Introduction to Symbolic 2-Plithogenic Probability Theory

Article · September 2023

DOI: 10.54216/GJMSA.070202

4 authors, including:

Mohamed Bisher Zeina
University of Aleppo

Mohammad Abobala
Tishreen University

Introduction to Symbolic 2-Plithogenic Probability Theory

Mohamed Bisher Zeina[*1], Nizar Altounji[2], Mohammad Abobala[3], Yasin Karmouta[4]

[1] Department of Mathematical Statistics, Faculty of Science, University of Aleppo, Aleppo, Syria
[2] Faculty of Science, Department of Mathematical Statistics, University of Aleppo, Aleppo, Syria
[3] Department of Mathematics, Faculty of Science, Tishreen University, Latakia, Syria
[4] Faculty of Science, Department of Mathematical Statistics, University of Aleppo, Aleppo, Syria

Emails: bisher.zeina@gmail.com ; nizar.altounji.94@hotmail.com;
mohammadabobala777gmail.com ; yassinkarmouta@gmail.com

Abstract

In this paper we present for the first time the concept of symbolic plithogenic random variables and study its properties including expected value and variance. We build the plithogenic formal form of two important distributions that are exponential and uniform distributions. We find its probability density function and cumulative distribution function in its plithogenic form. We also derived its expected values and variance and the formulas of its random numbers generating. We finally present the fundamental form of plithogenic probability density and cumulative distribution functions. All the theorems were proved depending on algebraic approach using isomorphisms. This paper can be considered the base of symbolic plithogenic probability theory.

Keywords: Plithogenic; Probability Density Function; Cumulative Distribution Function; Random Numbers Generation; Exponential Distribution; Uniform Distribution.

1. Introduction

As a generalization of fuzzy and intuitionistic fuzzy sets Smarandache presented neutrosophic sets which is more powerful definition of uncertain sets and which has been applied in many fields of science including machine learning, artificial intelligence, statistics, engineering, etc.[1]–[15].

Presenting new sets and studying its algebraic structures is very interesting subject in mathematics. As a generalization of neutrosophic sets Smarandache presented plithogenic sets which is defined depending on split indeterminacy and on absorbance law.[16]–[31].

Symbolic probability theory was first presented in [32] by M.B. Zeina and A. Hatip where symbolic (literal) neutrosophic random variable was defined in the form $X_N = X + I; I^2 = I$ and this definition was applied in many other fields of probability theory (for example see [33], [34]). This definition was generalized by M.B. Zeina and M. Abobala in [35] to the form $X_N = X + IY; I^2 = I$ which is the most general form of a neutrosophic random variable and applied in many other branches of neutrosophic probability theory [36]–[39].

In this work we are going to present for the first time the main definitions related to plithogenic probability theory defined on 2-S plithogenic sets which may be applied in many fields of probability theory and its

Doi: https://doi.org/10.54216/GJMSA.070202
Received: March 19, 2023 Revised: June 28, 2023 Accepted: August 17, 2023

related branches of science including queueing theory, reliability theory, stochastic processes, survival analysis, etc. [1]–[3], [5], [10], [13], [36], [40]–[55].

2. Preliminaries

Definition 2.1

Set of symbolic 2-plithogenic real numbers is defined as follows:

$$2 - SP_R = \{a + bP_1 + cP_2; a, b, c \in R\}$$

Where:

$$P_1^2 = P_1, P_2^2 = P_2, P_1P_2 = P_2P_1 = P_{\max(1,2)} = P_2$$

Definition 2.2

Let $a_0 + a_1P_1 + a_2P_2, b_0 + b_1P_1 + b_2P_2 \in 2 - SP_R$, then addition and multiplication are defined as follows:

$$(a_0 + a_1P_1 + a_2P_2) + (b_0 + b_1P_1 + b_2P_2) = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2$$

$$(a_0 + a_1P_1 + a_2P_2) \cdot (b_0 + b_1P_1 + b_2P_2) = a_0b_0 + P_1(a_0b_1 + a_1b_0 + a_1b_1) + P_2(a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)$$

Definition 2.3

AH-Isometry on symbolic $2 - SP_R$ sets and its inverse can be defined as follows:

$$T: 2 - SP_R \to R \times R \times R;$$

$$T(a_0 + a_1P_1 + a_2P_2) = (a_0, a_0 + a_1, a_0 + a_1 + a_2)$$

$$T^{-1}: R \times R \times R \to 2 - SP_R;$$

$$T^{-1}(a_0, a_1, a_2) = a_0 + (a_1 - a_0)P_1 + (a_2 - a_1)P_2$$

Definition 2.4

We say that $a_P \geq_P b_P$ if $a_0 \geq b_0$ and $a_0 + a_1 \geq b_0 + b_1$ and $a_0 + a_1 + a_2 \geq b_0 + b_1 + b_2$.

Definition 2.5

A plithogenic number a_P is said to be nonnegative if $a_0 \geq 0$ and $a_0 + a_1 \geq 0$ and $a_0 + a_1 + a_2 \geq 0$.

3. Plithogenic Random Variables

Definition 3.1

We define 2-SP random variable as follows:

$$X_P: \Omega_P \to 2 - SP_R; \Omega_P = \Omega_0 \times \Omega_1(P_1) \times \Omega_2(P_2)$$

$$X_P = X_0 + X_1P_1 + X_2P_2; P_1^2 = P_1, P_2^2 = P_2, P_1P_2 = P_2P_1 = P_2$$

Where X_0, X_1, X_2 are classical random variables defined on $\Omega_0, \Omega_1, \Omega_2$ respectively and each random variable takes its in R.

Remark

Taking AH-Isometry transfers X_P to three classical random variables $(X_0, X_0 + X_1, X_0 + X_1 + X_2)$.

Theorem 3.1

Let X_P be a plithogenic random variable then expectation and variance of it are:

Doi: https://doi.org/10.54216/GJMSA.070202
Received: March 19, 2023 Revised: June 28, 2023 Accepted: August 17, 2023

1. $E(X_P) = E(X_0) + E(X_1)P_1 + E(X_2)P_2$

2. $V(X_P) = V(X_0) + [V(X_0 + X_1) - V(X_0)]P_1 + [V(X_0 + X_1 + X_2) - V(X_0 + X_1)]P_2$

Proof

We will prove the theorem assuming that X_p is discrete random variable and same proof can be done when X_p is continuous.

1. $E(X_P) = E(X_0 + X_1P_1 + X_2P_2) = \sum(x_0 + x_1P_1 + x_2P_2)f(x_0 + x_1P_1 + x_2P_2)$

Taking AH-Isometry:

$$T[E(X_P)] = T\left[\sum(x_0 + x_1P_1 + x_2P_2)f(x_0 + x_1P_1 + x_2P_2)\right]$$
$$= \sum T[(x_0 + x_1P_1 + x_2P_2)f(x_0 + x_1P_1 + x_2P_2)]$$
$$= \left(\sum x_0 f(x_0), \sum(x_0 + x_1)f(x_0 + x_1), \sum(x_0 + x_1 + x_2)f(x_0 + x_1 + x_2)\right)$$
$$= \big(E(X_0), E(X_0 + X_1), E(X_0 + X_1 + X_2)\big)$$

Taking the inverse isometry:

$$E(X_P) = T^{-1}\big(E(X_0), E(X_0 + X_1), E(X_0 + X_1 + X_2)\big)$$
$$= E(X_0) + [E(X_0 + X_1) - E(X_0)]P_1 + [E(X_0 + X_1 + X_2) - E(X_0 + X_1)]P_2$$
$$= E(X_0) + E(X_1)P_1 + E(X_2)P_2$$

2. $E(X_P^2) = E(X_0 + X_1P_1 + X_2P_2)^2 = \sum(x_0 + x_1P_1 + x_2P_2)^2 f(x_0 + x_1P_1 + x_2P_2)$

Taking AH-Isometry:

$$T[E(X_P^2)] = T\left[\sum(x_0 + x_1P_1 + x_2P_2)^2 f(x_0 + x_1P_1 + x_2P_2)\right]$$
$$= \sum T[(x_0 + x_1P_1 + x_2P_2)^2 f(x_0 + x_1P_1 + x_2P_2)]$$
$$= \left(\sum x_0^2 f(x_0), \sum(x_0 + x_1)^2 f(x_0 + x_1), \sum(x_0 + x_1 + x_2)^2 f(x_0 + x_1 + x_2)\right)$$
$$= (E(X_0^2), E(X_0 + X_1)^2, E(X_0 + X_1 + X_2)^2)$$

Taking the inverse isometry:

$$E(X_P^2) = T^{-1}(E(X_0^2), E(X_0 + X_1)^2, E(X_0 + X_1 + X_2)^2)$$
$$= E(X_0^2) + [E(X_0 + X_1)^2 - E(X_0^2)]P_1 + [E(X_0 + X_1 + X_2)^2 - E(X_0 + X_1)^2]P_2$$

We also have:

$$[E(X_P)]^2 = [E(X_0) + E(X_1)P_1 + E(X_2)P_2]^2 = T^{-1}T[E(X_0) + E(X_1)P_1 + E(X_2)P_2]^2$$
$$= T^{-1}[[E(X_0)]^2, [E(X_0 + X_1)]^2, [E(X_0 + X_1 + X_2)]^2]$$
$$= [E(X_0)]^2 + [[E(X_0 + X_1)]^2 - [E(X_0)]^2]P_1 + [[E(X_0 + X_1 + X_2)]^2 - [E(X_0 + X_1)]^2]P_2$$

So, we can write:

$$V(X_P) = E(X_P^2) - [E(X_P)]^2$$
$$= E(X_0^2) + [E(X_0 + X_1)^2 - E(X_0^2)]P_1 + [E(X_0 + X_1 + X_2)^2 - E(X_0 + X_1)^2]P_2 - [E(X_0)]^2$$
$$- [[E(X_0 + X_1)]^2 - [E(X_0)]^2]P_1 - [[E(X_0 + X_1 + X_2)]^2 - [E(X_0 + X_1)]^2]P_2$$
$$= V(X_0) + [V(X_0 + X_1) - V(X_0)]P_1 + [V(X_0 + X_1 + X_2) - V(X_0 + X_1)]P_2$$

Definition 3.2

A function $f(x_P)$ defined on $R(P_1, P_2)$ is called a plithogenic probability density function if it satisfies the following conditions:

1. $f(x_p) \geq 0$.
2. $\int_{-\infty}^{+\infty} f(x_P)dx_P = 1$.

Example 3.1

Doi: https://doi.org/10.54216/GJMSA.070202
Received: March 19, 2023 Revised: June 28, 2023 Accepted: August 17, 2023

Let $f(x_P) = 2x_P; 0 \leq_P x_P \leq_P 1$, then $f(x_P)$ is a plithogenic probability density function because:

$$T[f(x_P)] = T[2x_P] = T[2x_0 + 2x_1P_1 + 2x_2P_2] = (2x_0, 2x_0 + 2x_1, 2x_0 + 2x_1 + 2x_2)$$

Which are all nonnegative functions, we also have:

$$T\left[\int_0^1 f(x_P)dx_P\right] = \left(\int_0^1 2x_0dx_0\,, \int_0^1 (2x_0 + 2x_1)d(x_0 + x_1)\,, \int_0^1 (2x_0 + 2x_1 + 2x_2)d(x_0 + x_1 + x_2)\right) = (1,1,1)$$

Which means that:

$$\int_0^1 f(x_P)dx_P = T^{-1}(1,1,1) = 1$$

Theorem 3.2

Let $X_P = X_0 + X_1P_1 + X_2P_2$ be a plithogenic random variable, the moments generating function of it takes the form:

$$M_{X_P}(t) = M_{X_0}(t) + \left[M_{X_0+X_1}(t) - M_{X_0}(t)\right]P_1 + \left[M_{X_0+X_1+X_2}(t) - M_{X_0+X_1}(t)\right]P_2$$

Proof

$$\begin{aligned}
M_{X_P}(t) = E(e^{tX_P}) = \int_{-\infty}^{+\infty} e^{tx_P}f(x_P)dx_P &= T^{-1}T\left[\int_{-\infty}^{+\infty} e^{tx_P}f(x_P)dx_P\right] \\
&= T^{-1}\left(\int_{-\infty}^{+\infty} e^{tx_0}f(x_0)dx_0\,, \int_{-\infty}^{+\infty} e^{t(x_0+x_1)}f(x_0 + x_1)d(x_0 \right. \\
&\left. + x_1)\,, \int_{-\infty}^{+\infty} e^{t(x_0+x_1+x_2)}f(x_0 + x_1 + x_2)d(x_0 + x_1 + x_2)\right) \\
&= T^{-1}\left(M_{X_0}(t), M_{X_0+X_1}(t), M_{X_0+X_1+X_2}(t)\right) \\
&= M_{X_0}(t) + \left[M_{X_0+X_1}(t) - M_{X_0}(t)\right]P_1 + \left[M_{X_0+X_1+X_2}(t) - M_{X_0+X_1}(t)\right]P_2
\end{aligned}$$

Theorem 3.3

Let $X_P = X_0 + X_1P_1 + X_2P_2$ be a plithogenic random variable with a moments generating function $M_{X_P}(t)$ then:

$$\frac{d^k}{dt^k}M_{X_P}(t)|_{t=0} = E(X_P^k)$$

Proof

Since:

$$M_{X_P}(t) = M_{X_0}(t) + \left[M_{X_0+X_1}(t) - M_{X_0}(t)\right]P_1 + \left[M_{X_0+X_1+X_2}(t) - M_{X_0+X_1}(t)\right]P_2$$

Taking kth derivative of both sides and substituting $t = 0$ yields to:

$$\begin{aligned}
\frac{d^k}{dt^k}M_{X_P}(t)|_{t=0} &= \frac{d^k}{dt^k}\left(M_{X_0}(t) + \left[M_{X_0+X_1}(t) - M_{X_0}(t)\right]P_1 + \left[M_{X_0+X_1+X_2}(t) - M_{X_0+X_1}(t)\right]P_2\right)|_{t=0} \\
&= \frac{d^k}{dt^k}M_{X_0}(t)|_{t=0} + \left[\frac{d^k}{dt^k}M_{X_0+X_1}(t)|_{t=0} - \frac{d^k}{dt^k}M_{X_0}(t)|_{t=0}\right]P_1 \\
&+ \left[\frac{d^k}{dt^k}M_{X_0+X_1+X_2}(t)|_{t=0} - \frac{d^k}{dt^k}M_{X_0+X_1}(t)|_{t=0}\right]P_2 \\
&= E(X_0^k) + [E(X_0 + X_1)^k - E(X_0^k)]P_1 + [E(X_0 + X_1 + X_2)^k - E(X_0 + X_1)^k]P_2 = E(X_P^k)
\end{aligned}$$

Doi: https://doi.org/10.54216/GJMSA.070202
Received: March 19, 2023 Revised: June 28, 2023 Accepted: August 17, 2023

4. Some Continuous Plithogenic Probability Distributions

Definition 4.1

A plithogenic random variable X_P is said to be exponentially distributed with the plithogenic parameter λ_P if its probability density function takes the form:

$$f(x_P) = \lambda_P e^{-\lambda_P x_P} \,;\, x_P, \lambda_P >_P 0$$

Theorem 4.1

The formal plithogenic form of probability density function of exponential distribution is:

$$f(x_P) = \lambda_0 e^{-\lambda_0 x_0} + \left[(\lambda_0+\lambda_1)e^{-(\lambda_0+\lambda_1)(x_0+x_1)} - \lambda_0 e^{-\lambda_0 x_0}\right]P_1 + \left[(\lambda_0+\lambda_1+\lambda_2)e^{-(\lambda_0+\lambda_1+\lambda_2)(x_0+x_1+x_2)} - (\lambda_0+\lambda_1)e^{-(\lambda_0+\lambda_1)(x_0+x_1)}\right]P_2$$

Proof

$$\begin{aligned} T[f(x_P)] &= T\left[\lambda_P e^{-\lambda_P x_P}\right] = T\left[(\lambda_0+\lambda_1P_1+\lambda_2P_2)e^{-(\lambda_0+\lambda_1P_1+\lambda_2P_2)\,(x_0+x_1P_1+x_2P_2)}\right] \\ &= T[\lambda_0+\lambda_1P_1+\lambda_2P_2\,]T\left[e^{-(\lambda_0+\lambda_1P_1+\lambda_2P_2)\,(x_0+x_1P_1+x_2P_2)}\right] \\ &= T[\lambda_0+\lambda_1P_1+\lambda_2P_2\,]\left[e^{-T(\lambda_0+\lambda_1P_1+\lambda_2P_2)\,T(x_0+x_1P_1+x_2P_2)}\right] \\ &= (\lambda_0, \lambda_0+\lambda_1, \lambda_0+\lambda_1+\lambda_2)e^{-(\lambda_0,\lambda_0+\lambda_1,\lambda_0+\lambda_1+\lambda_2)(x_0,x_0+x_1,x_0+x_1+x_2)} \\ &= \left(\lambda_0 e^{-\lambda_0 x_0}, (\lambda_0+\lambda_1)e^{-(\lambda_0+\lambda_1)(x_0+x_1)}, (\lambda_0+\lambda_1+\lambda_2)e^{-(\lambda_0+\lambda_1+\lambda_2)(x_0+x_1+x_2)}\right) \end{aligned}$$

So:

$$\begin{aligned} f(x_P) &= T^{-1}\left(\lambda_0 e^{-\lambda_0 x_0}, (\lambda_0+\lambda_1)e^{-(\lambda_0+\lambda_1)(x_0+x_1)}, (\lambda_0+\lambda_1+\lambda_2)e^{-(\lambda_0+\lambda_1+\lambda_2)(x_0+x_1+x_2)}\right) \\ &= \lambda_0 e^{-\lambda_0 x_0} + \left[(\lambda_0+\lambda_1)e^{-(\lambda_0+\lambda_1)(x_0+x_1)} - \lambda_0 e^{-\lambda_0 x_0}\right]P_1 \\ &+ \left[(\lambda_0+\lambda_1+\lambda_2)e^{-(\lambda_0+\lambda_1+\lambda_2)(x_0+x_1+x_2)} - (\lambda_0+\lambda_1)e^{-(\lambda_0+\lambda_1)(x_0+x_1)}\right]P_2 \end{aligned}$$

Example 4.1

$$f(x_P) = e^{-x_0} + \left[2e^{-2(x_0+x_1)} - e^{-x_0}\right]P_1 + \left[4e^{-4(x_0+x_1+x_2)} - 2e^{-2(x_0+x_1)}\right]P_2$$

Is a plithogenic probability density function of an exponentially distributed random variable with parameter $\lambda_P = 1 + P_1 + 2P_2$.

Theorem 4.2

The plithogenic cumulative distribution function of plithogenic exponential distribution is:

$$F(x_P) = 1 - e^{-\lambda_0 x_0} + \left[e^{-\lambda_0 x_0} - e^{-(\lambda_0+\lambda_1)(x_0+x_1)}\right]P_1 + \left[e^{-(\lambda_0+\lambda_1)(x_0+x_1)} - e^{-(\lambda_0+\lambda_1+\lambda_2)(x_0+x_1+x_2)}\right]P_2$$

Proof

$$F(x_P) = \int_0^{x_P} f(t_P)dt_P$$

$$\begin{aligned} T\big(F(x_P)\big) &= T\left(\int_0^{x_P} f(t_P)dt_P\right) \\ &= T\left(\int_0^{(x_0+x_1P_1+x_2P_2)} \left((\lambda_0+\lambda_1P_1+\lambda_2P_2)e^{-(\lambda_0+\lambda_1P_1+\lambda_2P_2)(t_0+t_1P_1+t_2P_2)}\right) d(t_0+t_1P_1 + t_2P_2)\right) \end{aligned}$$

$$= \left(\int_0^{x_0} \lambda_0 e^{-\lambda_0 t_0}dt_0\,, \int_0^{x_0+x_1} (\lambda_0+\lambda_1)e^{-(\lambda_0+\lambda_1)(t_0+t_1)}d(t_0+t_1), \int_0^{x_0+x_1+x_2} (\lambda_0+\lambda_1 + \lambda_2)e^{-(\lambda_0+\lambda_1+\lambda_2)(t_0+t_1+t_2)}d(t_0+t_1+t_2)\right)$$

$$= \left(1 - e^{-\lambda_0 x_0}, 1 - e^{-(\lambda_0+\lambda_1)(x_0+x_1)}, 1 - e^{-(\lambda_0+\lambda_1+\lambda_2)(x_0+x_1+x_2)}\right)$$

Doi: https://doi.org/10.54216/GJMSA.070202
Received: March 19, 2023 Revised: June 28, 2023 Accepted: August 17, 2023

Which yields by taking T^{-1} to:

$$F(x_P) = 1 - e^{-\lambda_0 x_0} + \left[1 - e^{-(\lambda_0+\lambda_1)(x_0+x_1)} - \left(1 - e^{-\lambda_0 x_0}\right)\right]P_1 + \left[1 - e^{-(\lambda_0+\lambda_1+\lambda_2)(x_0+x_1+x_2)} - \left(1 - e^{-(\lambda_0+\lambda_1)(x_0+x_1)}\right)\right]P_2$$

Or:

$$F(x_P) = 1 - e^{-\lambda_0 x_0} + \left[e^{-\lambda_0 x_0} - e^{-(\lambda_0+\lambda_1)(x_0+x_1)}\right]P_1 + \left[e^{-(\lambda_0+\lambda_1)(x_0+x_1)} - e^{-(\lambda_0+\lambda_1+\lambda_2)(x_0+x_1+x_2)}\right]P_2$$

Theorem 4.3

Let X_P be a plithogenic exponential random variable with plithogenic parameter $\lambda_p > 0$ then:

1. $E(X_P) = \frac{1}{\lambda_0} + \left(\frac{1}{\lambda_0+\lambda_1} - \frac{1}{\lambda_0}\right)P_1 + \left(\frac{1}{\lambda_0+\lambda_1+\lambda_2} - \frac{1}{\lambda_0+\lambda_1}\right)P_2$
2. $Var(X_P) = \frac{1}{\lambda_0^2} + \left[\frac{1}{(\lambda_0+\lambda_1)^2} - \frac{1}{\lambda_0^2}\right]P_1 + \left[\frac{1}{(\lambda_0+\lambda_1+\lambda_2)^2} - \frac{1}{(\lambda_0+\lambda_1)^2}\right]P_2$

Proof

1. $E(X_P) = T^{-1}T\left(\int_0^\infty x_P \lambda_P e^{-\lambda_P x_P} dx_P\right)$

$$= T^{-1}\left(\int_0^\infty x_0\lambda_0 e^{-\lambda_0 x_0} dx_0, \int_0^\infty (x_0+x_1)(\lambda_0+\lambda_1)e^{-(\lambda_0+\lambda_1)(x_0+x_1)} d(x_0+x_1), \int_0^\infty (x_0+x_1+x_2)(\lambda_0+\lambda_1+\lambda_2)e^{-(\lambda_0+\lambda_1+\lambda_2)(x_0+x_1+x_2)} d(x_0+x_1+x_2)\right) = T^{-1}\left(\frac{1}{\lambda_0}, \frac{1}{\lambda_0+\lambda_1}, \frac{1}{\lambda_0+\lambda_1+\lambda_2}\right)$$

$$= \frac{1}{\lambda_0} + \left(\frac{1}{\lambda_0+\lambda_1} - \frac{1}{\lambda_0}\right)P_1 + \left(\frac{1}{\lambda_0+\lambda_1+\lambda_2} - \frac{1}{\lambda_0+\lambda_1}\right)P_2$$

2. $Var(X_P) = T^{-1}T\left(\int_0^\infty [x_P - E(X_P)]^2 \lambda_P e^{-\lambda_P x_P} dx_P\right) = T^{-1}\left(\int_0^\infty \left(x_0 - \frac{1}{\lambda_0}\right)^2 \lambda_0 e^{-\lambda_0 x_0} dx_0, \int_0^\infty \left(x_0 + x_1 - \frac{1}{\lambda_0+\lambda_1}\right)^2 (\lambda_0+\lambda_1)e^{-(\lambda_0+\lambda_1)(x_0+x_1)} d(x_0+x_1), \int_0^\infty \left(x_0+x_1+x_2 - \frac{1}{\lambda_0+\lambda_1+\lambda_2}\right)^2 (\lambda_0+\lambda_1+\lambda_2)e^{-(\lambda_0+\lambda_1+\lambda_2)(x_0+x_1+x_2)} d(x_0+x_1+x_2)\right) = T^{-1}\left(\frac{1}{\lambda_0^2}, \frac{1}{(\lambda_0+\lambda_1)^2}, \frac{1}{(\lambda_0+\lambda_1+\lambda_2)^2}\right) = \frac{1}{\lambda_0^2} + \left[\frac{1}{(\lambda_0+\lambda_1)^2} - \frac{1}{\lambda_0^2}\right]P_1 + \left[\frac{1}{(\lambda_0+\lambda_1+\lambda_2)^2} - \frac{1}{(\lambda_0+\lambda_1)^2}\right]P_2$

Definition 4.2

A plithogenic random variable X_P is said to be uniformly distributed with the plithogenic parameters a_P, b_p if its probability density function takes the form:

$$f(x_P) = \frac{1}{b_P - a_P};\, a_P < x_P < b_P$$

Theorem 4.4

The formal plithogenic form of probability density function of uniform distribution is:

$$f(x_P) = \frac{1}{b_0 - a_0} + \left[\frac{1}{b_0 + b_1 - a_0 - a_1} - \frac{1}{b_0 - a_0}\right]P_1 + \left[\frac{1}{b_0 + b_1 + b_2 - a_0 - a_1 - a_2} - \frac{1}{b_0 + b_1 - a_0 - a_1}\right]P_2$$

Doi: https://doi.org/10.54216/GJMSA.070202
Received: March 19, 2023 Revised: June 28, 2023 Accepted: August 17, 2023

Proof

$$f(x_P)=T^{-1}T[f(x_P)]=T^{-1}T\left[\frac{1}{b_P-a_P}\right]=T^{-1}T\left[\frac{1}{b_0+b_1P_1+b_2P_2-a_0-a_1P_1-a_2P_2}\right]$$
$$=T^{-1}T\left[\frac{1}{b_0-a_0+(b_1-a_1)P_1+(b_2-a_2)P_2}\right]$$
$$=T^{-1}\left(\frac{1}{b_0-a_0},\frac{1}{b_0+b_1-a_0-a_1},\frac{1}{b_0+b_1+b_2-a_0-a_1-a_2}\right)$$
$$=\frac{1}{b_0-a_0}+\left[\frac{1}{b_0+b_1-a_0-a_1}-\frac{1}{b_0-a_0}\right]P_1+\left[\frac{1}{b_0+b_1+b_2-a_0-a_1-a_2}-\frac{1}{b_0+b_1-a_0-a_1}\right]P_2$$

Theorem 4.5

Plithogenic cumulative distribution function of plithogenic uniform distribution is:

$$F(x_P)=\frac{x_0-a_0}{b_0-a_0}+\left[\frac{x_0+x_1-a_0-a_1}{b_0-a_0+b_1-a_1}-\frac{x_0-a_0}{b_0-a_0}\right]P_1+\left[\frac{x_0+x_1+x_2-a_0-a_1-a_2}{b_0-a_0+b_1-a_1+b_2-a_2}-\frac{x_0+x_1-a_0-a_1}{b_0-a_0+b_1-a_1}\right]P_2$$

Proof

$$F(x_P)=\int_{a_P}^{x_P}f(t_P)dt_P$$

$$T\big(F(x_P)\big)=T\left(\int_{a_P}^{x_P}f(t_P)dt_P\right)$$
$$=T\left(\int_{(a_0+a_1P_1+a_2P_2)}^{(x_0+x_1P_1+x_2P_2)}\left(\frac{1}{b_0+b_1P_1+b_2P_2-a_0-a_1P_1-a_2P_2}\right)d(t_0+t_1P_1+t_2P_2)\right)$$
$$=\left(\int_{a_0}^{x_0}\frac{1}{b_0-a_0}dt_0,\int_{a_0+a_1}^{x_0+x_1}\frac{1}{b_0-a_0+b_1-a_1}d(t_0+t_1),\int_{a_0+a_1+a_2}^{x_0+x_1+x_2}\frac{1}{b_0-a_0+b_1-a_1+b_2-a_2}d(t_0+t_1+t_2)\right)$$
$$=\left(\frac{x_0-a_0}{b_0-a_0},\frac{x_0+x_1-a_0-a_1}{b_0-a_0+b_1-a_1},\frac{x_0+x_1+x_2-a_0-a_1-a_2}{b_0-a_0+b_1-a_1+b_2-a_2}\right)$$

Taking T^{-1} yields to:

$$F(x_P)=\frac{x_0-a_0}{b_0-a_0}+\left[\frac{x_0+x_1-a_0-a_1}{b_0-a_0+b_1-a_1}-\frac{x_0-a_0}{b_0-a_0}\right]P_1+\left[\frac{x_0+x_1+x_2-a_0-a_1-a_2}{b_0-a_0+b_1-a_1+b_2-a_2}-\frac{x_0+x_1-a_0-a_1}{b_0-a_0+b_1-a_1}\right]P_2$$

Theorem 4.6

Let X_P be a plithogenic uniform random variable with plithogenic parameters a_P, b_p then:

1. $E(X_P)=\frac{a_0+b_0}{2}+\left[\frac{a_0+a_1+b_0+b_1}{2}-\frac{a_0+b_0}{2}\right]P_1+\left[\frac{a_0+a_1+a_2+b_0+b_1+b_2}{2}-\frac{a_0+a_1+b_0+b_1}{2}\right]P_2$
2. $Var(X_P)=\frac{(b_0-a_0)^2}{12}+\left[\frac{(b_0+b_1-a_0-a_1)^2}{12}-\frac{(b_0-a_0)^2}{12}\right]P_1+\left[\frac{(b_0+b_1+b_2-a_0-a_1-a_2)^2}{12}-\frac{(b_0+b_1-a_0-a_1)^2}{12}\right]P_2$

Proof

1. $E(X_P)=T^{-1}T\left(\int_{a_P}^{b_P}\frac{x_P}{b_P-a_P}dx_P\right)$

Doi: https://doi.org/10.54216/GJMSA.070202
Received: March 19, 2023 Revised: June 28, 2023 Accepted: August 17, 2023

$$= T^{-1}\left(\int_{a_0}^{b_0} \frac{x_0}{b_0 - a_0} dx_0\,, \int_{a_0+a_1}^{b_0+b_1} \frac{x_0 + x_1}{b_0 + b_1 - a_0 - a_1} d(x_0 + x_1)\,, \int_{a_0+a_1+a_2}^{b_0+b_1+b_2} \frac{x_0 + x_1 + x_2}{b_0 + b_1 + b_2 - a_0 - a_1 - a_2} d(x_0 + x_1 + x_2)\right)$$

$$= T^{-1}\left(\frac{a_0 + b_0}{2}, \frac{a_0 + a_1 + b_0 + b_1}{2}, \frac{a_0 + a_1 + a_2 + b_0 + b_1 + b_2}{2}\right)$$

$$= \frac{a_0 + b_0}{2} + \left[\frac{a_0 + a_1 + b_0 + b_1}{2} - \frac{a_0 + b_0}{2}\right] P_1 + \left[\frac{a_0 + a_1 + a_2 + b_0 + b_1 + b_2}{2} - \frac{a_0 + a_1 + b_0 + b_1}{2}\right] P_2$$

2.

$$Var(X_P) = T^{-1}T\left(\int_{a_P}^{b_P} \frac{[x_P - E(X_P)]^2}{b_P - a_P} dx_P\right)$$

$$= T^{-1}\left(\int_{a_0}^{b_0} \left(x_0 - \frac{a_0 + b_0}{2}\right)^2 \frac{1}{b_0 - a_0} dx_0\,, \int_{a_0+a_1}^{b_0+b_1} \left(x_0 + x_1 - \frac{a_0 + a_1 + b_0 + b_1}{2}\right)^2 \frac{1}{b_0 + b_1 - a_0 - a_1} d(x_0 + x_1)\,, \int_{0}^{\infty} \left(x_0 + x_1 + x_2 - \frac{a_0 + a_1 + a_2 + b_0 + b_1 + b_2}{2}\right)^2 \frac{1}{b_0 + b_1 + b_2 - a_0 - a_1 - a_2} d(x_0 + x_1 + x_2)\right) = T^{-1}\left(\frac{(b_0 - a_0)^2}{12}, \frac{(b_0 + b_1 - a_0 - a_1)^2}{12}, \frac{(b_0 + b_1 + b_2 - a_0 - a_1 - a_2)^2}{12}\right)$$

$$= \frac{(b_0 - a_0)^2}{12} + \left[\frac{(b_0 + b_1 - a_0 - a_1)^2}{12} - \frac{(b_0 - a_0)^2}{12}\right] P_1 + \left[\frac{(b_0 + b_1 + b_2 - a_0 - a_1 - a_2)^2}{12} - \frac{(b_0 + b_1 - a_0 - a_1)^2}{12}\right] P_2$$

5. Fundamental Form of Continuous Plithogenic Probability Densities and Plithogenic Cumulative Distribution Functions

Let X_P be a plithogenic random variable that has a probability density function $f(x_P;\Theta_P)$ and cumulative distribution function $F(x_P;\Theta_P)$ with $\Theta_P = (\theta_{1P}, \dots, \theta_{kP}); \theta_{iP} = \theta_{i0} + \theta_{i1}P_1 + \theta_{i2}P_2\,; i = 1,2, \dots, k$ a vector of parameters, in this section we provide two fundamental theorems in plithogenic probability theory:

Theorem 5.1

Formal form of $f(x_P;\Theta_P)$ is:

$$f(x_P;\Theta_P) = f(x_0;\Theta_0) + [f(x_0 + x_0;\Theta_0 + \Theta_1) - f(x_0;\Theta_0)]\, P_1 + [f(x_0 + x_1 + x_2;\Theta_0 + \Theta_1 + \Theta_2) - f(x_0 + x_1;\Theta_0 + \Theta_1)]P_2$$

Where: $\Theta_0 = (\theta_{10}, \dots, \theta_{k0})$, $\Theta_0 + \Theta_1 = (\theta_{10} + \theta_{11}, \dots, \theta_{k0} + \theta_{k1})$, $\Theta_0 + \Theta_1 + \Theta_2 = (\theta_{10} + \theta_{11} + \theta_{12}, \dots, \theta_{k0} + \theta_{k1} + \theta_{k2})$

Proof

Using one dimensional AH-Isometry:

$$T(f(x_P;\Theta_P)) = T(f(x_0 + x_1P_1 + x_2P_2;\Theta_0 + \Theta_1P_1 + \Theta_2P_2))$$
$$= f(T(x_0 + x_1P_1 + x_2P_2;\Theta_0 + \Theta_1P_1 + \Theta_2P_2))$$
$$= f\big((x_0, x_0 + x_1, x_0 + x_1 + x_2); (\Theta_0, \Theta_0 + \Theta_1, \Theta_0 + \Theta_1 + \Theta_2)\big)$$
$$= (f(x_0;\Theta_0), f(x_0 + x_1;\Theta_0 + \Theta_1), f(x_0 + x_1 + x_2;\Theta_0 + \Theta_1 + \Theta_2))$$

Taking T^{-1} of both sides:

$$\Rightarrow f(x_P;\Theta_P) = T^{-1}\big((f(x_0;\Theta_0), f(x_0 + x_1;\Theta_0 + \Theta_1), f(x_0 + x_1 + x_2;\Theta_0 + \Theta_1 + \Theta_2))\big)$$
$$= f(x_0;\Theta_0) + [f(x_0 + x_1;\Theta_0 + \Theta_1) - f(x_0;\Theta_0)]P_1 + [f(x_0 + x_1 + x_2;\Theta_0 + \Theta_1 + \Theta_2) - f(x_0 + x_1;\Theta_0 + \Theta_1)\,]P_2$$

Doi: https://doi.org/10.54216/GJMSA.070202
Received: March 19, 2023 Revised: June 28, 2023 Accepted: August 17, 2023

Theorem 5.2

Formal form of continuous plithogenic cumulative distribution function is:

$$F(x_P;\Theta_P) = F(x_0;\Theta_0) + [F(x_0+x_1;\Theta_0+\Theta_1) - F(x_0;\Theta_0)]P_1 + [F(x_0+x_1+x_2;\Theta_0+\Theta_1+\Theta_2) - F(x_0+x_1;\Theta_0+\Theta_1)]P_2$$

Proof

Straightforward by integration of results in theorem 5.1.

6. Plithogenic Random Numbers Generation

Let X_P be a random variable that has a plithogenic pdf $f(x_P)$ and plithogenic cdf $F(x_P)$, if we assumed that $U_P = F(x_P)$, then U_P is a plithogenic random variable uniformly distributed on [0,1], i.e., $U_P \sim Unif(0,1)$.

We can write X_P as a function of U_P to generate random plithogenic numbers as we do in the classical case, let's work with the already presented distributions in section 4.

6.1 Random numbers generation according to plithogenic uniform distribution

We have:

$$F(x_P) = \frac{x_0-a_0}{b_0-a_0} + \left[\frac{x_0+x_1-a_0-a_1}{b_0-a_0+b_1-a_1} - \frac{x_0-a_0}{b_0-a_0}\right]P_1 + \left[\frac{x_0+x_1+x_2-a_0-a_1-a_2}{b_0-a_0+b_1-a_1+b_2-a_2} - \frac{x_0+x_1-a_0-a_1}{b_0-a_0+b_1-a_1}\right]P_2$$

By setting $U_P = F(x_P) \Longrightarrow T(U_P) = T(F(x_P))$, then:

$$(u_0, u_0+u_1, u_0+u_1+u_2) = \left(\frac{x_0-a_0}{b_0-a_0}, \frac{x_0+x_1-a_0-a_1}{b_0-a_0+b_1-a_1}, \frac{x_0+x_1+x_2-a_0-a_1-a_2}{b_0-a_0+b_1-a_1+b_2-a_2}\right)$$

$$u_0 = \frac{x_0-a_0}{b_0-a_0}$$

$$\Longrightarrow x_0 = u_0(b_0-a_0) + a_0$$

$$u_0+u_1 = \frac{x_0+x_1-a_0-a_1}{b_0-a_0+b_1-a_1}$$

$$\Longrightarrow x_0+x_1 = (u_0+u_1)(b_0-a_0+b_1-a_1) + (a_0+a_1)$$

$$u_0+u_1+u_2 = \frac{x_0+x_1+x_2-a_0-a_1-a_2}{b_0-a_0+b_1-a_1+b_2-a_2}$$

$$\Longrightarrow x_0+x_1+x_2 = (u_0+u_1+u_2)(b_0-a_0+b_1-a_1+b_2-a_2) + (a_0+a_1+a_2)$$

Hence:

$$(x_0, x_0+x_1, x_0+x_1+x_2) = \big(u_0(b_0-a_0)+a_0\,, (u_0+u_1)(b_0-a_0+b_1-a_1) + (a_0+a_1), (u_0+u_1+u_2)(b_0-a_0+b_1-a_1+b_2-a_2) + (a_0+a_1+a_2)\big)$$

Now we take T^{-1} of both sides and get:

$$x_P = u_0(b_0-a_0)+a_0 + [(u_0+u_1)(b_0-a_0+b_1-a_1) + (a_0+a_1) - u_0(b_0-a_0)+a_0]P_1 + [(u_0+u_1+u_2)(b_0-a_0+b_1-a_1+b_2-a_2) + (a_0+a_1+a_2) - (u_0+u_1)(b_0-a_0+b_1-a_1) + (a_0+a_1)]P_2$$

6.2 Random numbers generation according to plithogenic exponential distribution

We proved that:

$$F(x_P) = 1 - e^{-\lambda_0 x_0} + \left[1 - e^{-(\lambda_0+\lambda_1)(x_0+x_1)} - (1 - e^{-\lambda_0 x_0})\right]P_1 + \left[1 - e^{-(\lambda_0+\lambda_1+\lambda_2)(x_0+x_1+x_2)} - (1 - e^{-(\lambda_0+\lambda_1)(x_0+x_1)})\right]P_2$$

$$\Longrightarrow (u_0, u_0+u_1, u_0+u_1+u_2) = \left(1 - e^{-\lambda_0 x_0}, 1 - e^{-(\lambda_0+\lambda_1)(x_0+x_1)}, 1 - e^{-(\lambda_0+\lambda_1+\lambda_2)(x_0+x_1+x_2)}\right)$$

Doi: https://doi.org/10.54216/GJMSA.070202
Received: March 19, 2023 Revised: June 28, 2023 Accepted: August 17, 2023

$$u_0 = 1 - e^{-\lambda_0 x_0}$$
$$\Rightarrow x_0 = -\frac{\ln(1-u_0)}{\lambda_0}$$
$$u_0 + u_1 = 1 - e^{-(\lambda_0+\lambda_1)(x_0+x_1)}$$
$$\Rightarrow x_0 + x_1 = -\frac{\ln(1-(u_0+u_1))}{\lambda_0+\lambda_1}$$
$$u_0 + u_1 + u_2 = 1 - e^{-(\lambda_0+\lambda_1+\lambda_2)(x_0+x_1+x_2)}$$
$$\Rightarrow x_0 + x_1 + x_2 = -\frac{\ln(1-(u_0+u_1+u_2))}{\lambda_0+\lambda_1+\lambda_2}$$

Hence:

$$(x_0, x_0+x_1, x_0+x_1+x_2) = \left(-\frac{\ln(1-u_0)}{\lambda_0}, -\frac{\ln\big(1-(u_0+u_1)\big)}{\lambda_0+\lambda_1}, -\frac{\ln\big(1-(u_0+u_1+u_2)\big)}{\lambda_0+\lambda_1+\lambda_2}\right)$$

Now we take T^{-1} of both sides and get:

$$x_P = -\frac{\ln(1-u_0)}{\lambda_0} + \left[-\frac{\ln\big(1-(u_0+u_1)\big)}{\lambda_0+\lambda_1} - \left(-\frac{\ln(1-u_0)}{\lambda_0}\right)\right] P_1 + \left[-\frac{\ln(1-(u_0+u_1+u_2))}{\lambda_0+\lambda_1+\lambda_2} - \left(-\frac{\ln(1-(u_0+u_1))}{\lambda_0+\lambda_1}\right)\right] P_2$$
$$\Rightarrow x_P = -\frac{\ln(1-u_0)}{\lambda_0} + \left[\frac{\ln(1-u_0)}{\lambda_0} - \frac{\ln\big(1-(u_0+u_1)\big)}{\lambda_0+\lambda_1}\right] P_1 + \left[\frac{\ln(1-(u_0+u_1))}{\lambda_0+\lambda_1} - \frac{\ln(1-(u_0+u_1+u_2))}{\lambda_0+\lambda_1+\lambda_2}\right] P_2$$

7. Conclusion

We have presented the formal form of plithogenic random variable and plithogenic probability density functions and studied its important properties including plithogenic expectation, plithogenic variance, plithogenic moments generating function, some plithogenic probability distributions and its probabilistic properties, plithogenic random numbers generation. Many theorems were presented and proved with an algebraic approach depending on algebraic isomorphisms. In future researches we are looking forward to study applications of this new definition in many branches of probability theory like reliability theory, queueing theory, dynamic systems, stochastic processes, etc.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

Acknowledgement: The authors are thankful for the editorial team and reviewers for their important comments and valuable review of the paper.

References

[1] F. Smarandache, "Introduction to Neutrosophic Measure, Neutrosophic Integral, and Neutrosophic Probability," *ArXiv*, 2013.

[2] F. Smarandache, "Introduction to Neutrosophic Statistics," *Branch Mathematics and Statistics Faculty and Staff Publications*, Jan. 2014, Accessed: Feb. 21, 2023. [Online]. Available: https://digitalrepository.unm.edu/math_fsp/33

[3] F. Smarandache, "Indeterminacy in neutrosophic theories and their applications," *International Journal of Neutrosophic Science*, vol. 15, no. 2, 2021, doi: 10.5281/zenodo.5295819.

[4] F. Smarandache, "(T, I, F)-Neutrosophic Structures," *Applied Mechanics and Materials*, vol. 811, 2015, doi: 10.4028/www.scientific.net/amm.811.104.

[5] F. Smarandache, "Neutrosophic theory and its applications," *Brussels*, vol. I, 2014.

Doi: https://doi.org/10.54216/GJMSA.070202
Received: March 19, 2023 Revised: June 28, 2023 Accepted: August 17, 2023

[6] W. B. Vasantha Kandasamy and Florentin. Smarandache, "Fuzzy cognitive maps and neutrosophic cognitive maps," p. 211, 2003.

[7] M. Akram, Shumaiza, and F. Smarandache, "Decision-making with bipolar neutrosophic TOPSIS and bipolar neutrosophic ELECTRE-I," *Axioms*, vol. 7, no. 2, 2018, doi: 10.3390/axioms7020033.

[8] F. Smarandache *et al.*, "Introduction to neutrosophy and neutrosophic environment," *Neutrosophic Set in Medical Image Analysis*, pp. 3–29, 2019, doi: 10.1016/B978-0-12-818148-5.00001-1.

[9] S. Broumi, M. B. Zeina, M. Lathamaheswari, A. Bakali, and M. Talea, "A Maple Code to Perform Operations on Single Valued Neutrosophic Matrices," *Neutrosophic Sets and Systems*, vol. 49, 2022.

[10] M. Miari, M. T. Anan, and M. B. Zeina, "Single Valued Neutrosophic Kruskal-Wallis and Mann Whitney Tests," *Neutrosophic Sets and Systems*, vol. 51, 2022, doi: 10.5281/zenodo.7163297.

[11] M. Mullai, K. Sangeetha, R. Surya, G. M. Kumar, R. Jeyabalan, and S. Broumi, "A Single Valued neutrosophic Inventory Model with Neutrosophic Random Variable," *International Journal of Neutrosophic Science*, vol. 1, no. 2, 2020, doi: 10.5281/zenodo.3679510.

[12] M. B. Zeina, O. Zeitouny, F. Masri, F. Kadoura, and S. Broumi, "Operations on single-valued trapezoidal neutrosophic numbers using (α, β, γ)-cuts 'maple package,'" *International Journal of Neutrosophic Science*, vol. 15, no. 2, 2021, doi: 10.54216/IJNS.150205.

[13] D. Nagarajan and J. Kavikumar, "Single-Valued and Interval-Valued Neutrosophic Hidden Markov Model," *Math Probl Eng*, vol. 2022, 2022, doi: 10.1155/2022/5323530.

[14] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst*, vol. 20, no. 1, pp. 87–96, 1986, doi: 10.1016/S0165-0114(86)80034-3.

[15] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965, doi: 10.1016/S0019-9958(65)90241-X.

[16] P. K. Singh, "Complex Plithogenic Set," *International Journal of Neutrosophic Science*, vol. 18, no. 1, 2022, doi: 10.54216/IJNS.180106.

[17] S. Alkhazaleh, "Plithogenic Soft Set," *Neutrosophic Sets and Systems*, 2020.

[18] N. Martin and F. Smarandache, "Introduction to Combined Plithogenic Hypersoft Sets," *Neutrosophic Sets and Systems*, vol. 35, 2020, doi: 10.5281/zenodo.3951708.

[19] F. Smarandache, *Plithogeny, Plithogenic Set, Logic, Probability, and Statistics*. Belgium: Pons, 2018. Accessed: Feb. 23, 2023. [Online]. Available: http://arxiv.org/abs/1808.03948

[20] F. Smarandache, "Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)," *Neutrosophic Sets and Systems* , vol. 53, Jan. 2023, doi: 10.5281/ZENODO.7536105.

[21] N. M. Taffach and A. Hatip, "A Review on Symbolic 2-Plithogenic Algebraic Structures," *Galoitica: Journal of Mathematical Structures and Applications*, vol. 5, no. 1, pp. 08–16, 2023, doi: 10.54216/GJMSA.050101.

[22] R. Ali and Z. Hasan, "An Introduction To The Symbolic 3-Plithogenic Modules," *Galoitica: Journal of Mathematical Structures and Applications*, vol. 6, no. 1, pp. 13–17, 2023, doi: 10.54216/GJMSA.060102.

[23] F. Smarandache, "Introduction to Plithogenic Logic as generalization of MultiVariate Logic," *Neutrosophic Sets and Systems*, vol. 45, 2021.

[24] R. Ali and Z. Hasan, "An Introduction to The Symbolic 3-Plithogenic Vector Spaces," *Galoitica: Journal of Mathematical Structures and Applications*, vol. 6, no. 1, pp. 08–12, 2023, doi: 10.54216/GJMSA.060101.

[25] F. Smarandache, "Plithogenic Probability & Statistics are generalizations of MultiVariate Probability & Statistics," *Neutrosophic Sets and Systems*, vol. 43, 2021.

[26] F. Smarandache, "Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets," *Neutrosophic Sets and Systems*, vol. 21, pp. 153–166, 2018.

Doi: https://doi.org/10.54216/GJMSA.070202
Received: March 19, 2023 Revised: June 28, 2023 Accepted: August 17, 2023

[27] M. Abdel-Basset and R. Mohamed, "A novel plithogenic TOPSIS- CRITIC model for sustainable supply chain risk management," *J Clean Prod*, vol. 247, Feb. 2020, doi: 10.1016/J.JCLEPRO.2019.119586.

[28] M. Abdel-Basset, M. El-hoseny, A. Gamal, and F. Smarandache, "A novel model for evaluation Hospital medical care systems based on plithogenic sets," *Artif Intell Med*, vol. 100, Sep. 2019, doi: 10.1016/J.ARTMED.2019.101710.

[29] N. M. Taffach and A. Hatip, "A Brief Review on The Symbolic 2-Plithogenic Number Theory and Algebraic Equations," *Galoitica: Journal of Mathematical Structures and Applications*, vol. 5, no. 1, pp. 36–44, 2023, doi: 10.54216/GJMSA.050103.

[30] F. Sultana *et al.*, "A study of plithogenic graphs: applications in spreading coronavirus disease (COVID-19) globally," *J Ambient Intell Humaniz Comput*, p. 1, 2022, doi: 10.1007/S12652-022-03772-6.

[31] M. Abdel-Basset, R. Mohamed, A. E. N. H. Zaied, and F. Smarandache, "A hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics," *Symmetry (Basel)*, vol. 11, no. 7, Jul. 2019, doi: 10.3390/SYM11070903.

[32] M. B. Zeina and A. Hatip, "Neutrosophic Random Variables," *Neutrosophic Sets and Systems*, vol. 39, 2021, doi: 10.5281/zenodo.4444987.

[33] C. Granados and J. Sanabria, "On Independence Neutrosophic Random Variables," *Neutrosophic Sets and Systems*, vol. 47, 2021.

[34] C. Granados, "New Notions On Neutrosophic Random Variables," *Neutrosophic Sets and Systems*, vol. 47, 2021.

[35] M. B. Zeina and M. Abobala, "A novel approach of neutrosophic continuous probability distributions using AH-isometry with applications in medicine," *Cognitive Intelligence with Neutrosophic Statistics in Bioinformatics*, pp. 267–286, Jan. 2023, doi: 10.1016/B978-0-323-99456-9.00014-3.

[36] M. B. Zeina and Y. Karmouta, "Introduction to Neutrosophic Stochastic Processes," *Neutrosophic Sets and Systems*, vol. 54, 2023.

[37] A. Astambli, M. B. Zeina, and Y. Karmouta, "Algebraic Approach to Neutrosophic Confidence Intervals," *Journal of Neutrosophic and Fuzzy Systems*, vol. 5, no. 2, pp. 08–22, 2023, doi: 10.54216/JNFS.050201.

[38] M. B. Zeina, M. Abobala, A. Hatip, S. Broumi, and S. Jalal Mosa, "Algebraic Approach to Literal Neutrosophic Kumaraswamy Probability Distribution," *Neutrosophic Sets and Systems*, vol. 54, pp. 124–138, 2023.

[39] A. Astambli, M. B. Zeina, and Y. Karmouta, "On Some Estimation Methods of Neutrosophic Continuous Probability Distributions Using One-Dimensional AH-Isometry," *Neutrosophic Sets and Systems*, vol. 53, 2023.

[40] S. Alias and D. Mohamad, "A Review on Neutrosophic Set and Its Development," 2018.

[41] M. Aslam and M. Albassam, "Presenting post hoc multiple comparison tests under neutrosophic statistics," *J King Saud Univ Sci*, vol. 32, no. 6, 2020, doi: 10.1016/j.jksus.2020.06.008.

[42] M. Aslam, "Neutrosophic analysis of variance: application to university students," *Complex and Intelligent Systems*, vol. 5, no. 4, 2019, doi: 10.1007/s40747-019-0107-2.

[43] F. Smarandache, "Symbolic Neutrosophic Theory," *ArXiv*, 2015, doi: 10.5281/ZENODO.32078.

[44] R. A. K. Sherwani, T. Arshad, M. Albassam, M. Aslam, and S. Abbas, "Neutrosophic entropy measures for the Weibull distribution: theory and applications," *Complex and Intelligent Systems*, vol. 7, no. 6, 2021, doi: 10.1007/s40747-021-00501-y.

[45] K. F. Alhasan, A. A. Salama, and F. Smarandache, "Introduction to neutrosophic reliability theory," *International Journal of Neutrosophic Science*, vol. 15, no. 1, 2021, doi: 10.5281/zenodo.5033829.

[46] H. Rashad and M. Mohamed, "Neutrosophic Theory and Its Application in Various Queueing Models: Case Studies," *Neutrosophic Sets and Systems*, vol. 42, 2021, doi: 10.5281/zenodo.4711516.

Doi: https://doi.org/10.54216/GJMSA.070202
Received: March 19, 2023 Revised: June 28, 2023 Accepted: August 17, 2023

[47] R. A. K. Sherwani, M. Naeem, M. Aslam, M. A. Raza, M. Abid, and S. Abbas, “Neutrosophic Beta Distribution with Properties and Applications,” *Neutrosophic Sets and Systems*, vol. 41, 2021.

[48] M. B. Zeina, “Neutrosophic M/M/1, M/M/c, M/M/1/b Queueing Systems,” *Research Journal of Aleppo University*, vol. 140, 2020, Accessed: Feb. 21, 2023. [Online]. Available: https://www.researchgate.net/publication/343382302_Neutrosophic_MM1_MMc_MM1b_Queueing_Systems

[49] M. B. Zeina, “Linguistic Single Valued Neutrosophic M/M/1 Queue,” *Research Journal of Aleppo University*, vol. 144, 2021, Accessed: Feb. 21, 2023. [Online]. Available: https://www.researchgate.net/publication/348945390_Linguistic_Single_Valued_Neutrosophic_MM1_Queue

[50] W. Q. Duan, Z. Khan, M. Gulistan, and A. Khurshid, “Neutrosophic Exponential Distribution: Modeling and Applications for Complex Data Analysis,” *Complexity*, vol. 2021, 2021, doi: 10.1155/2021/5970613.

[51] Z. Khan, M. Gulistan, N. Kausar, and C. Park, “Neutrosophic Rayleigh Model with Some Basic Characteristics and Engineering Applications,” *IEEE Access*, vol. 9, pp. 71277–71283, 2021, doi: 10.1109/ACCESS.2021.3078150.

[52] R. A. K. Sherwani, M. Aslam, M. A. Raza, M. Farooq, M. Abid, and M. Tahir, “Neutrosophic Normal Probability Distribution—A Spine of Parametric Neutrosophic Statistical Tests: Properties and Applications,” *Neutrosophic Operational Research*, pp. 153–169, 2021, doi: 10.1007/978-3-030-57197-9_8.

[53] M. B. Zeina, “Erlang Service Queueing Model with Neutrosophic Parameters,” *International Journal of Neutrosophic Science*, vol. 6, no. 2, pp. 106–112, 2020, doi: 10.54216/IJNS.060202.

[54] M. B. Zeina, “Neutrosophic Event-Based Queueing Model,” *International Journal of Neutrosophic Science*, vol. 6, no. 1, 2020, doi: 10.5281/zenodo.3840771.

[55] M. Miari, M. T. Anan, and M. B. Zeina, “Neutrosophic Two Way ANOVA,” *International Journal of Neutrosophic Science*, vol. 18, no. 3, 2022, doi: 10.54216/IJNS.180306.

[56] Ben Othman, K., Von Shtawzen, O., Khaldi, A., and Ali, R., "On The Concept Of Symbolic 7-Plithogenic Real Matrices", Pure Mathematics For Theoretical Computer Science, Vol.1, 2023.

[57] Ben Othman, K., Von Shtawzen, O., Khaldi, A., and Ali, R., "On The Symbolic 8-Plithogenic Matrices", Pure Mathematics For Theoretical Computer Science, Vol.1, 2023.

Doi: https://doi.org/10.54216/GJMSA.070202
Received: March 19, 2023 Revised: June 28, 2023 Accepted: August 17, 2023