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A note on Smarandache number related triangles

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Abstract The triangle T(a, b, c) with angles a, b, c, and the triangle $T(a', b', c')$ with angles a', b', c' are said to be pseudo Smarandache related if Z(a) =Z(a′), Z(b) =Z(b′), Z(c) =Z(c′), and the pair of triangles T(a,b, c) and $T(a', b', c')$ are said to be Smarandache related if $S(a) = S(a'), S(b) = S(b'), S(c) = S(c')$, where Z(.) is the pseudo Smarandache function, and S(.) is the Smarandache function. This paper lists all the dissimilar pseudo Smarandache related triangles, under the additional condition that a =a′, found by computer search.

Keywords Pseudo Smarandache function, Smarandache function, pseudo Smarandache related triangles, Smarandache function related triangles.

§1. Introduction

The pseudo Smarandache function, denoted by Z(n), has been introduced by Kashihara [1]. Since then, the pseudo Smarandache function has seen several generalizations in different directions. One such generalization is the concept of the pseudo Smarandache related triangles, proposed by Ashbacher [2]. Actually, the idea of the Smarandache related triangles was introduced by Sastry [3], and Ashcacher [2] extended the idea to include the pseudo Smarandache function as well. The formal definitions of the pseudo Smarandache function, Smarandache function, and the Smarandache number related triangles are given below.

Definition 1.1. For any integer n ≥1, the pseudo Smarandache function, Z(n), is the minimum integer m such that 1+2+$\cdots$ +m is divisible by n, that is

$$Z(n) = \min\left\{m : m \in \mathrm{Z}^+, n|\frac{m(m+1)}{2}\right\}, n \geq 1,$$

where Z^+ is the set of all positive integers.

Definition 1.2. Two triangles T(a, b, c) (with angles a, b and c) and $T(a', b', c')$ (with angles a', b' and c'), are said to be Smarandache related if

$$Z(a) = Z(a'), Z(b) = Z(b'), Z(c) = Z(c'),$$

where $a + b + c = 180 = a' + b' + c'$.

Definition 1.3. The Smarandache function S(n) is defined as follows:

$$S(n) = \min\left\{m : m \in \mathrm{Z}^+, n|m!\right\}, n \geq 1,$$

and two triangles T(a, b, c) and $T(a', b', c')$ are said to be Smarandache related if

$$S(a) = S(a'), S(b) = S(b'), S(c) = S(c').$$

Definition 1.4. A triangle is said to be Pythagorean if and only if one of its angle is 90^0.

Definition 1.5. Two triangles are said to be similar if the angles of one triangle are equal to the corresponding angles of the second triangle, in any order.

So far as the authors know, not much work has been done in connection with the Smarandache number related triangles. Ashbacher [2] reports some of the dissimilar pairs of Smarandache number related triangles, found using a computer search.

This paper reports all pairs of dissimilar pseudo Smarandache related triangles, under the additional restriction that a = a'. Though such a restricted search might be unwanted, nevertheless, the number of such dissimilar triangles reduces very dramatically, only to 59. On the other hand, the number of Smarandache related triangles under the same condition is 1072. We believe that a closer study of these triangles would be helpful in further research. These triangles are given in the next Section 2. We conclude this paper with some discussion in the final Section 3.

§2. Computational results

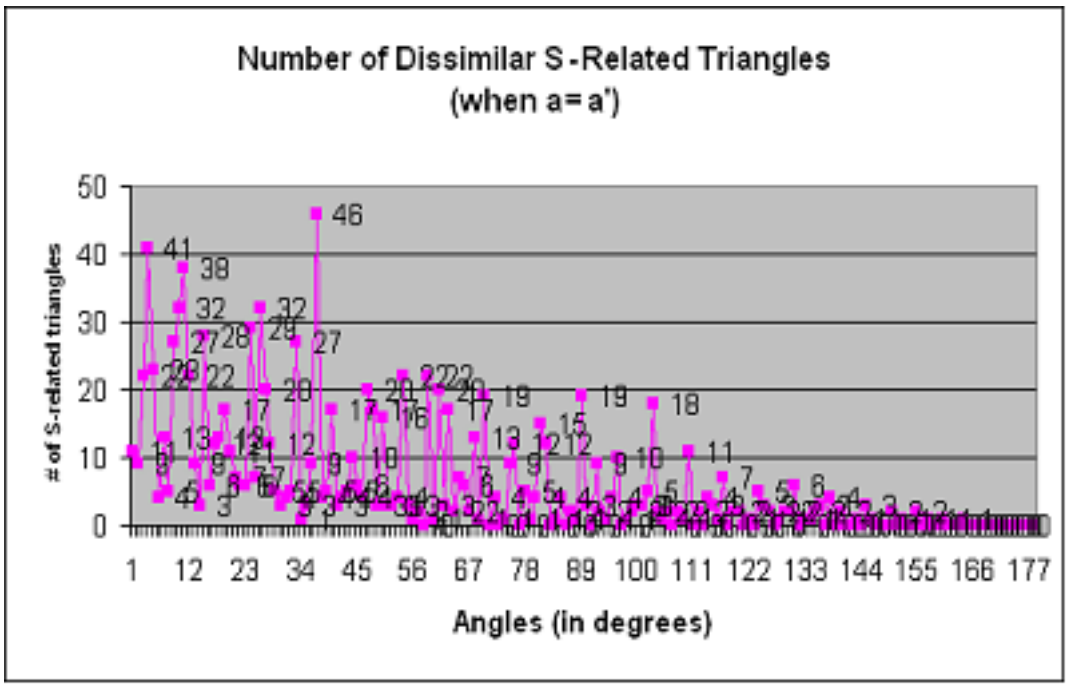
We searched for all dissimilar pseudo Smarandache function related triangles T(a,b,c) (with angles a, b and c) and $T(a', b', c')$, under the additional restriction that a =a′, on a computer. Our findings are given below.

(1) a =a′ =4; T(4,8,168), T(4,120,56) with Z(8) =15 =Z(120), Z(168) =48 =Z(56),
(2) a =a′) =4; T(4,44,132), T(4,88,88) with Z(44) =32 =Z(88) =Z(132),
(3) a =a′ =4; T(4,80,96), T(4,104,72) with Z(80) =64 =Z(104), Z(96) =63 =Z(72),
(4) a =a′ =5; T(5,25,150), T(5,100,75) with Z(25) =24 =Z(100), Z(150) =24 =Z(75),
(5) a =a′ =9; T(9,38,133), T(9,95,76) with Z(38) =19 =Z(95), Z(133) =56 =Z(76),
(6) a =a′ =10; T(10,20,150), T(10,120,50) with Z(20) =15 =Z(120), Z(150) =24 =Z(50),
(7) a =a′ =11; T(11,13,156), T(11,39,130) with Z(13) =12 =Z(39), Z(156) =39 =Z(130),
(8) a =a′ =12; T(12,14,154), T(12,28,140) with Z(14) =7 =Z(28), Z(154) =55 =Z(140),
(9) a =a′ =19; T(19,7,154), T(19,21,140) with Z(7) =6 =Z(21), Z(154) =55 =Z(140),
(10) a =a′ =20; T(20,48,112), T(20,88,72) with Z(48) =32 =Z(88), Z(112) =63 =Z(72),
(11) a =a′ =25; T(25,31,124), T(25,93,62) with Z(31) =30 =Z(93), Z(124) =31 =Z(62),
(12) a =a′ =26; T(26,16,138), T(26,62,92) with Z(16) =31 =Z(62), Z(138) =23 =Z(92),
(13) a =a′ =26; T(26,22,132), T(26,66,88) with Z(22) =11 =Z(66), Z(132) =32 =Z(88),
(14) a =a′ =27; T(27,34,119), T(27,68,85) with Z(34) =16 =Z(68), Z(119) =34 =Z(85),
(15) a =a′ =28; T(28,8,144), T(28,40,112) with Z(8) =15 =Z(40), Z(144) =63 =Z(112),
(16) a =a′ =28; T(28,8,144), T(28,120,32) with Z(8) =15 =Z(120), Z(144) =63 =Z(32),
(17) a =a′ =28; T(28,32,120), T(28,112,40) with Z(32) =63 =Z(112), Z(120) =15 =Z(40),
(18) a =a′ =30; T(30,50,100), T(30,75,75) with Z(50) =24 =Z(75) =Z(100),
(19) a =a′ =32; T(32,37,111), T(32,74,74) with Z(37) =36 =Z(111) =Z(74),
(20) a =a′ =33; T(33,27,120), T(33,117,30) with Z(27) =26 =Z(117), Z(120) =15 =Z(30),

(21) a =a′ =36; T(36,32,112), T(36,72,72) with Z(32) =63 =Z(72) =Z(112),
(22) a =a′ =37; T(37,11,132), T(37,55,88) with Z(11) =10 =Z(55), Z(132) =32 =Z(88),
(23) a =a′ =42; T(42,46,92), T(42,69,69) with Z(46) =23 =Z(69) =Z(92),
(24) a =a′ =44; T(44,24,112), T(44,40,96) with Z(24) =15 =Z(40), Z(112) =63 =Z(96),
(25) a =a′ =47; T(47,19,114), T(47,57,76) with Z(19) =18 =Z(57), Z(114) =56 =Z(76),
(26) a =a′ =48; T(48,20,112), T(48,60,72) with Z(20) =15 =Z(60), Z(112) =63 =Z(72),
(27) a =a′ =48; T(48,22,110), T(48,33,99) with Z(22) =11 =Z(33), Z(110) =44 =Z(99),
(28) a =a′ =54; T(54,42,84), T(54,70,56) with Z(42) =20 =Z(70), Z(84) =48 =Z(56),
(29) a =a′ =55; T(55,25,100), T(55,50,75) with Z(25) =24 =Z(75) =Z(50) =Z(100),
(30) a =a′ =60; T(60,8,112), T(60,24,96) with Z(8) =15 =Z(24), Z(112) =63 =Z(96),
(31) a =a′ =60; T(60,32,88), T(60,72,48) with Z(32) =63 =Z(72), Z(88) =32 =Z(48),
(32) a =a′ =63; T(63,9,108), T(63,36,81) with Z(9) =8 =Z(36), Z(81) =80 =Z(108),
(33) a =a′ =64; T(64,12,104), T(64,36,80) with Z(12) =8 =Z(36), Z(104) =64 =Z(80),
(34) a =a′ =68; T(68,14,98), T(68,28,84) with Z(14) =7 =Z(28), Z(84) =48 =Z(98),
(35) a =a′ =70; T(70,22,88), T(70,66,44) with Z(22) =11 =Z(66), Z(88) =32 =Z(44),
(36) a =a′ =72; T(72,4,104), T(72,28,80) with Z(4) =7 =Z(28), Z(104) =64 =Z(80),
(37) a =a′ =72; T(72,12,96), T(72,36,72) with Z(12) =8 =Z(36), Z(96) =63 =Z(72),
(38) a =a′ =72; T(72,16,92), T(72,62,46) with Z(16) =31 =Z(62), Z(92) =23 =Z(46),
(39) a =a′ =72; T(72,20,88), T(72,60,48) with Z(20) =15 =Z(60), Z(88) =32 =Z(48),
(40) a =a′ =75; T(75,7,98), T(75,21,84) with Z(7) =6 =Z(21), Z(98) =48 =Z(84),
(41) a =a′ =80; T(80,4,96), T(80,28,72) with Z(4) =7 =Z(28), Z(96) =63 =Z(72),
(42) a =a′ =80; T(80,25,75), T(80,50,50) with Z(25) =24 =Z(50) =Z(75),
(43) a =a′ =81; T(81,11,88), T(81,55,44) with Z(11) =10 =Z(55), Z(88) =32 =Z(44),
(44) a =a′ =88; T(88,20,72), T(88,60,32) with Z(20) =15 =Z(60), Z(72) =63 =Z(32),
(45) a =a′ =94; T(94,8,78), T(94,60,26) with Z(8) =15 =Z(60), Z(78) =12 =Z(26),
(46) a =a′ =100; T(100,20,60), T(100,40,40) with Z(20) =15 =Z(40) =Z(60),
(47) a =a′ =103; T(103,11,66), T(103,55,22) with Z(11) =10 =Z(55), Z(66) =11 =Z(22),
(48) a =a′ =108; T(108,9,63), T(108,18,54) with Z(9) =8 =Z(18), Z(63) =27 =Z(54),
(49) a =a′ =112; T(112,20,48), T(112,24,44) with Z(20) =15 =Z(24), Z(48) =32 =Z(44),
(50) a =a′ =120; T(120,20,40), T(120,30,30) with Z(20) =15 =Z(30) =Z(40),
(51) a =a′ =128; T(128,13,39), T(128,26,26) with Z(13) =12 =Z(26) =Z(39),
(52) a =a′ =130; T(130,2,48), T(130,6,44) with Z(2) =3 =Z(6), Z(48) =32 =Z(44),
(53) a =a′ =132; T(132,8,40), T(132,24,24) with Z(8) =15 =Z(24) =Z(40),
(54) a =a′ =136; T(136,4,40), T(136,14,30) with Z(4) =7 =Z(14), Z(40) =15 =Z(30),
(55) a =a′ =138; T(138,12,30), T(138,18,24) with Z(12) =8 =Z(18), Z(30) =15 =Z(24),
(56) a =a′ =140; T(140,4,36), T(140,28,12) with Z(4) =7 =Z(28), Z(36) =8 =Z(12),
(57) a =a′ =145; T(145,7,28), T(145,21,14) with Z(7) =6 =Z(21), Z(28) =7 =Z(14),
(58) a =a′ =146; T(146,4,30), T(146,14,20) with Z(4) =7 =Z(14), Z(30) =15 =Z(20),
(59) a =a′ =154; T(154,2,24), T(154,6,20) with Z(2) =3 =Z(6), Z(24) =15 =Z(20).

We also looked for all dissimilar Smarandache function related triangles T(a,b,c) (with angles a, b and c) and T(a′,b′,c′), under the same condition that a =a′, on a computer. Our findings are given below, both in the tabular and graphical forms. For the values of a =a′, not

listed in the table, the number of such a pair of triangles is 0 in each case. Thus, for example, there is no pair of dissimilar Smarandache function related triangles each with one angle fixed at 58 degrees.



§3. Some observations and remarks

In Section 2, we report all the pseudo Smarandache function related dissimilar triangles T(a,b,c) and $T(a', b', c')$ (with a+b+c =180 = $a' + b' + c'$), under the additional condition that $a = a'$. The cases that do not appear in the list are either cannot occur or lead to similar triangles.

Ashbacher [2], based on an exhaustive computer search for pairs of all dissimilar pseudo Smarandache function related triangles with values of a in the range 1 ≤a ≤178, reports that a cannot take the following values (1):

1, 15, 23, 35, 41, 45, 51, 59, 65, 67, 71, 73, 77, 79, 82, 83, 86, 87, 89,
90, 91, 97, 101, 102, 105, 107, 109, 113, 115, 116, 118, 121, 123, 125, 126, 127,
131, 134, 135, 137, 139, 141, 142, 143, 148, 149, 151, 152, 153, 157, 158, 159,
161, 163, 164, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178.

The values of a =a′ for which we get pairs of dissimilar triangles can be compared with the values given in (1). Ashbacher also gives the values of a in 1 ≤a ≤178 for which there are no dissimilar Smarandache function related pairs of triangles.

a = a.′	Number of S-related triangles	a = a.′	Number of S-related triangles	a = a.′	Number of S-related triangles	a = a.′	Number of S-related triangles	a = a.′	Number of S-related triangles
1	11	28	12	55	3	88	1	122	1
2	9	29	5	56	1	89	19	124	5
3	22	30	3	57	2	90	3	125	3
4	41	31	4	59	22	92	9	126	2
5	23	32	5	60	1	93	2	128	1
6	4	33	27	61	20	94	1	129	2
7	13	34	1	62	3	95	4	130	2
8	5	35	3	63	17	96	10	131	6
9	27	36	9	64	2	98	1	133	1
10	32	37	46	65	7	99	2	134	1
11	38	38	4	66	6	100	3	135	2
12	22	39	5	67	2	101	3	136	3
13	9	40	17	68	13	102	5	138	4
14	3	41	3	69	1	103	18	140	2
15	28	42	4	70	19	104	2	143	1
16	6	43	5	72	4	105	1	145	3
17	12	44	10	74	1	106	2	146	1
18	13	45	6	75	9	108	2	150	2
19	17	46	4	76	12	109	1	152	1
20	11	47	20	78	5	110	11	155	2
21	7	48	17	79	1	112	2	157	1
22	6	49	3	80	4	114	4	160	1
23	6	50	16	81	15	115	3	164	1
24	29	51	3	82	12	116	1		
25	7	52	3	84	1	117	7		
26	32	53	4	85	4	119	2		
27	20	54	22	87	2	120	2		

We get two pairs of 60 degrees triangles, namely, the pairs T(60, 8, 112), T(60, 24, 96) and T(60, 32, 88), T(60, 72, 48), which are pseudo Smarandache function related. There is only one pair of 120 degrees dissimilar triangles, T(120, 20, 40) and T(120, 30, 30), which are pseudo Smarandache function related, while there is no Pythogorean dissimilar pseudo Smarandache function related triangles.

Looking for the pairs of triangles which are Smarandache function related, we get 25. The pairs of triangles which are both Smarandache function related and pseudo Smarandache function related are as follows :

T(4,44,132), T(4,88,88) with S(44) =11 =S(88) =S(132),
T(5,25,150), T(5,100,75) with S(25) =10 =S(100) =S(150) =S(75),
T(9,76,95), T(9,133,38) with S(76) =19 =S(133) =S(95) =S(38),
T(10,20,150), T(10,120,50) with S(20) =5 =S(120), S(150) =10 =S(50),
T(11,13,156), T(11,39,130) with s(13) =13 =S(39) =S(156) =S(130),
T(25,31,124), T(25,93,62) with S(31) =31 =S(93) =Z(124) =S(62),
T(26,22,132), T(26,66,88) with S(22) =11 =S(66) =S(132) =S(88),
T(27,34,119), T(27,68,85) with S(34) =17 =S(68) =S(119) =S(85),
T(30,50,100), T(30,75,75) with S(50) =10 =S(75) =S(100),
T(32,37,111), T(32,74,74) with S(37) =37 =S(111) =S(74),
T(37,11,132), T(37,55,88) with S(11) =11 =S(55) =S(132) =S(88),
T(42,46,92), T(42,69,69) with S(46) =23 =S(69) =S(92),
T(47,19,114), T(47,57,76) with S(19) =19 =S(57) =S(114) =S(76),
T(48,22,110), T(48,33,99) with S(22) =11 =S(33) =S(110) =S(99),
T(54,42,84), T(54,70,56) with S(42) =7 =S(70) =S(84) =S(56),
T(55,25,100), T(55,50,75) with S(25) =10 =S(75) =S(50) =S(100),
T(63,9,108), T(63,36,81) with S(9) =6 =S(36), S(81) =9 =S(108),
T(70,22,88), T(70,66,44) with S(22) =11 =S(66) =S(88) =S(44),
T(80,25,75), T(80,50,50) with S(25) =10 =S(50) =S(75),
T(81,11,88), T(81,55,44) with S(11) =11 =S(55) =S(88) =S(44),
T(100,20,60), T(100,40,40) with S(20) =5 =S(40) =S(60),
T(103,11,66), T(103,55,22) with S(11) =11 =S(55) =S(66) =S(22),
T(120,20,40), T(120,30,30) with S(20) =5 =S(30) =S(40),
T(128,13,39), T(128,26,26) with S(13) =13 =S(26) =S(39),
T(145,7,28), T(145,21,14) with S(7) =7 =S(21) =S(28) =S(14).

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