

D numbers theory: a generalization of Dempster-Shafer theory

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Abstract

Dempster-Shafer theory is widely applied to uncertainty modelling and knowledge reasoning due to its ability of expressing uncertain information. However, some conditions, such as exclusiveness hypothesis and completeness constraint, limit its development and application to a large extent. To overcome these shortcomings in Dempster-Shafer theory and enhance its capability of representing uncertain information, a novel theory called D numbers theory is systematically proposed in this paper. Within the proposed theory, uncertain information is expressed by D numbers, reasoning and synthesis of information are implemented by D numbers combination rule. The proposed D numbers theory is an generalization of Dempster-Shafer theory, which inherits the advantage of Dempster-Shafer theory and strengthens its capability of uncertainty modelling.

Keywords: D numbers theory, Dempster-Shafer theory, D numbers,

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1. Introduction

Since first proposed by Dempster [1] and then developed by Shafer [2], Dempster-Shafer theory of evidence, also called Dempster-Shafer theory or evidence theory, has been paid much attentions for a long time and continually attracted growing interests. This theory needs weaker conditions than the Bayesian theory of probability, so it is often regarded as an extension of the Bayesian theory [3, 4, 5, 6]. Many studies have been devoted to further improve and perfect this theory in many aspects, for instance combination of evidences [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19], conflict management [20, 21, 22, 23, 24, 25, 26, 27], independence of evidence [28, 29, 30, 31], generation of mass function [32, 33, 34, 35], similarity measure between evidences [36, 37, 38], uncertainty measure of evidences [39, 40, 41, 42, 43, 44, 45, 46, 47], and so on [48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73]. Due to its ability to handle uncertain information, Dempster-Shafer theory has been extensively used in many fields, such as statistical learning [74, 75, 76, 77, 78, 79, 80, 81], classification and clustering [82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98], Granular computing [99, 100, 101, 102], uncertainty and knowledge reasoning [103, 104, 105, 106, 107, 108, 109, 110, 111], decision making [112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126], risk assessment and evaluation [127, 128, 129, 130], knowledge-based systems and expert systems [131, 132, 133, 134, 135], and so forth [136, 137, 138, 139, 140, 141, 142, 143,

144, 145, 146].

Even as a theory of reasoning under the uncertain environment, Dempster-Shafer theory has an advantage of directly expressing the “uncertainty” by assigning the probability to the subsets of the set composed of multiple objects, rather than to each of the individual objects. However, it is also constrained by many strong hypotheses and hard constraints which limit its development and application to a large extent. For one hand, the elements in a frame of discernment (FOD) are required to be mutually exclusive. It is called exclusiveness hypothesis. For another, the sum of basic probabilities of a mass function must be equal to 1, which is called completeness constraint. In the following of this paper, we will show how these conditions limit the application of Dempster-Shafer theory.

To overcome these shortcomings in Dempster-Shafer theory and strengthen its capability of representing uncertain information, a novel theory called D numbers theory is systematically proposed in this paper. A novel data representation called D numbers [147, 148, 149] is used to model uncertain information. What’s more, a D numbers combination rule is proposed to synthesize all the information expressed by D numbers and implement the uncertainty and knowledge reasoning. Actually, D numbers and D numbers combination rule is an extension of mass function and Dempster’s rule of combination, respectively. If meeting certain conditions, they will degenerate to classical mass function and Dempster’s rule of combination. Consequently, the proposed D numbers theory is an generalization of Dempster-Shafer theory.

The rest of this paper is organized as follows. Section 2 gives a brief

introduction about the Dempster-Shafer theory. In Section 3, the proposed D numbers theory is presented, mainly including D numbers and D numbers combination rule. Some numerical examples are given to show the application of D numbers theory in Section 4. Finally, conclusions are given in Section 5.

2. Dempster-Shafer theory

For completeness of the explanation, a few basic concepts about Dempster-Shafer theory are introduced as follows.

For a finite nonempty set $\Omega = \{H_1, H_2, \dots, H_N\}$, Ω is called a frame of discernment (FOD) when satisfying

$$H_i \cap H_j = \emptyset, \quad \forall i, j = \{1, \dots, N\}. \quad (1)$$

Let 2^Ω be the set of all subsets of Ω , namely

$$2^\Omega = \{A \mid A \subseteq \Omega\}. \quad (2)$$

2^Ω is called the power set of Ω . For a FOD Ω , a mass function is a mapping m from 2^Ω to $[0, 1]$, formally defined by:

$$m : 2^\Omega \rightarrow [0, 1] \quad (3)$$

which satisfies the following condition:

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{A \in 2^\Omega} m(A) = 1 \quad (4)$$

In Dempster-Shafer theory, a mass function is also called a basic probability assignment (BPA). Given a BPA, the belief function $Bel : 2^\Omega \rightarrow [0, 1]$ is defined as

$$Bel(A) = \sum_{B \subseteq A} m(B) \quad (5)$$

The plausibility function $Pl : 2^\Omega \rightarrow [0, 1]$ is defined as

$$Pl(A) = 1 - Bel(\bar{A}) = \sum_{B \cap A \neq \emptyset} m(B) \quad (6)$$

where $\bar{A} = \Omega - A$. These functions Bel and Pl express the lower bound and upper bound in which subset A has been supported, respectively.

Given two independent BPAs m_1 and m_2 , Dempster's rule of combination, denoted by $m = m_1 \oplus m_2$, is used to combine them and it is defined as follows

$$m(A) = \begin{cases} \frac{1}{1-K} \sum_{B \cap C = A} m_1(B)m_2(C), & A \neq \emptyset; \\ 0, & A = \emptyset. \end{cases} \quad (7)$$

with

$$K = \sum_{B \cap C = \emptyset} m_1(B)m_2(C) \quad (8)$$

Note that the Dempster's rule of combination is only applicable to such two BPAs which satisfy the condition $K < 1$.

3. D numbers theory

In the mathematical framework of Dempster-Shafer theory, there are several strong hypotheses and constraints on the FOD and BPA. However, these hypotheses and constraints limit the ability of Dempster-Shafer theory to represent uncertain information.

First, a FOD must be a mutually exclusive and collectively exhaustive set, the elements of FOD are required to be mutually exclusive, as shown in Eq.(1). In many situations, however, it is very difficult to be satisfied. Take assessment as an example. In evaluating one object, it often uses linguistic variables to express the assessment result, such as “Very Good”, “Good”, “Fair”, “Bad” and “Very Bad”. Due to given by human, it inevitably exists intersections among these linguistic variables. Therefore, the exclusiveness hypothesis cannot be guaranteed precisely so that the application of Dempster-Shafer theory is questionable for such situations. There are already some studies about FOD with non-exclusive hypotheses [54, 150].

Second, the sum of basic probabilities of a normal BPA must be equal to 1, as shown in Eq.(4). We call it as completeness constraint. But in some cases, due to lack of knowledge and information, it is possible to obtain an incomplete BPA whose sum of basic probabilities is less than 1. For example, if an assessment is based on little partial information, the lack of information may result in a complete BPA cannot be obtained. Furthermore, in an open world [52], the incompleteness of FOD may also lead to the incompleteness of BPA. Hence the completeness constraint is hard to completely meet in some cases and it restricts the application of Dempster-Shafer theory.

To overcome these existing shortcomings in Dempster-Shafer theory and enhance its capability of expressing uncertain information, a novel theory, named as D numbers theory, is systematically proposed. D numbers theory looses FOD’s exclusiveness hypothesis and BPA’s completeness constraint, which is a generalization of Dempster-Shafer theory.

Definition 1. Let Θ be a nonempty set $\Theta = \{F_1, F_2, \dots, F_N\}$ satisfying $F_i \neq F_j$ if $i \neq j, \forall i, j = \{1, \dots, N\}$, D numbers is a mapping formulated by

$$D : 2^\Theta \rightarrow [0, 1] \quad (9)$$

with

$$\sum_{B \subseteq \Theta} D(B) \leq 1 \quad \text{and} \quad D(\emptyset) = 0 \quad (10)$$

where \emptyset is the empty set and B is a subset of Θ .

It is found that the definition of D numbers is similar with the definition of BPA. But note that, at first, differ from the definition of FOD in Dempster-Shafer theory, the exclusiveness hypothesis is removed, i.e., the elements in set Θ don't require mutually exclusive in D numbers. At second, the completeness constraint is released in D numbers. If $\sum_{B \subseteq \Theta} D(B) = 1$, the information is said to be complete; if $\sum_{B \subseteq \Theta} D(B) < 1$, the information is said to be incomplete. The degree of information's completeness is defined as below.

Definition 2. Let D be a D number on a finite nonempty set Θ , the degree of information's completeness in D is quantified by

$$Q = \sum_{B \subseteq \Theta} D(B) \quad (11)$$

In Dempster-Shafer theory, Dempster's rule of combination has played a central role to synthesize all the knowledge of the initial BPAs. Correspondingly, in D numbers theory which is treat as a generalization of Dempster-Shafer theory, a D numbers combination rule is proposed to combine the information indicated by D numbers.

Definition 3. Let D_1 and D_2 be two D numbers, the combination of D_1 and D_2 , indicated by $D = D_1 \odot D_2$, is defined by

$$\begin{cases} D(\emptyset) = 0 \\ D(B) = \frac{1}{1-K_D} \sum_{B_1 \cap B_2 = B} D_1(B_1)D_2(B_2), \quad B \neq \emptyset \end{cases} \quad (12)$$

with

$$K_D = \frac{1}{Q_1 Q_2} \sum_{B_1 \cap B_2 = \emptyset} D_1(B_1)D_2(B_2) \quad (13)$$

$$Q_1 = \sum_{B_1 \subseteq \Theta} D_1(B_1) \quad (14)$$

$$Q_2 = \sum_{B_2 \subseteq \Theta} D_2(B_2) \quad (15)$$

The proposed D numbers combination rule is a generalization of Dempster's rule of combination. If D_1, D_2 are defined on a FOD and $Q_1 = 1, Q_2 = 1$, the D numbers combination rule will degenerate to the Dempster's rule of combination. This combination rule provides a practical and feasible scheme to synthesize the uncertain information modeled by D numbers.

So far, D numbers theory has been proposed. In summary, it includes two main aspects. On the one hand, with respect to representation of uncertain information, D numbers provide a useful model. On the other hand, with respect to knowledge and uncertainty reasoning, D numbers combination rule can be employed to synthesize uncertain information.

4. Numerical Examples

In this section, some numerical examples are given to show the applications of proposed D numbers theory.

Example 1. Assume a local government plans to build a hydropower station nearby a river. Before to implement this project, environmental impact assessment (EIA) is carried out, which is to identify and assess the consequences or potential impacts of human activities to the environment. Two groups of experts are employed to execute the task, independently. Assume the evaluation result is expressed by linguistic variables *High*, *Medium* and *Low*. One group evaluates that the damage of this project to the environment is *High*. The other group's is *Medium*.

If these results are modeled by using Dempster-Shafer theory, two BPAs can be obtained that $m_1(\textit{High}) = 1$, $m_2(\textit{Medium}) = 1$. The Dempster's rule of combination is then used to combine the evaluations given by these two groups. However, due to m_1 and m_2 are completely conflicting, i.e., $K = 1$, the Dempster's rule of combination is unable to handle this situation. Actually, in Dempster-Shafer theory there is a hypothesis that *High* and *Medium* are mutually exclusive, i.e., $\textit{High} \cap \textit{Medium} = \emptyset$, as shown in Figure 1.

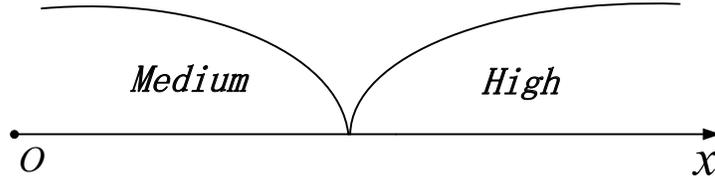


Figure 1: The linguistic variables of *High* and *Medium* in Dempster-Shafer theory

But in the real situation, it inevitably exists intersections among linguistic variables given by human beings. D numbers theory abandons the exclusive-

ness hypothesis that elements must be mutually exclusive, as shown in Figure 2. In D numbers theory, these evaluation results can be indicated by two D numbers that $D_1(High) = 1$, $D_2(Medium) = 1$. The combination of D_1 and D_2 is expressed by $D(High \cap Medium) = 1$. Therefore, D numbers theory is more reasonable and capable to model the imprecise, ambiguous, and vague information.

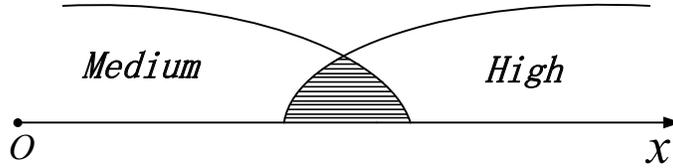


Figure 2: The linguistic variables of *High* and *Medium* in D numbers theory

Example 2. Medical diagnosis is a typical field that involves various types of uncertainties. Assume a patient is with the symptoms of fever, polypnea, cough. According to previous cases, it is likely caused by flu (F), Bacterial or fungal pneumonia (B), and upper respiratory infection (U). Two independent diagnostic reports are submitted by two doctors. One doctor diagnoses that *the patient got F with a possibility of 0.7, and got B or U with a possibility of 0.2, the remainder 0.1 is unknown.* The other doctor's diagnostic report shows that *it is 0.5 sure that the patient got F and 0.3 sure that the patient got B, the remainder 0.2 is also unknown.* The problem is what disease the patient got.

Let's consider this problem in the framework of Dempster-Shafer theory first. According to these two diagnostic reports, two BPAs can be obtained

$$m_1(F) = 0.7, m_1(B, U) = 0.2, m_1(F, B, U) = 0.1;$$

$$m_2(F) = 0.5, m_2(B) = 0.3, m_2(F, B, U) = 0.2.$$

Table 1: Intersection table to combine m_1 and m_2

$m_1 \oplus m_2$	$m_2(F) = 0.5$	$m_2(B) = 0.3$	$m_1(F, B, U) = 0.2$
$m_1(F) = 0.7$	$\{F\}(0.35)$	$\emptyset(0.21)$	$\{F\}(0.14)$
$m_1(B, U) = 0.2$	$\emptyset(0.10)$	$\{B\}(0.06)$	$\{B, U\}(0.04)$
$m_1(F, B, U) = 0.1$	$\{F\}(0.05)$	$\{B\}(0.03)$	$\{F, B, U\}(0.02)$

The intersection table of $m_1 \oplus m_2$ is shown in Table 1. Then, we can obtain that

$$K = m_1(B, U)m_2(F) + m_1(F)m_2(B) = 0.31;$$

$$m(F) = \frac{1}{1-K} (m_1(F)m_2(F) + m_1(F, B, U)m_2(F) + m_1(F)m_2(F, B, U)) = 0.7826;$$

$$m(B) = \frac{1}{1-K} (m_1(B, U)m_2(B) + m_1(F, B, U)m_2(B)) = 0.1304;$$

$$m(B, U) = \frac{1}{1-K} m_1(B, U)m_2(F, B, U) = 0.0580;$$

$$m(F, B, U) = \frac{1}{1-K} m_1(F, B, U)m_2(F, B, U) = 0.0290;$$

In the above calculating process, there is an invisible hypothesis that the possibility of unknown is equal to that of $\{F, B, U\}$. In other words, the set of all diseases causing the symptoms of fever, polypnea and cough, is seen as equivalent to the set $\{F, B, U\}$ which only contains three types of diseases. However, it can not be obtained according to the doctors' diagnostic reports. This invisible hypothesis obviously is not reasonable. In the real world, there may be other reasons resulting in these symptoms, but they are unknown due

to the limitation of human beings' current knowledge and cognitive level. For example, until 2003, SARS (Severe Acute Respiratory Syndrome) is found and then the knowledge about disease whose symptoms are fever, polypnea and cough is updated.

Maybe there are some debate about the above discussion. Someone would argue that a set X which includes all unknown factors can be imported in the construction of BPAs. For example, the first BPA can be constructed as $m_1(F) = 0.7$, $m_1(B, U) = 0.2$, $m_1(F, B, U, X) = 0.1$. By this means, the invisible hypothesis is removed. However, the situation is not as good as thought. At first, the complexity of this problem has greatly increased if introduce X . At second, actually the invisible hypothesis is almost ignored by people in the applications of Dempster-Shafer theory in order to reduce the complexity. At third, the proposed D numbers theory is congenitally able to well handle the situation of information incompleteness. In the following, we will investigate this example using D numbers theory.

Now let us consider this problem by using D numbers theory. According to the two pieces of information given by the diagnostic reports, two D numbers can be derived that

$$D_1(F) = 0.7, D_1(B, U) = 0.2;$$

$$D_2(F) = 0.5, D_2(B) = 0.3.$$

It is noted that the unknown information is not assigned to any set. The constructed two D numbers are in the forms of information incompleteness.

$$Q_1 = D_1(F) + D_1(B, U) = 0.9;$$

$$Q_2 = D_2(F) + D_2(B) = 0.8.$$

Table 2: Intersection table to combine D_w and D_s

$D_1 \odot D_2$	$D_2(F) = 0.5$	$D_2(B) = 0.3$
$D_1(F) = 0.7$	$\{F\}(0.35)$	$\emptyset(0.21)$
$D_1(B, U) = 0.2$	$\emptyset(0.10)$	$\{B\}(0.06)$

The intersection table of $D_1 \odot D_2$ is shown in Table 2. According to Table 2, we can calculate that

$$K_D = \frac{1}{Q_1 Q_2} (D_1(B, U)D_2(F) + D_1(F)D_2(B)) = 0.4306;$$

$$D(F) = \frac{1}{1-K_D} D_1(F)D_2(F) = 0.6147;$$

$$D(B) = \frac{1}{1-K_D} D_1(B, U)D_2(B) = 0.1054.$$

with $Q = D(F) + D(B) = 0.72$.

The result also shows the flu is the most probable disease the patient got. By comparison with Dempster-Shafer theory, however, in the proposed D numbers theory the unknown is inherited during the reasoning. D numbers theory has inherent advantage to handle the situation of information incompleteness, which is more natural and reasonable.

Example 3. Pattern recognition is a key technology in machine learning and many other fields. Supposing there is an object X which certainly belongs to one of three classes indicated by $\{A, B, C\}$. The weight sensor reports that X belongs to class A with a certainty of 0.6, and belongs to class C with a certainty of 0.4. The shape sensor reports that it is 0.7 sure that X belongs to classes A, B and the reminder 0.3 is with completely ignorance.

Dempster-Shafer theory can be used in this case. Due to the object

certainly belongs to one of these three classes, the FOD can be determined that $\Omega = \{A, B, C\}$. According to the reports of weight sensor and shape sensor, two BPAs are obtained that:

$$\begin{aligned} m_w(A) &= 0.6, m_w(C) = 0.4; \\ m_s(A, B) &= 0.7, m_s(A, B, C) = 0.3. \end{aligned}$$

Table 3: Intersection table to combine m_w and m_s

$m_w \oplus m_s$	$m_s(A, B) = 0.7$	$m_s(A, B, C) = 0.3$
$m_w(A) = 0.6$	$\{A\}(0.42)$	$\{A\}(0.18)$
$m_w(C) = 0.4$	$\emptyset(0.28)$	$\{C\}(0.12)$

The intersection table of $m_w \oplus m_s$ is shown in Table 3. Then, we can obtain that

$$\begin{aligned} K &= m_w(C)m_s(A, B) = 0.28; \\ m(A) &= \frac{1}{1-K} (m_w(A)m_s(A, B) + m_w(A)m_s(A, B, C)) = 0.8333; \\ m(C) &= \frac{1}{1-K} m_w(C)m_s(A, B, C) = 0.1667. \end{aligned}$$

Namely, the object X is with 0.8333 certainty belonging to class A , and with 0.1667 certainty belonging to class C by using Dempster-Shafer theory to combine the reports of weight sensor and shape sensor.

Now let's consider this problem using D numbers theory. At first, two D numbers can be derived to express the reports of weight sensor and shape sensor, respectively.

$$\begin{aligned} D_w(A) &= 0.6, D_w(C) = 0.4; \\ D_s(A, B) &= 0.7, D_s(A, B, C) = 0.3. \end{aligned}$$

Table 4: Intersection table to combine D_w and D_s

$D_w \odot D_s$	$D_s(A, B) = 0.7$	$D_s(A, B, C) = 0.3$
$D_w(A) = 0.6$	$\{A\}(0.42)$	$\{A\}(0.18)$
$D_w(C) = 0.4$	$\emptyset(0.28)$	$\{C\}(0.12)$

Note that due to X belongs to one of $\{A, B, C\}$ with 100% certainty, for the shape sensor's report, the remaining possibility 0.3 can be assigned to set $\{A, B, C\}$, namely $D_s(A, B, C) = 0.3$. It is reasonable. The intersection table of $D_w \odot D_s$ is shown in Table 4. So, we can calculate that

$$Q_w = D_w(A) + D_w(C) = 1.0;$$

$$Q_s = D_s(A, B) + D_s(A, B, C) = 1.0;$$

$$K_D = \frac{1}{Q_w Q_s} D_w(C) D_s(A, B) = 0.28;$$

$$D(A) = \frac{1}{1-K_D} (D_w(A) D_s(A, B) + D_w(A) D_s(A, B, C)) = 0.8333;$$

$$D(C) = \frac{1}{1-K_D} D_w(C) D_s(A, B, C) = 0.1667.$$

The result derived from D numbers theory is identical with that of Dempster-Shafer theory. In this example, the set $\{A, B, C\}$ is mutually exclusive and collectively exhaustive for this problem, and the two pieces of information are complete, therefore, both Dempster-Shafer theory and D numbers theory can handle this case, and D numbers theory has degenerated to Dempster-Shafer theory.

5. Conclusions

In this paper, a novel theory called D numbers theory is systematically proposed. The proposed D numbers theory is an generalization of Dempster-Shafer theory, which releases the FOD's exclusiveness hypothesis and BPA's completeness constraint in Dempster-Shafer theory. In the D numbers theory, D number is an extension of BPA, D numbers combination rule is an extension of Dempster's rule of combination. Some numerical examples have been given to show the application of the proposed theory. In the future research direction, one the one hand, the properties of D numbers theory will be further studied. On the other hand, this theory will be applied to many real applications.

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Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this article.

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