,.8) & (.4,.6) & (.3,.6) & (.5,.6)\\ (.8,.3) & (.7,.4) & (.7,.3) & (.7,.8)\\ (.3,.6) & (.5,.7) & (.6,.5) & (.5,.4)\end{bmatrix}$$

Using Eq. 7, as shown in the equation at the bottom of the page.

For, as shown in the equation at the bottom of the page.

Demonstrated specific properties for q-ROFHSNs of the q-ROFHSIWG operator by Equation 7.

B. PROPERTIES OF Q-ROFHSIWG OPERATOR

1) IDEMPOTENCY

$\mathfrak{J}_{\check{d}_{ij}}=\mathfrak{J}_{\check{d}}=\left(\mathcal{T}_{\check{d}_{ij}},\mathcal{J}_{\check{d}_{ij}}\right)\forall i,j$, then, $q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{nm}}\right)=\mathfrak{J}_{\check{d}_{\delta}}$.

Proof: As we know that, as shown in the equation at the bottom of the next page.

2) BOUNDEDNESS

Let $\mathfrak{J}_{\check{d}_{ij}}$ be a collection of q-ROFHSNs and $\mathfrak{J}^{-}_{\check{d}_{ij}}=\left\langle \min_j\min_i\left\{\mathcal{T}_{\check{d}_{ij}}\right\},\max_j\max_i\left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right\rangle$ and $\mathfrak{J}^{+}_{\check{d}_{ij}}=\left\langle \max_j\max_i\left\{\mathcal{T}_{\check{d}_{ij}}\right\},\min_j\min_i\left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right\rangle$, then $\mathfrak{J}^{-}_{\check{d}_{ij}}\le q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{nm}}\right)\le\mathfrak{J}^{+}_{\check{d}_{ij}}$.

Proof: As we know that $\mathfrak{J}_{\check{d}_{ij}}=\left(\mathcal{T}_{\check{d}_{ij}},\mathcal{J}_{\check{d}_{ij}}\right)$ be a q-ROFHSN, then, (8), as shown at the bottom of the next page.

Similarly, (9), as shown at the bottom of the next page.

$$\begin{aligned}
&\otimes_{j=1}^{\beta_1+1}\left(\otimes_{i=1}^{\beta_2+1}\left(\mathfrak{J}_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}\\
&=\otimes_{j=1}^{\beta_1+1}\left(\otimes_{i=1}^{\beta_2}\left(\mathfrak{J}_{\check{d}_{ij}}\right)^{\Omega_i}\otimes\left(\mathfrak{J}_{\check{d}_{(\beta_2+1)j}}\right)^{\Omega_{\beta_2+1}}\right)^{\gamma_j}\\
&=\otimes_{j=1}^{\beta_1+1}\left(\otimes_{i=1}^{\beta_2}\left(\mathfrak{J}_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}\otimes_{j=1}^{\beta_1+1}\left(\left(\mathfrak{J}_{\check{d}_{(\beta_2+1)j}}\right)^{\Omega_{\beta_2+1}}\right)^{\gamma_j}\\
&=\left\langle\begin{array}{c}\sqrt[q]{\prod_{j=1}^{\beta_1+1}\left(\prod_{i=1}^{\beta_2}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}-\prod_{j=1}^{\beta_1+1}\left(\prod_{i=1}^{\beta_2}\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{J}^q_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j}}\otimes\sqrt[q]{\prod_{j=1}^{\beta_1+1}\left(\left(1-\mathcal{J}^q_{\check{d}_{(\beta_2+1)j}}\right)^{\Omega_{(\beta_2+1)j}}\right)^{\gamma_j}-\prod_{j=1}^{\beta_1+1}\left(\left[1-\left(\mathcal{T}^q_{\check{d}_{(\beta_2+1)j}}+\mathcal{J}^q_{\check{d}_{(\beta_2+1)j}}\right)\right]^{\Omega_{(\beta_2+1)j}}\right)^{\gamma_j}},\\ \sqrt[q]{1-\prod_{j=1}^{\beta_1+1}\left(\prod_{i=1}^{\beta_2}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}}\otimes\sqrt[q]{1-\prod_{j=1}^{\beta_1+1}\left(\left(1-\mathcal{J}^q_{\check{d}_{(\beta_2+1)j}}\right)^{\Omega_{\beta_2+1}}\right)^{\gamma_j}}\end{array}\right\rangle\\
&=\left\langle\sqrt[q]{\prod_{j=1}^{\beta_1+1}\left(\prod_{i=1}^{\beta_2+1}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}-\prod_{j=1}^{\beta_1+1}\left(\prod_{i=1}^{\beta_2+1}\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{J}^q_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j}},\sqrt[q]{1-\prod_{j=1}^{\beta_1+1}\left(\prod_{i=1}^{\beta_2+1}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}}\right\rangle
\end{aligned}$$

$$\begin{aligned}
&q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{34}}\right)\\
&=\left\langle\sqrt[q]{\prod_{j=1}^{4}\left(\prod_{i=1}^{3}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}-\prod_{j=1}^{4}\left(\prod_{i=1}^{3}\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{J}^q_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j}},\sqrt[q]{1-\prod_{j=1}^{4}\left(\prod_{i=1}^{3}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}}\right\rangle
\end{aligned}$$

$$\begin{aligned}
&q=3\\
&=\left\langle\begin{array}{c}\sqrt[3]{\left\{\begin{array}{c}\left(\begin{array}{c}\left\{(0.49)^{0.143}(0.97)^{0.514}(0.78)^{0.343}\right\}^{0.35}\left\{(0.78)^{0.143}(0.06)^{0.514}(0.34)^{0.343}\right\}^{0.15}\\ \left\{(0.78)^{0.143}(0.97)^{0.514}(0.88)^{0.343}\right\}^{0.2}\left\{(0.78)^{0.143}(0.49)^{0.514}(0.94)^{0.343}\right\}^{0.3}\end{array}\right)\\ -\left(\begin{array}{c}\left\{(0.46)^{0.143}(0.46)^{0.514}(0.76)^{0.343}\right\}^{0.35}\left\{(0.72)^{0.143}(0.59)^{0.514}(0.53)^{0.343}\right\}^{0.15}\\ \left\{(0.76)^{0.143}(0.63)^{0.514}(0.66)^{0.343}\right\}^{0.2}\left\{(0.66)^{0.143}(0.15)^{0.514}(0.81)^{0.343}\right\}^{0.3}\end{array}\right)\end{array}\right\}},\\ \sqrt[3]{1-\left(\begin{array}{c}\left\{(0.49)^{0.143}(0.97)^{0.514}(0.78)^{0.343}\right\}^{0.35}\left\{(0.78)^{0.143}(0.06)^{0.514}(0.34)^{0.343}\right\}^{0.15}\\ \left\{(0.78)^{0.143}(0.97)^{0.514}(0.88)^{0.343}\right\}^{0.2}\left\{(0.78)^{0.143}(0.49)^{0.514}(0.94)^{0.343}\right\}^{0.3}\end{array}\right)}\end{array}\right\rangle\\
&=\left\langle\begin{array}{c}\sqrt[3]{\{(0.9315)(0.7575)(0.9812)(0.7429)\}-\{(0.5465)(0.5851)(0.9196)(0.3306)\}},\\ \sqrt[3]{1-\{(0.9315)(0.7575)(0.9812)(0.7429)\}}\end{array}\right\rangle\\
&=\left\langle\sqrt[3]{0.5143-0.0972},\sqrt[3]{1-0.5143}\right\rangle\\
&=\langle 0.7493,0.7879\rangle.
\end{aligned}$$

Let $\check{d}_k = q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) = \left\langle \mathcal{T}_{\check{d}_k}, \mathcal{J}_{\check{d}_k} \right\rangle = \mathfrak{J}_{\check{d}_k}$, then utilizing Equation 1.

$$\mathfrak{S}(\check{d}_k) = \mathcal{T}^q_{\check{d}_k} - \mathcal{J}^q_{\check{d}_k} + \left(\frac{e^{\mathcal{T}^q_{\check{d}_k} - \mathcal{J}^q_{\check{d}_k}}}{e^{\mathcal{T}^q_{\check{d}_k} - \mathcal{J}^q_{\check{d}_k}} + 1} - \frac{1}{2} \right) \pi^q_{\check{d}_k} \leq \left(\max_j \max_i \left\{ \mathcal{T}_{\check{d}_{ij}} \right\} \right)^q - \left(\min_j \min_i \left\{ \mathcal{J}_{\check{d}_{ij}} \right\} \right)^q + \left(\frac{e^{\left(\max_j \max_i \left\{ \mathcal{T}_{\check{d}_{ij}} \right\} \right)^q - \left(\min_j \min_i \left\{ \mathcal{J}_{\check{d}_{ij}} \right\} \right)^q}}{e^{\left(\max_j \max_i \left\{ \mathcal{T}_{\check{d}_{ij}} \right\} \right)^q - \left(\min_j \min_i \left\{ \mathcal{J}_{\check{d}_{ij}} \right\} \right)^q} + 1} - \frac{1}{2} \right) \pi^q_{\mathfrak{J}^+_{\check{d}_{ij}}}$$

$$= \mathfrak{S}(\mathfrak{J}^+_{\check{d}_{ij}}) \Rightarrow \mathfrak{S}(\check{d}_k) \leq \mathfrak{S}(\mathfrak{J}^+_{\check{d}_{ij}})$$

and

$$\mathfrak{S}(\check{d}_k) = \mathcal{T}^q_{\check{d}_k} - \mathcal{J}^q_{\check{d}_k} + \left(\frac{e^{\mathcal{T}^q_{\check{d}_k} - \mathcal{J}^q_{\check{d}_k}}}{e^{\mathcal{T}^q_{\check{d}_k} - \mathcal{J}^q_{\check{d}_k}} + 1} - \frac{1}{2} \right) \pi^q_{\check{d}_k} \geq \left(\min_j \min_i \left\{ \mathcal{T}_{\check{d}_{ij}} \right\} \right)^q - \left(\max_j \max_i \left\{ \mathcal{J}_{\check{d}_{ij}} \right\} \right)^q$$

$$q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) = \left\langle \sqrt[q]{\prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \mathcal{J}^q_{\check{d}_{ij}} \right)^{\Omega_i} \right)^{\gamma_j} - \prod_{j=1}^m \left(\prod_{i=1}^n \left[1 - \left(\mathcal{T}^q_{\check{d}_{ij}} + \mathcal{J}^q_{\check{d}_{ij}} \right) \right]^{\Omega_i} \right)^{\gamma_j}}, \sqrt[q]{1 - \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \mathcal{J}^q_{\check{d}_{ij}} \right)^{\Omega_i} \right)^{\gamma_j}} \right\rangle$$

$$q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right) = \left\langle \sqrt[q]{\left(\left(1 - \mathcal{J}^q_{\check{d}_{ij}} \right)^{\sum_{i=1}^n \Omega_i} \right)^{\sum_{j=1}^m \gamma_j} - \left(\left[1 - \left(\mathcal{T}^q_{\check{d}_{ij}} + \mathcal{J}^q_{\check{d}_{ij}} \right) \right]^{\sum_{i=1}^n \Omega_i} \right)^{\sum_{j=1}^m \gamma_j}}, \sqrt[q]{1 - \left(\left(1 - \mathcal{J}^q_{\check{d}_{ij}} \right)^{\sum_{i=1}^n \Omega_i} \right)^{\sum_{j=1}^m \gamma_j}} \right\rangle$$
$$= \left\langle \sqrt[q]{\left(1 - \mathcal{J}^q_{\check{d}_{ij}} \right) - \left[1 - \left(\mathcal{T}^q_{\check{d}_{ij}} + \mathcal{J}^q_{\check{d}_{ij}} \right) \right]}, \sqrt[q]{1 - \left(1 - \mathcal{J}^q_{\check{d}_{ij}} \right)} \right\rangle$$
$$= \left\langle \mathcal{T}_{\check{d}_{ij}}, \mathcal{J}_{\check{d}_{ij}} \right\rangle = \mathfrak{J}_{\check{d}_{ij}}.$$

$$\min_j \min_i \left\{ \mathcal{J}^q_{\check{d}_{ij}} \right\} \leq \mathcal{J}^q_{\check{d}_{ij}} \leq \max_j \max_i \left\{ \mathcal{J}^q_{\check{d}_{ij}} \right\}$$
$$\Rightarrow 1 - \max_j \max_i \left\{ \mathcal{J}^q_{\check{d}_{ij}} \right\} \leq 1 - \mathcal{J}^q_{\check{d}_{ij}} \leq 1 - \min_j \min_i \left\{ \mathcal{J}^q_{\check{d}_{ij}} \right\}$$
$$\Leftrightarrow \left(1 - \max_j \max_i \left\{ \mathcal{J}^q_{\check{d}_{ij}} \right\} \right)^{\Omega_i} \leq \left(1 - \mathcal{J}^q_{\check{d}_{ij}} \right)^{\Omega_i} \leq \left(1 - \min_j \min_i \left\{ \mathcal{J}^q_{\check{d}_{ij}} \right\} \right)^{\Omega_i}$$
$$\Leftrightarrow \left(1 - \max_j \max_i \left\{ \mathcal{J}^q_{\check{d}_{ij}} \right\} \right)^{\sum_{i=1}^n \Omega_i} \leq \prod_{i=1}^n \left(1 - \mathcal{J}^q_{\check{d}_{ij}} \right)^{\Omega_i} \leq \left(1 - \min_j \min_i \left\{ \mathcal{J}^q_{\check{d}_{ij}} \right\} \right)^{\sum_{i=1}^n \Omega_i}$$
$$\Leftrightarrow \left(1 - \max_j \max_i \left\{ \mathcal{J}^q_{\check{d}_{ij}} \right\} \right)^{\sum_{j=1}^m \gamma_j} \leq \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \mathcal{J}^q_{\check{d}_{ij}} \right)^{\Omega_i} \right)^{\gamma_j} \leq \left(1 - \min_j \min_i \left\{ \mathcal{J}^q_{\check{d}_{ij}} \right\} \right)^{\sum_{j=1}^m \gamma_j}$$
$$\Leftrightarrow 1 - \max_j \max_i \left\{ \mathcal{J}^q_{\check{d}_{ij}} \right\} \leq \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \mathcal{J}^q_{\check{d}_{ij}} \right)^{\Omega_i} \right)^{\gamma_j} \leq 1 - \min_j \min_i \left\{ \mathcal{J}^q_{\check{d}_{ij}} \right\}$$
$$\Leftrightarrow \min_j \min_i \left\{ \mathcal{J}^q_{\check{d}_{ij}} \right\} \leq 1 - \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \mathcal{J}^q_{\check{d}_{ij}} \right)^{\Omega_i} \right)^{\gamma_j} \leq \max_j \max_i \left\{ \mathcal{J}^q_{\check{d}_{ij}} \right\}$$
$$\Leftrightarrow \min_j \min_i \left\{ \mathcal{J}_{\check{d}_{ij}} \right\} \leq \sqrt[q]{1 - \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \mathcal{J}^q_{\check{d}_{ij}} \right)^{\Omega_i} \right)^{\gamma_j}} \leq \max_j \max_i \left\{ \mathcal{J}_{\check{d}_{ij}} \right\} \tag{8}$$

$$\min_j \min_i \left\{ \mathcal{T}_{\check{d}_{ij}} \right\} \leq \sqrt[q]{\prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \mathcal{J}^q_{\check{d}_{ij}} \right)^{\Omega_i} \right)^{\gamma_j} - \prod_{j=1}^m \left(\prod_{i=1}^n \left[1 - \left(\mathcal{T}^q_{\check{d}_{ij}} + \mathcal{J}^q_{\check{d}_{ij}} \right) \right]^{\Omega_i} \right)^{\gamma_j}} \leq \max_j \max_i \left\{ \mathcal{T}_{\check{d}_{ij}} \right\}. \tag{9}$$

$$+\left(\frac{e^{\left(\min_j\min_i\left\{\mathcal{T}_{\check{d}_{ij}}\right\}\right)^q-\left(\max_j\max_i\left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right)^q}}{e^{\left(\min_j\min_i\left\{\mathcal{T}_{\check{d}_{ij}}\right\}\right)^q-\left(\max_j\max_i\left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right)^q}+1}-\frac{1}{2}\right)\pi^q_{\mathfrak{J}^-_{\check{d}_{ij}}}$$
$$=\mathfrak{S}(\mathfrak{J}^-_{\check{d}_{ij}})$$
$$\Rightarrow \mathfrak{S}(\check{d}_k)\geq\mathfrak{S}(\mathfrak{J}^-_{\check{d}_{ij}})$$

Seeing the overhead process, we have the subsequent cases:

If $\mathfrak{S}(\check{d}_k)<\mathfrak{S}(\mathfrak{J}^+_{\check{d}_{ij}})$ and $\mathfrak{S}(\check{d}_k)>\mathfrak{S}(\mathfrak{J}^-_{\check{d}_{ij}})$, then by comparing two q-ROFHSNs, we obtain

$$\mathfrak{J}^-_{\check{d}_{ij}}<q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{nm}}\right)<\mathfrak{J}^+_{\check{d}_{ij}}.$$

If $\mathfrak{S}(\check{d}_k)=\mathfrak{S}(\mathfrak{J}^+_{\check{d}_{ij}})$, i.e.,

$$\mathcal{T}^q_{\check{d}_k}-\mathcal{J}^q_{\check{d}_k}+\left(\frac{e^{\mathcal{T}^q_{\check{d}_k}-\mathcal{J}^q_{\check{d}_k}}}{e^{\mathcal{T}^q_{\check{d}_k}-\mathcal{J}^q_{\check{d}_k}}+1}-\frac{1}{2}\right)\pi^q_{\check{d}_k}$$
$$=\left(\max_j\max_i\left\{\mathcal{T}_{\check{d}_{ij}}\right\}\right)^q-\left(\min_j\min_i\left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right)^q$$
$$+\left(\frac{e^{\left(\max_j\max_i\left\{\mathcal{T}_{\check{d}_{ij}}\right\}\right)^q-\left(\min_j\min_i\left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right)^q}}{e^{\left(\max_j\max_i\left\{\mathcal{T}_{\check{d}_{ij}}\right\}\right)^q-\left(\min_j\min_i\left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right)^q}+1}-\frac{1}{2}\right)\pi^q_{\mathfrak{J}^+_{\check{d}_{ij}}},$$

we get $\mathcal{T}_{\check{d}_k}=\max_j\max_i\left\{\mathcal{T}_{\check{d}_{ij}}\right\}$, and $\mathcal{J}_{\check{d}_k}=\min_j\min_i\left\{\mathcal{J}_{\check{d}_{ij}}\right\}$. Hence, $\pi^q_{\check{d}_k}=\pi^q_{\mathfrak{J}^+_{\check{d}_{ij}}}$. Then by comparing two q-ROFHSNs, we obtain

$$q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{nm}}\right)=\mathfrak{J}^+_{\check{d}_{ij}}.$$

If $\mathfrak{S}(\check{d}_k)=\mathfrak{S}(\mathfrak{J}^-_{\check{d}_{ij}})$, i.e.,

$$\mathcal{T}^q_{\check{d}_k}-\mathcal{J}^q_{\check{d}_k}+\left(\frac{e^{\mathcal{T}^q_{\check{d}_k}-\mathcal{J}^q_{\check{d}_k}}}{e^{\mathcal{T}^q_{\check{d}_k}-\mathcal{J}^q_{\check{d}_k}}+1}-\frac{1}{2}\right)\pi^q_{\check{d}_k}$$
$$=\left(\min_j\min_i\left\{\mathcal{T}_{\check{d}_{ij}}\right\}\right)^q-\left(\max_j\max_i\left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right)^q$$
$$+\left(\frac{e^{\left(\min_j\min_i\left\{\mathcal{T}_{\check{d}_{ij}}\right\}\right)^q-\left(\max_j\max_i\left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right)^q}}{e^{\left(\min_j\min_i\left\{\mathcal{T}_{\check{d}_{ij}}\right\}\right)^q-\left(\max_j\max_i\left\{\mathcal{J}_{\check{d}_{ij}}\right\}\right)^q}+1}-\frac{1}{2}\right)\pi^q_{\mathfrak{J}^+_{\check{d}_{ij}}},$$

we get $\mathcal{T}_{\check{d}_k}=\min_j\min_i\left\{\mathcal{T}_{\check{d}_{ij}}\right\}$, and $\mathcal{J}_{\check{d}_k}=\max_j\max_i\left\{\mathcal{J}_{\check{d}_{ij}}\right\}$. Hence, $\pi^q_{\check{d}_k}=\pi^q_{\mathfrak{J}^-_{\check{d}_{ij}}}$, then by comparing two q-ROFHSNs, we achieve

$$q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{nm}}\right)=\mathfrak{J}^-_{\check{d}_{ij}}.$$

So, it proved that

$$\mathfrak{J}^-_{\check{d}_{ij}}\leq q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{nm}}\right)\leq\mathfrak{J}^+_{\check{d}_{ij}}.$$

3) SHIFT INVARIANCE

If $\mathfrak{J}_{\check{d}_k}=\left\langle\mathcal{T}_{\check{d}_k},\mathcal{J}_{\check{d}_k}\right\rangle$ be a q-ROFHSN. Then,

$$q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}}\otimes\mathfrak{J}_{\check{d}_k},\mathfrak{J}_{\check{d}_{12}}\otimes\mathfrak{J}_{\check{d}_k},\ldots,\mathfrak{J}_{\check{d}_{nm}}\otimes\mathfrak{J}_{\check{d}_k}\right)$$
$$=q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{nm}}\right)\otimes\mathfrak{J}_{\check{d}_k}.$$

Proof: Consider $\mathfrak{J}_{\check{d}_k}$ and $\mathfrak{J}_{\check{d}_{ij}}$ be two q-ROFHSNs. Then by Definition 1 (2):

$$\mathfrak{J}_{\check{d}_{11}}\otimes\mathfrak{J}_{\check{d}_{12}}=\left\langle\sqrt[q]{\mathcal{T}^q_{11}+\mathcal{T}^q_{12}-\mathcal{T}^q_{11}\mathcal{T}^q_{12}-\mathcal{T}^q_{11}\mathcal{T}^q_{12}-\mathcal{J}^q_{11}\mathcal{T}^q_{12}},\right.$$
$$\left.\sqrt[q]{\mathcal{J}^q_{11}+\mathcal{J}^q_{12}-\mathcal{J}^q_{11}\mathcal{J}^q_{12}}\right\rangle$$

Therefore, as shown in the equation at the bottom of the next page.

4) HOMOGENEITY

Prove that $q-ROFHSIWG\left(\alpha\mathfrak{J}_{\check{d}_{11}},\alpha\mathfrak{J}_{\check{d}_{12}},\ldots,\alpha\mathfrak{J}_{\check{d}_{nm}}\right)=\alpha q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}},\mathfrak{J}_{\check{d}_{12}},\ldots,\mathfrak{J}_{\check{d}_{nm}}\right)$ for any $\alpha>0$.

Proof: Let $\mathfrak{J}_{\check{d}_{ij}}$ be a q-ROFHSN and $\alpha>0$, then by Definition 1 (3), we have

$$\alpha\mathfrak{J}_{\check{d}_{ij}}=\left\langle\sqrt{1-\left(1-\mathcal{T}^q_{\check{d}_{ij}}\right)^\alpha},\right.$$
$$\left.\sqrt{\left(1-\mathcal{T}^q_{\check{d}_{ij}}\right)^\alpha-\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{J}^q_{\check{d}_{ij}}\right)\right]^\alpha}\right\rangle.$$

So, as shown in the equation at the bottom of the next page.

IV. MCGDM MODEL UNDER Q-ROFHSS INFORMATION

To authenticate the implication of the prearranged interaction AOs, a DM method has been present to resolve the MCGDM obstacles. Also, a statistical illustration will be carried out to confirm the pragmatism of the settled approach.

A. PROPOSED MCGDM APPROACH

Let us consider $\aleph=\left\{\aleph^1,\aleph^2,\aleph^3,\ldots,\aleph^s\right\}$ be a set of s alternatives $\mathfrak{U}=\{\delta_1,\delta_2,\delta_3,\ldots,\delta_n\}$ be a set of n specialists. The weights of experts are given as $\Omega=(\Omega_1,\Omega_1,\ldots,\Omega_n)^T$ and $\Omega_i>0$, $\sum_{i=1}^n\Omega_i=1$. Let $\mathfrak{L}=\{d_1,d_2,\ldots,d_m\}$ expressed the set of attributes with their corresponding multi sub-attributes such as $\mathfrak{L}'=\left\{\left(d_{1\rho}\times d_{2\rho}\times\ldots\times d_{m\rho}\right)\text{for all}\rho\in\{1,2,\ldots,t\}\right\}$ with weights $\gamma=\left(\gamma_{1\rho},\gamma_{2\rho},\gamma_{3\rho},\ldots,\gamma_{m\rho}\right)^T$ such as $\gamma_\rho>0$, $\sum_{\rho=1}^t\gamma_\rho=1$ and can be indicated as $\mathfrak{L}'=\left\{\check{d}_\partial:\partial\in\{1,2,\ldots,m\}\right\}$. The team of professionals $\{\kappa^i: i=1,2,\ldots,n\}$ assess the substitutes $\{\aleph^{(z)}: z=1,2,\ldots,s\}$ under the preferred sub-attributes $\{\check{d}_\partial:\partial=1,2,\ldots,k\}$ in the term of q-ROFHSNs such as $\left(\aleph^{(z)}_{\check{d}_{ik}}\right)_{n\times m}=\left(\mathcal{T}_{\check{d}_{ij}},\mathcal{J}_{\check{d}_{ij}}\right)_{n\times m}$, where $0\leq\mathcal{T}_{\check{d}_{ij}},\mathcal{J}_{\check{d}_{ij}}\leq 1$ and $0\leq\left(\mathcal{T}_{\check{d}_{ij}}\right)^q+\left(\mathcal{J}_{\check{d}_{ij}}\right)^q\leq 1$

$\forall i, k$. The specialists deliver their judgment in the form of q-ROFHSNs $\mathcal{L}_{\phi}$ for individually substituting and contemporary the stepwise procedure to acquire the most appropriate alternate.

Step 1. Develop decision matrices $D^{(z)} = \left(\mathcal{T}_{\check{d}_{ij}}, \mathcal{J}_{\check{d}_{ij}}\right)_{n\times m}$ in the form of q-ROFHSNs for each alternative.

$$\left(\aleph^{(z)}, \mathfrak{L}'\right)_{n\times\partial} = \begin{matrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_n \end{matrix} \begin{pmatrix} \left(\mathcal{T}^{(z)}_{\check{d}_{11}}, \mathcal{J}^{(z)}_{\check{d}_{11}}\right) & \left(\mathcal{T}^{(z)}_{\check{d}_{12}}, \mathcal{J}^{(z)}_{\check{d}_{12}}\right) & \cdots & \left(\mathcal{T}^{(z)}_{\check{d}_{1\partial}}, \mathcal{J}^{(z)}_{\check{d}_{1\partial}}\right) \\ \left(\mathcal{T}^{(z)}_{\check{d}_{21}}, \mathcal{J}^{(z)}_{\check{d}_{21}}\right) & \left(\mathcal{T}^{(z)}_{\check{d}_{22}}, \mathcal{J}^{(z)}_{\check{d}_{22}}\right) & \cdots & \left(\mathcal{T}^{(z)}_{\check{d}_{2\partial}}, \mathcal{J}^{(z)}_{\check{d}_{2\partial}}\right) \\ \vdots & \vdots & \vdots & \vdots \\ \left(\mathcal{T}^{(z)}_{\check{d}_{n1}}, \mathcal{J}^{(z)}_{\check{d}_{n1}}\right) & \left(\mathcal{T}^{(z)}_{\check{d}_{n2}}, \mathcal{J}^{(z)}_{\check{d}_{n2}}\right) & \cdots & \left(\mathcal{T}^{(z)}_{\check{d}_{n\partial}}, \mathcal{J}^{(z)}_{\check{d}_{n\partial}}\right) \end{pmatrix}$$

Step 2. Developed the normalized decision matrices by transforming the cost type aspects to benefit type.

$$h_{ij} = \begin{cases} \mathfrak{J}^{c}_{\check{d}_{ij}}; & \text{cost type parameter} \\ \mathfrak{J}_{\check{d}_{ij}}; & \text{benefit type parameter} \end{cases}$$

Step 3. By using established interaction AOs, calculate the communal decision matrix $\mathcal{L}_k$.

Step 4. Evaluate the score values for each alternative using equation 1.

Step 5. Specify the finest alternative over a superlative score value $\mathcal{L}_k$.

Step 6. Rank all the alternatives.

B. APPLICATION OF PROPOSED MCGDM MODEL

There are thousands of dissimilar CRCs in exchange, each with changed values. The first cryptocurrency, BTC, was

$$\begin{aligned}
&q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}}\otimes\mathfrak{J}_{\check{d}_k}, \mathfrak{J}_{\check{d}_{12}}\otimes\mathfrak{J}_{\check{d}_k}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\otimes\mathfrak{J}_{\check{d}_k}\right) \\
&= \otimes_{j=1}^{m}\gamma_j\left(\otimes_{i=1}^{n}\Omega_i\left(\mathfrak{J}_{\check{d}_{ij}}\otimes\mathfrak{J}_{\check{d}_k}\right)\right) \\
&= \left\langle \sqrt[q]{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{J}^q_{\check{d}_{ij}}\right)\right]^{\Omega_i}\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{J}^q_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j}}, \right. \\
&\qquad \left. \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}} \right\rangle \\
&= \left\langle \sqrt[q]{\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j} - \left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{J}^q_{\check{d}_{ij}}\right)\right]\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{J}^q_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j}}, \right. \\
&\qquad \left. \sqrt[q]{1-\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}} \right\rangle \\
&= \left\langle \sqrt[q]{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{J}^q_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}} \right\rangle \\
&\quad \otimes\left\langle\mathcal{T}_{\check{d}_k}, \mathcal{J}_{\check{d}_k}\right\rangle \\
&= q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right)\otimes\mathfrak{J}_{\check{d}_k}.
\end{aligned}$$

$$\begin{aligned}
&q-ROFHSIWG\left(\alpha\mathfrak{J}_{\check{d}_{11}}, \alpha\mathfrak{J}_{\check{d}_{12}}, \ldots, \alpha\mathfrak{J}_{\check{d}_{nm}}\right) \\
&= \left\langle \sqrt[q]{\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\alpha\Omega_i}\right)^{\gamma_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{J}^q_{\check{d}_{ij}}\right)\right]^{\alpha\Omega_i}\right)^{\gamma_j}}, \sqrt[q]{1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\alpha\Omega_i}\right)^{\gamma_j}} \right\rangle \\
&= \left\langle \sqrt[q]{\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}\right)^{\alpha} - \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left[1-\left(\mathcal{T}^q_{\check{d}_{ij}}+\mathcal{J}^q_{\check{d}_{ij}}\right)\right]^{\Omega_i}\right)^{\gamma_j}\right)^{\alpha}}, \right. \\
&\qquad \left. \sqrt[q]{1-\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{J}^q_{\check{d}_{ij}}\right)^{\Omega_i}\right)^{\gamma_j}\right)^{\alpha}} \right\rangle \\
&= \alpha q-ROFHSIWG\left(\mathfrak{J}_{\check{d}_{11}}, \mathfrak{J}_{\check{d}_{12}}, \ldots, \mathfrak{J}_{\check{d}_{nm}}\right).
\end{aligned}$$

established in 2009 by a systems analyst using the pseudonym of Satoshi Nakamoto. Nakamoto delivers the first depiction of the blockchain. Blockchain is the expertise that permits cryptocurrency to work like administration-delivered (fiat) coins deprived of the envelopment of any essential bank or trusted third party. Especially blockchain elucidates the ''double-spending delinquent'' accompanying digital money. Meanwhile, digital data is clichéd, and digital currency needs a contrivance that consistently averts a coinage component from existence, ''replicated'' or consumed more than once. As a communal object, the worldwide economic organization has generally been liable for launching and confirming the acceptability of financial contacts. The rationality of cryptocurrency is demonstrated and preserved, deprived of any participation by the world's central banks. As a substitute, records of cryptocurrency trades are openly retained. Dealings corroborated by blockchain expertise are unassailable; their importance cannot be altered. That stops hackers from generating deceitful contract archives and founds reliance between customers.

Cryptocurrency is a digital disbursement structure that doesn't trust banks to authenticate communications. As an alternative to actuality cash conceded about and substituted in the real world, cryptocurrency payments are only digital items to a virtual catalog that labels definite contacts. The contacts are documented in an open record when you transfer cryptocurrency capital. Cryptocurrency is deposited in digital files. Cryptocurrency expected its label since it practices encryption to confirm dealings. This capital's innovative coding is convoluted in packing and conducting cryptocurrency statistics among files and opening archives. Encryption intends to deliver sanctuary and protection. Generally, a person's concern in CRCs is to craft for income, with entrepreneurs at periods dynamic values upward. CRCs track a scattered open record entitled blockchain, which records all dealings rationalized and seized by currency owners. Components of cryptocurrency are generated over a mining procedure, which contains computer influence to resolve convoluted scientific complications that engender coins. Consumers can also purchase the exchanges from traders, then stock and devote them to consuming cryptographic files. If your particular cryptocurrency, you don't possess something perceptible. All you have is a sign that permits you to transfer entities of record or measurement from one person to another starved of a reliable third party. While BTC has been about since 2009, the presentation of CRCs and blockchain expertise is tranquil developing on the economic facade, with extra routine estimated in the prospect. Communications containing bonds, shares, and other monetary resources can ultimately be exported via expertise. There are many types of CRCs; in the following, we will discuss some of them.

Bitcoin (BTC): BTC is a cryptocurrency, an alphanumeric asset that practices cryptography to resistor its formation and administration slightly than depend on a dominant authority. Initially premeditated as a moderate interchange, BTC is currently mainly noticed as a stock of assessment. The antiquity of BTC is in progress with its development and execution by Satoshi Nakamoto, who incorporated several standing philosophies from cryptography professionals. Above the growth of BTC's antiquity, it has endured prompt progress to convert a noteworthy stock of significance equally online and offline. Since the middle of 2010, some dealings initiated forbearing bitcoin in accumulation to old-fashioned exchanges. The trend of BTC cryptocurrency is given in the following figure 1.

FIGURE 1. 1BTC=46004.15 USD (source: price index data from CoinDesk (https://coinmarketcap.com/currencies/bitcoin/?period=7d).

Ethereum (ETH): ETH is an open-source, community package that works blockchain knowledge to permit smooth conventions and cryptocurrency transactions devoid of the association of a broker, but anywhere did it originate from? The cryptocurrency domain is a beginning zone underway with BTC's inauguration in 2009. BTC patented into a spectacle as an estimate scheduling two components an internet assembled assistance and the innovative blockchain ability on which that asset scores. From there, people used the effective interchange and blockchain philosophies to consequent up with other structures and assets. ETH is a blockchain that swarms a substantial range of functionality for originators accumulating explanations on ETH as contemptible. The ETH blockchain has an ordinary coin recognized as Ether, which helps to compensate for activity on the ETH blockchain. The coin also employs crypto interactions and vacillates in cost. Other assets fictitious on the ETH blockchain, such as ERC-20 signs, need ETH as expenditure for controls connected with any transactions of those belongings. The ETH blockchain was transliterated in the solidity encrypting philologist. A non-income item, the ETH ingredient, assists one of the administrators of the ETH scheme. Contrasting BTC with its enigmatic formation and originator(s), ETH's antiquity is extra forthright. Vitalik Buterin and numerous others co-created ETH, but the particulars contiguous to the huge blockchain's backstory warranty extra description. The trend of ETH cryptocurrency is given in the following figure 2.

Tether (TET): Willett circulated a whitepaper that labeled the probability of the construction of innovative cryptocurrencies on the best of the BTC blockchain. Willett went on to support device this impression in the cryptocurrency Mastercoin, which had a supplementary Mastercoin substance to stimulate the usage of this novel second layer.

TABLE 1. Q-ROFHS decision matrix for $\aleph_1$.

	$\check{d}_1$	$\check{d}_2$	$\check{d}_3$	$\check{d}_4$	$\check{d}_5$	$\check{d}_6$	$\check{d}_7$	$\check{d}_8$
$\mathcal{H}_1$	(0.79, 0.98)	(0.79, 0.68)	(0.95, 0.92)	(0.83, 0.91)	(0.89, 0.98)	(0.76, 0.98)	(0.83, 0.94)	(0.95, 0.94)
$\mathcal{H}_2$	(0.83, 0.84)	(0.88, 0.69)	(0.98, 0.76)	(0.78, 0.81)	(0.84, 0.99)	(0.93, 0.88)	(0.86, 0.81)	(0.97, 0.86)
$\mathcal{H}_3$	(0.88, 0.72)	(0.95, 0.78)	(0.84, 0.78)	(0.98, 0.91)	(0.85, 0.89)	(0.95, 0.89)	(0.88, 0.81)	(0.81, 0.84)
$\mathcal{H}_4$	(0.99, 0.88)	(0.69, 0.97)	(0.89, 0.73)	(0.71, 0.88)	(0.98, 0.79)	(0.98, 0.84)	(0.91, 0.88)	(0.71, 0.88)

FIGURE 2. 1ETH=3760.10 USD (source: price index data from CoinDesk (https://coinmarketcap.com/currencies/ethereum/? Period=7d).

The master coin procedure suited the scientific basis of the TET cryptocurrency and was one of the unique participants of the master coin foundation. Brock pierce, developed a co-founder of TET, and founder, Craig Sellars, befitted the CTO of the master coin foundation. The pioneer of TET, formerly called "Realcoin," was publicized in July 2014 by co-founders Brock Pierce, Reeve Collins, and Craig Sellars as a Santa Monica-based startup. The first tokens were dispensed on 6 October 2014 on the BTC blockchain. The Omni layer Procedure completed this. Tether CEO Reeve Collins stated the task was retitled to "Tether." The enterprise also proclaimed it was inflowing reserved beta, which sustained a "TET+ token" for three currencies: US Tether for USD, EuroTET for euros, and YenTET for Japanese yen. Tether said, "Every TET+ token is assisted 100% by its imaginative exchange and can be converted at any time with no revelation to altercation hazard." The enterprise's website states that it is amalgamated in Hong Kong with headquarters in Switzerland, lacking bountiful information. The trend of TET cryptocurrency is given in the following figure 3.

FIGURE 3. 1Tether=1.00 USD (https://coinmarketcap.com/currencies/tether/?period=7d).

Binance Coin (BNB): BNB is a cryptocurrency interchange propelled in the midsummer of 2017 and quickly raised to convert one of the world's major dealing capacities, assembling a revenue of $200 million in its second full quarter of process. BNB is an ERC-20 token dispensed as a portion of a preliminary coin subscription to be used for promotional transaction charges, with a proportion of the indications signed every quarter. ERC-20 tokens were changed by intrinsic tokens when Binance propelled Binance Chain, its particular communal blockchain, in April 2019. The group assembled Binance DEX, a regionalized altercation that consents dealers to retain the protection of their money. Over time, the token has enlarged surplus functionality, such as used in the Binance Launchpad lottery scheme and dissimilar expenditures in the Binance system. CEO Changpeng Zhao has also specified that 90% of Binance workers elect to receive a share of their earnings in BNB. The trend of BNB cryptocurrency is given in the following figure 4.

FIGURE 4. 1BNB=508.32 USD (https://coinmarketcap.com/currencies/bnb/?period=7d).

Let $\{\aleph^{(1)} = \text{Tether}, \aleph^{(2)} = \text{Binance Coin}, \aleph^{(3)} = \text{Ethereum}, \aleph^{(4)} = \text{Bitcoin}\}$ be a set of alternatives that represents the CRCs and $\mathfrak{L} = \{d_1 = \{\text{security}, d_2 = \text{decentralization}, d_3 = \text{demand}\}$ be a collection of considered attributes given as security $= d_1 = \{d_{11} = \text{strong}, d_{12} = \text{poor}\}$, Decentralization $= d_2 = \{d_{21} = \text{decentralized application}, d_{22} = \text{decentralized autonomous organization}\}$, Demand $= d_3 = \{d_{31} = more, d_{32} = less\}$. Let $\mathfrak{L}' = d_1 \times d_2 \times d_3$ be a set of sub-attributes

$$\mathfrak{L}' = d_1 \times d_2 \times d_3 = \{d_{11}, d_{12}\} \times \{d_{21}, d_{22}\} \times \{d_{31}, d_{32}\}$$
$$= \left\{ \begin{array}{c} (d_{11}, d_{21}, d_{31}), (d_{11}, d_{21}, d_{32}), \\ (d_{11}, d_{22}, d_{31}), (d_{11}, d_{22}, d_{32}), \\ (d_{12}, d_{21}, d_{31}), (d_{12}, d_{21}, d_{32}), \\ (d_{12}, d_{22}, d_{31}), (d_{12}, d_{22}, d_{32}) \end{array} \right\},$$
$$\mathfrak{L}' = \left\{ \check{d}_1, \check{d}_2, \check{d}_3, \check{d}_4, \check{d}_5, \check{d}_6, \check{d}_7, \check{d}_8 \right\}$$

TABLE 2. Q-ROFHS decision matrix for $\aleph_2$.

	$\breve{d}_1$	$\breve{d}_2$	$\breve{d}_3$	$\breve{d}_4$	$\breve{d}_5$	$\breve{d}_6$	$\breve{d}_7$	$\breve{d}_8$
$\mathcal{H}_1$	(0.78, 0.94)	(0.76, 0.98)	(0.95, 0.98)	(0.84, 0.95)	(0.93, 0.84)	(0.76, 0.94)	(0.86, 0.97)	(0.89, 0.98)
$\mathcal{H}_2$	(0.86, 0.87)	(0.85, 0.88)	(0.98, 0.99)	(0.87, 0.87)	(0.77, 0.65)	(0.93, 0.98)	(0.88, 0.85)	(0.84, 0.99)
$\mathcal{H}_3$	(0.77, 0.78)	(0.85, 0.89)	(0.77, 0.78)	(0.95, 0.79)	(0.87, 0.88)	(0.92, 0.89)	(0.86, 0.89)	(0.84, 0.99)
$\mathcal{H}_4$	(0.97, 0.87)	(0.76, 0.89)	(0.87, 0.93)	(0.69, 0.88)	(0.98, 0.79)	(0.98, 0.84)	(0.94, 0.86)	(0.79, 0.84)

TABLE 3. Q-ROFHS decision matrix for $\aleph_3$.

	$\breve{d}_1$	$\breve{d}_2$	$\breve{d}_3$	$\breve{d}_4$	$\breve{d}_5$	$\breve{d}_6$	$\breve{d}_7$	$\breve{d}_8$
$\mathcal{H}_1$	(0.71, 0.98)	(0.81, 0.91)	(0.91, 0.98)	(0.79, 0.68)	(0.81, 0.98)	(0.77, 0.98)	(0.73, 0.94)	(0.79, 0.98)
$\mathcal{H}_2$	(0.81, 0.88)	(0.78, 0.88)	(0.88, 0.98)	(0.85, 0.99)	(0.85, 0.89)	(0.83, 0.88)	(0.83, 0.87)	(0.99, 0.78)
$\mathcal{H}_3$	(0.78, 0.89)	(0.98, 0.69)	(0.77, 0.69)	(0.88, 0.77)	(0.88, 0.89)	(0.98, 0.89)	(0.79, 0.98)	(0.88, 0.79)
$\mathcal{H}_4$	(0.67, 0.87)	(0.69, 0.88)	(0.87, 0.99)	(0.97, 0.86)	(0.87, 0.96)	(0.68, 0.89)	(0.97, 0.87)	(0.69, 0.88)

TABLE 4. Q-ROFHS decision matrix for $\aleph_4$.

	$\breve{d}_1$	$\breve{d}_2$	$\breve{d}_3$	$\breve{d}_4$	$\breve{d}_5$	$\breve{d}_6$	$\breve{d}_7$	$\breve{d}_8$
$\mathcal{H}_1$	(0.74, 0.98)	(0.76, 0.98)	(0.95, 0.98)	(0.81, 0.91)	(0.89, 0.98)	(0.76, 0.98)	(0.83, 0.94)	(0.95, 0.93)
$\mathcal{H}_2$	(0.83, 0.87)	(0.98, 0.88)	(0.98, 0.88)	(0.78, 0.88)	(0.84, 0.99)	(0.93, 0.88)	(0.87, 0.85)	(0.97, 0.76)
$\mathcal{H}_3$	(0.78, 0.79)	(0.85, 0.89)	(0.74, 0.79)	(0.98, 0.69)	(0.85, 0.89)	(0.95, 0.89)	(0.83, 0.89)	(0.89, 0.74)
$\mathcal{H}_4$	(0.97, 0.87)	(0.76, 0.89)	(0.87, 0.93)	(0.69, 0.88)	(0.98, 0.79)	(0.98, 0.84)	(0.94, 0.86)	(0.79, 0.84)

be a set of all sub-attributes with weights $\gamma_j = (0.22, 0.1, 0.04, 0.07, 0.13, 0.14, 0.11, 0.17)^T$. Let $\{\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3, \mathcal{H}_4\}$ be a team of specialists with weights $\Omega_i = (0.16, 0.25, 0.33, 0.26)^T$. Professionals give their predilections in q-ROFHSNs under multi sub-attributes of considered attributes. The numerical values are considered from [55].

C. BY USING THE Q-ROFHSIWA OPERATOR

Step 1. The professionals encapsulate their urgencies and score values in Table 1-Table 4 in q-ROFHSNs.

Step 2. No need to normalize.

Step 3. Aggregated values of professionals from prearranged tables 1-4 are deliberate by the particular q-ROFHSIWA operator as follows: $\mathcal{L}_1 = \langle 0.8325, 0.9381\rangle$, $\mathcal{L}_2 = \langle 0.8993, 0.8336\rangle$, $\mathcal{L}_3 = \langle 0.8884, 0.7954\rangle$, $\mathcal{L}_4 = \langle 0.9169, 0.7289\rangle$.

Step 4. Employing the score function to compute the score values such as $\mathfrak{S}(\mathcal{L}_1) = -0.2827$, $\mathfrak{S}(\mathcal{L}_2) = 0.1561$, $\mathfrak{S}(\mathcal{L}_3) = 0.2134$, $\mathfrak{S}(\mathcal{L}_4) = 0.4108$.

Step 5. $\aleph^4$ has a maximum score value, so $\aleph^4$ is the finest choice.

Step 6. the ranking of the alternatives is given as follows:

$\mathfrak{S}(\mathcal{L}_4) > \mathfrak{S}(\mathcal{L}_3) > \mathfrak{S}(\mathcal{L}_2) > \mathfrak{S}(\mathcal{L}_1)$. So, $\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$. It is observed that the most appropriate cryptocurrency among selected CRCs. The effect of q is given in Table 5 on decision outcomes under the q-ROFHSIWA operator. Also, the graphical representation of the effect of the parameter q shown in Figure. 5.

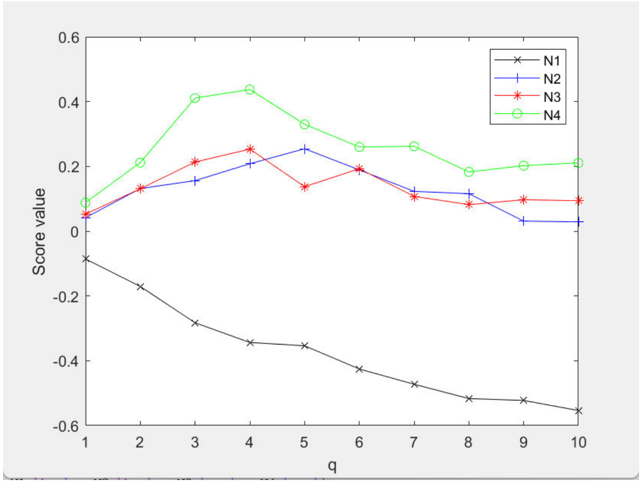

FIGURE 5. Score values of the alternatives for $1 \leq q \leq 10$ under q-ROFHSIWA.

D. EFFECT ON ALTERNATIVES RANKING BY THE VARIATION OF Q PARAMETER UNDER Q-ROFHSIWA OPERATOR

The classification instruction directs that the finest substitute is $\aleph^{(4)}$ and $\aleph^{(1)}$ is the worst one. Even though these outcomes

TABLE 5. Effects of parameter q on decision results using Q-ROFHSIWA operator.

Parameter	Score value	Ranking
q=1	(-0.0860, 0.0418, 0.0527, 0.0882)	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$
q=2	(-0.1708, 0.1315, 0.1309, 0.2119)	$\aleph^{(4)} > \aleph^{(2)} > \aleph^{(3)} > \aleph^{(1)}$
q=3	(-0.2827, 0.1561, 0.2134,0.4108)	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$
q=4	(-0.3436, 0.2081, 0.2533, 0.4373)	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$
q=5	(-0.3538, 0.2544, 0.1374, 0.3299)	$\aleph^{(4)} > \aleph^{(2)} > \aleph^{(3)} > \aleph^{(1)}$
q=6	(-0.4255, 0.1891, 0.1928, 0.2599)	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$
q=7	(-0.4726, 0.1229, 0.1073, 0.2620)	$\aleph^{(4)} > \aleph^{(2)} > \aleph^{(3)} > \aleph^{(1)}$
q=8	(-0.5168, 0.1156, 0.0820, 0.1829)	$\aleph^{(4)} > \aleph^{(2)} > \aleph^{(3)} > \aleph^{(1)}$

are identical to those resolute in [55], Table 5 discloses that there is an alteration in the classification order of $\aleph^{(2)}$ and $\aleph^{(3)}$ while 'q' is between 1 and 10. The graphical representation of the effect of parameter "q" on the outcome of the decision is shown in Figure 5. It can be observed from Figure 5 that when $q =$ 1-10, the order of the substitution classification is the same. Although, there is a difference in the ranking order of $\aleph^{(2)}$ and $\aleph^{(3)}$, but does not affect the best and worst alternatives. Furthermore, IFHSS [49] and PFHSS [52] are considered by *MD* and *NMD*. These sets cannot handle the case of $(MD)^2 + (NMD)^2 > 1$. It has been perceived that the methods established in [55] can describe fuzzy data, but the use of parameter q does make the data collection process more flexible. The planned technique is easier to define ambiguous data and makes assembling facts by parameters more flexible. After accumulating some settings, several hybrid configurations of FS convert in exceptional cases of q-ROFHSS (see Table 7). The parameter "q" assists professionals in reviewing any item more broadly. So, it is recommended that experts should pick the value of "q" conferring to their inclinations. Over this study and assessment, we concluded that the consequences gained through the projected method are extra precise than the results attained by other approaches.

E. BY USING THE Q-ROFHSIWG OPERATOR

Step 1 and step 2 are similar to 4.3.

Step 3. Aggregated values of professionals from prearranged Tables 1-4 are deliberate by the particular q-ROFHSIWG operator as follows: $\mathcal{L}_1 = \langle 0.7997, 0.9628\rangle$, $\mathcal{L}_2 = \langle 0.8241, 0.9073\rangle$, $\mathcal{L}_3 = \langle 0.8154, 0.8714\rangle$, $\mathcal{L}_4 = \langle 0.7854, 0.8765\rangle$.

Step 4. Employing the score function to compute the score values such as $\mathfrak{S}(\mathcal{L}_1) = -0.4123$, $\mathfrak{S}(\mathcal{L}_2) = -0.1989$, $\mathfrak{S}(\mathcal{L}_3) = -0.1266$, $\mathfrak{S}(\mathcal{L}_4) = -0.2055$.

Step 5. $\aleph^3$ has a maximum score value, so $\aleph^3$ is the finest choice.

Step 6. Consuming the deliberated operator, the classification of the substitutes is given as follows: $\mathfrak{S}(\mathcal{L}_3) > \mathfrak{S}(\mathcal{L}_2) > \mathfrak{S}(\mathcal{L}_4) > \mathfrak{S}(\mathcal{L}_1)$. So, $\aleph^{(3)} > \aleph^{(2)} > \aleph^{(4)} > \aleph^{(1)}$. The effect of q is given in Table 6 on decision outcomes under the q-ROFHSIWG operator. Also, the graphical representation of the effect of the parameter q shown in Figure. 6.

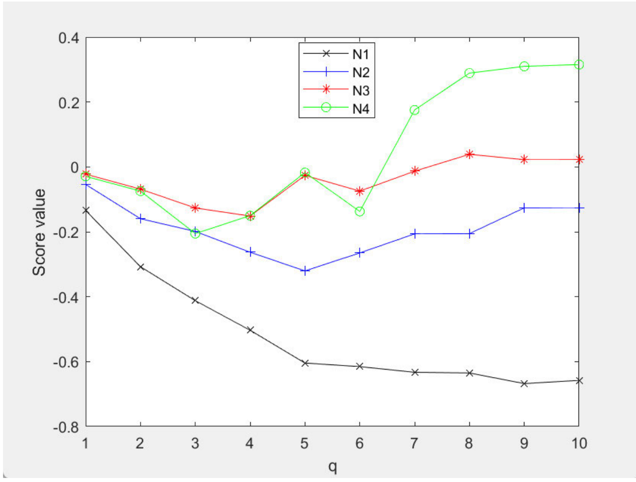

FIGURE 6. Score values of the alternatives for $1 \le q \le 10$ under q-ROFHSIWG.

F. EFFECT ON ALTERNATIVES RANKING BY THE VARIATION OF Q PARAMETER UNDER Q-ROFHSIWG OPERATOR

To deliberate the influence of the factor q on the outcome of the decision, we used different values of q, as a classification order for substitutes. A suitable alternative is $\aleph^{(3)}$, when $q =$ 1-3, with the ranking order of all alternatives $\aleph^{(3)} > \aleph^{(4)} > \aleph^{(2)} > \aleph^{(1)}$. When $q =$ 4-10, the best alternatives are slightly different, which is $\aleph^{(4)}$ with the ranking order of alternatives $\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$ which is different than $q =$ 1-3. One more fascinating fact detected in this investigation is that for $q =$ 1-10, the worst alternative is the same. With this analysis, we have found that if the value of the parameter q changes, it will responsively affect the hierarchical order of the substitutes. Therefore, professionals can pick the appropriate q value to assess the objective delivered consequently. It is endorsed that experts need to consider the parameter values when the ranking order of alternatives is stable.

V. COMPARATIVE ANALYSIS AND DISCUSSION

The following section compares the presented model and prevalent methods to substantiate the rationality of the technique delivered.

TABLE 6. Effects of parameter *q* on decision results using Q-ROFHSIWG operator.

Parameter	Score value	Ranking
q=1	(-0.1342,-0.0541,-0.0223,-0.0291)	$\aleph^{(3)} > \aleph^{(4)} > \aleph^{(2)} > \aleph^{(1)}$
q=2	(-0.3076,-0.1597,-0.0685,-0.0746)	$\aleph^{(3)} > \aleph^{(4)} > \aleph^{(2)} > \aleph^{(1)}$
q=3	(-0.4123,-0.1989,-0.1266,-0.2055)	$\aleph^{(3)} > \aleph^{(2)} > \aleph^{(4)} > \aleph^{(1)}$
q=4	(-0.5034,-0.2630,-0.1514,-0.1503)	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$
q=5	(-0.6045,-0.3205,-0.0262,-0.0174)	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$
q=6	(-0.6155,-0.2651,-0.2461,-0.1373)	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$
q=7	(-0.6330,-0.2064,-0.0132,0.1755)	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$
q=8	(-0.6351,-0.2056,0.0388,0.2891)	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$
q=9	(-0.6678,-0.1262,0.0222,0.3099)	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$
q=10	(-0.6579,-0.1276,0.0227,0.3154)	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$

TABLE 7. Feature analysis of different models with a proposed model.

	Fuzzy information	MD	NMD	Parametrization	Sub-parameters	Advantages
FS [6]	✓	×	✓	×	×	Contracts hesitation via MD
IFS [7]	✓	×	✓	×	×	Contracts hesitation via MD + NMD> 1
PFS [13]	✓	×	✓	×	×	Contracts hesitation via MD and NMD
q-ROFS [22]	✓	✓	✓	×	×	Contracts hesitation $(\mathrm{MD})^2 + (NMD)^2 > 1$
FSS [27]	✓	✓	×	✓	×	Contracts hesitation using parametric values of MD
IFSS [31]	✓	✓	×	×	×	Contracts hesitation via parametric values of MD and NMD; MD + NMD > 1
PFSS [35]	✓	✓	×	×	×	Contracts hesitation if $(\mathrm{MD})^2 + (NMD)^2 > 1$
q-ROFSS [43]	✓	✓	✓	✓	×	Contracts hesitation, if $(\mathrm{MD})^q + (NMD)^q > 1$
IFHSS [46]	✓	✓	✓	✓	✓	Contracts hesitation of multi sub-attributes via MD and NMD. MD + NMD > 1
PFHSS [50]	✓	✓	✓	✓	✓	Contracts hesitation of multi sub-attributes $(\mathrm{MD})^2 + (NMD)^2 > 1$
q-ROFHSS	✓	✓	✓	✓	✓	Contracts hesitation of multi sub-attributes $(\mathrm{MD})^q + (NMD)^q > 1$

A. SUPREMACY OF THE PLANNED TECHNIQUE

The estimated technique is skillful and convincing. In the q-ROFHSS setup, we developed an advanced MCGDM method using q-ROFHSIWA and q-ROFHSIWG operators. Our scheduled technique is extra talented than predominant techniques and may have the subtlest effect on MCGDM obstacles. The supportive method is multi-purpose and conversant, amended permitting inconsistency, obligations, and varying productivity. Dissimilar replicas have specific classification behaviors, so there are direct amendments among ratings of expected techniques to meet their prospects. Methodical studies and evaluations are resolute that the results obtained from the existing methods were comparable to mixed organizations. In addition, several hybrid structures of FS, such as IFS, PFS, IFSS, IFSS, FHSS, FHSS, and PFHSS, are reduced to q-ROFHSS under some specific conditions. It is informal to combine scarce and unclear information in DM agendas. Information around affluence can be expressed more fully and logically in it. In the DM process, false and disturbing facts are mixed. As a result, our progressive approach will be more expert, important, superior, and better than many hybrid FS configurations. Table 7 below clarifies some prevailing replicas' prediction technique and feature analysis.

TABLE 8. Comparative analysis with existing operators.

Operator	$\aleph^{(1)}$	$\aleph^{(2)}$	$\aleph^{(3)}$	$\aleph^{(4)}$	Ranking order
PFSIWA [38]	0.2887	0.2964	0.3338	0.4228	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$
PFSIWG [38]	0.1928	0.2609	0.3529	0.2885	$\aleph^{(3)} > \aleph^{(4)} > \aleph^{(2)} > \aleph^{(1)}$
IFHSWA [49]	0.4094	0.4174	0.4618	0.4983	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$
IFHSWG [49]	0.3564	0.4079	0.4262	0.3616	$\aleph^{(3)} > \aleph^{(2)} > \aleph^{(4)} > \aleph^{(1)}$
PFHSIWA [56]	0.1759	0.2348	0.2457	0.2673	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$
PFHSIWG [56]	−0.0264	0.0127	−0.0145	0.0159	$\aleph^{(4)} > \aleph^{(2)} > \aleph^{(3)} > \aleph^{(1)}$
q-ROFHSWA [55]	0.0125	0.0187	0.0247	0.0749	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$
q-ROFHSWG [55]	−0.0254	−0.0179	−0.0121	−0.0246	$\aleph^{(3)} > \aleph^{(2)} > \aleph^{(4)} > \aleph^{(1)}$
q-ROFHSIWA	−0.2827	0.1516	0.2134	0.4108	$\aleph^{(4)} > \aleph^{(3)} > \aleph^{(2)} > \aleph^{(1)}$
q-ROFHSIWG	−0.4123	−0.1989	−0.1266	−0.2055	$\aleph^{(3)} > \aleph^{(2)} > \aleph^{(4)} > \aleph^{(1)}$

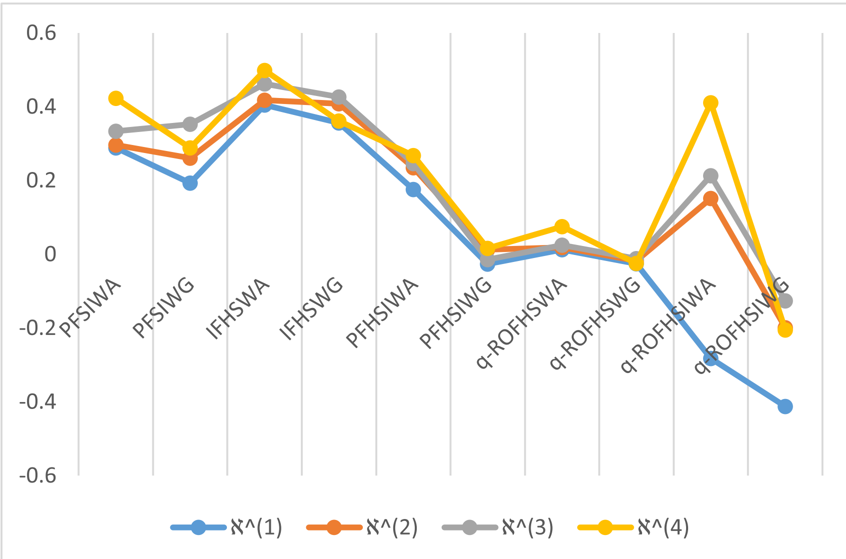

FIGURE 7. Comparative analysis.

B. COMPARATIVE ANALYSIS

To substantiate the efficacy of the premeditated scheme, we connect the achieved significance with some dominant methods under the setting of PFS, IFSS, PFSS, q-ROFSS, IFHSS, and PFHSS. A summary of all concerns is stated in Table 8. Zulqarnain et al. [38] developed PFSIWA and PFSIWG operators to compute the parameterized values of the alternatives. But, these interaction AOs impotent to compact with the sub-attributes of the alternatives. The AOs for IFHSS [49] can contract with the sub-parameterized values of alternatives, but when the sum of $MD+ > 1$. Then these AOs unable to accommodate the decision results. Zulqarnain et al. [56] introduced the interaction AOs for PFHSS and operated their presented AOs to resolve MCDM hurdles considering the parameterized values of the sub-attributes. Also, these AOs fail to handle the scenario when the $(MD)^2 + (NMD)^2 > 1$. The AOs developed by Khan et al. [55] can expertly compact with the multi-sub-attributes of the alternatives. But, in some cases, these AOs unable to carry some undesirable results. Therefore, to address these complex issues, we settled interaction AOs for q-ROFHSSS, which is accomplished by handling multi sub-attributes equated to prevailing AOs. Therefore, q-ROFHSS is the broadest form of PFHSS. Therefore, based on the facts above, the operators expected in this article are more significant, reliable, and flourishing. Table 8 below compares the anticipated model with the prevalent replicas.

So, we have the prerogative impending surprising to the prevailing operators we have established to report the misuse and the incomprehensible concerns in the general DM process. Intentionally supporting measures associated with the present methodology are suppression consequences for undesirable causes. Thus, it comforts the syndicate of indefinite and undeclared facts in the DM development. The graphical results of comparative studies are given in the following figure 7.

C. ADVANTAGES OF PROPOSED RESEARCH

In the following subsection, we will designate the recompenses of the planned methodology.

- The planned method practices the idea of parameterization in conjunction with q-ROFHSS to address the importance of DM obstacles. Consistency-parameterized MD and NMD imitate the prospect that there is a level of salutation and justification. This correspondence holds astonishing capacity in computing effective demonstrations in the interpolation universe with these aspects.
- Because the model emphasizes in-depth surveillance of parameters and the set of values of their respective sub-parameters, it supports decision-makers make balanced and consistent judgments over DM.

It authenticates all the forms and features of predominant theories, so it's not unreasonable to contemplate it as a general system of existing ideas.

VI. CONCLUSION

To control a variety of CRCs, equivocate deceives, and remain transactions on the operational flea market, it is compulsory to perform an operative and suitable exploration of the cryptocurrency market. Exploration of the cryptocurrency market shows that sanctuary, reorganization, and ultimatum are the fundamental peak features of Bitcoin's speculation objectives, monitored by economic encouragements, with slight modification. Sub-factors' classification is a high sanctuary, reorganized solicitations, and amplified mandate for cryptocurrency arcades. Consequently, many scholars and investigators originated from studying CRCs. Many intellectuals are spiraling to FS and its amalgam configuration to decipher the troubles of reviewing the Bitcoin market, as obscurity occurs in virtually all practical structures. The deficiency of meditation about unclear circumstances among attributes may obstruct some of MCGDM's complex implications. Mathematical demonstration in MCGDM exploits all effects while assimilation intention under fiscal, superior, and welfare boundaries. Surveys must be constrained for superlative decisions and access to decision necessities. In genuine DM, the estimation of alternative facts conceded by the professional is frequently imprecise, irregular, and imprudent, so q-ROFHSNs can be used to comport this unreliable data. The essential dispassionate of this exertion is to execute the interactive operational laws for the q-ROFHSS. Considering the settled algebraic interactional operational laws, we presented the q-ROFHSIWA and q-ROFHSIWG operators for q-ROFHSS with their desired properties. Also, a DM technique has been scheduled to address MCGDM obstacles based on the validated operators. To state the stoutness of the demonstrated methodology, we deliver a comprehensive mathematical illustration of the most suitable cryptocurrency. A comprehensive analysis of some existing procedures is offered. Lastly, based on the consequences achieved, it is unconquerable that the method proposed in this study is the most concrete and operative way to explain the problem of MCGDM.

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RANA MUHAMMAD ZULQARNAIN received the Ph.D. degree from Northwest University, Xi'an, China. He is currently working at Zhejiang Normal University, Jinhua, Zhejiang, China. His research interests include artificial intelligence, fuzzy algebra, and soft sets.

IMRAN SIDDIQUE is currently working as a Full Professor at the University of Management and Technology, Lahore, Pakistan. He is a reviewer of several well-known SCI and ESCI journals. His research interests include artificial intelligence, fuzzy algebra, soft sets, fuzzy fluid dynamics, fluid mechanics, lubrication theory, soliton theory, and graph theory.

SAYED M. ELDIN was the Dean of the Faculty of Engineering, Cairo University, where he achieved many unique signs of progress in both academia and research on the impact of emerging technologies in electrical engineering. He is currently with the Faculty of Engineering and Technology, Future University in Egypt, on leave from Cairo University after nearly 30 years of service to the Faculty of Engineering, Cairo University. He was a PI of several nationally and internationally funded projects. He has many publications in highly refereed international journals and specialized conferences in the applications of artificial intelligence on protection of electrical power networks. In addition, he is in the editorial boards of several international journals.

SHAHID HUSSAIN GURMANI was born in 1989. He received the Ph.D. degree from the School of Mathematical Sciences, Anhui University, Hefei, China, in 2022. He is currently working with Zhejiang Normal University, Jinhua, Zhejiang, China. His research works have been published in *Applied Intelligence*, *Soft Computing*, *International Journal of Fuzzy Systems*, *Kybernetes*, and the *Journal of Intelligent and Fuzzy Systems*. His research interests include group decision-making, artificial intelligence, fuzzy set and extension of fuzzy set theory, and aggregation operators.

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