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On a Smarandache Closed and Completely Filter of a Smarandache BH-algebra

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Abstract

In this paper, the notions of a Smarandache closed filter and Smarandache completely closed filter of a Smarandache BH-Algebra are introduced Also, Some properties of these notions are studied.

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Keywords: BCK-algebra, BH-algebra, BH-algebra, Smarandache a filter of Smarandache BH-algebra.

1 Introduction

The notion of BCK-algebras was formulated first in 1966 [1]. At the same year another algebraic structure called BCI-algebra which was a generalization of a BCK-algebra was given by K. Iseki[2]. In 1983, Q.P.Hu and X. Li introduced the notion of a BCH- algebra which was a generalization of BCK/BCI -algebras [3]. In1991, C. S. Hoo introduced the notions of an ideal, a closed ideal and a filter in a BCI-algebra [4]. A BH- algebra is an algebraic structure introduced by Y.B. Jun et al in 1998 which was a generalization of BCH/BCI/BCK-algebras[5]. The notions of a Smarandache BCI-algebra, Smarandache ideal of a Smarandache BCI-algebra are given by Y.B.Jun in 2005 [6]. A.B.Saeid and A.Namdar introduced the notion of a Smarandache BCH-algebra and Smarandache ideal of Smarandache BCH-algebra in 2009 [7]. In 2012, H.H.Abbass and

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H.A. Dahham discussed the concept of completely closed filter of a BH-algebra, and completely closed filter with respect to an element of BH-algebra[8]. In 2013, H. H. Abbass and S. J. Mohammed introduced notions of the Smarandache BH-algebra, Smarandache (ideal, closed ideal, completely closed ideal) of a Smarandache BH-algebra[9]. In 2019, H. H. Abbass and Q. M. Luhaib introduced the notion of Smarandache filter of a Smarandache BH-Algebra [10]. In this paper, the notions of Smarandache closed filter and Smarandache completely closed filter of a Smarandache BH-Algebra.

2 Preliminaries

In this section, some basic concepts about a BCI-algebra, a BCK-algebra, a BCH-algebra, a BH-algebra, a Smarandache BH-algebra, and a Smarandache filter of a Smarandache BH-Algebra are viewed.

Definition 2.1. [6]. A BCI-algebra is an algebra $(X, *, 0)$, where X is a nonempty set, $*$ is a binary operation and 0 is a constant, satisfying the following axioms: for all $x, y, z \in X$:

i. $((x * y) * (x * z)) * (z * y) = 0$,

ii. $(x * (x * y)) * y = 0$,

iii. $x * x = 0$,

iv. $x * y = 0$ and $y * x = 0$ imply $x = y$.

Definition 2.2. [3]. BCK-algebra is a BCI-algebra satisfying the axiom: $0 * x = 0$ for all $x \in X$.

Definition 2.3. [5]. A BH-algebra is a nonempty set X with a constant 0 and a binary operation * satisfying the following conditions:

i. $x * x = 0, \forall x \in X$.

ii. $x * y = 0$ and $y * x = 0$ imply $x = y, \forall x, y \in X$.

iii. $x * 0 = x, \forall x \in X$.

Definition 2.4. [13]

A BH-algebra is said to be normal BH-algebra if it satisfying the following conditions:

i. $0 * (x * y) = (0 * x) * (0 * y)$, for all $x, y \in X$

ii. $(x * y) * x = 0 * y$, for all $x, y \in X$

iii. $(x * (x * y)) * y = 0$ for all $x, y \in X$

Definition 2.5. [10] A subset R of a BH-algebra X is said to be regular if it satisfies: $(\forall x \in R)(\forall y \in X)(x * y \in R \Rightarrow y \in R)$

Definition 2.6. [8] A filter of a BH-algebra X is a non-empty subset F of X such that:

(F1**)** If $x \in F$, and $y \in F$, then $y * (y * x) \in F$ and $x * (x * y) \in F$.

(F2**)** If $x \in F$ and $x * y = 0$ then $y \in F$:

Further F is a closed filter if $0 * x \in F$ for all $x \in F$. We shall denote $y * (y * x)$ by $x \wedge y$.

Definition 2.7. [8] Let X be a BH-algebra, and F be a filter. Then F is completely closed filter if $x * y \in F$, $\forall x; y \in F$:

Definition 2.8. [10]. A Smarandache BH-algebra is defined to be a BH-algebra X in which there exists a proper subset Q of X such that

i. $0 \in Q$ and $|Q| \geq 2$.

ii. Q is a BCK-algebra under the operation of X.

Definition 2.9. [5]

A nonempty subset S of a BH-algebra X is called a sub algebra of X if $x * y \in S$; $\forall x; y \in S$:

Definition 2.10. [9] A non-empty subset F of a Smarandache BH-algebra X is called a Smarandache filter of X, if it satisfies (F1) and

(F3) If $x \in F$ and $x * y = 0$ then $y \in F$, $\forall\, y \in Q$.

Proposition 2.11. [9] Let X be a Smarandache BH-algebra. Then every filter of X is a Smarandache filter of X.

Theorem 2.12. [9] Let X be a Smarandache BH-algebra, and F be a S-marandache filter of X such that $x * y \neq 0$, for all $y \notin F$ and $x \in F$. Then F is a filter of X.

Theorem 2.13. [8] Every normal subset N of a BH-algebra X is subalgebra of X.

Proposition 2.14. [9] Let X be a Smarandache BH-algebra and let $\{F_i,\ i \in \lambda\}$ be a family of a Smarandache filter of X Then $\bigcap_{i\in\lambda} F_i$ is a Smarandache filter of X.

Proposition 2.15. [9] Let X be a Smarandache filter and let $\{F_i,\ i \in\ \}$ be a chain of Smarandache filters of X. Then $\bigcup_{i\in\lambda} F_i$ is a Smarandache filter of X.

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3 Main results

In this section, the concepts of a Smarandache closed filter of a Smarandache BH-algebra and Smarandache completely closed filter are introduced, also study some properties of this concept are studied .

Definition 3.1. Smarandache filter of a Smarandache BH-algebra X is called a Smarandache closed filter of X if: $0 * x \in F$, for all $x \in F$.

Example 3.2. Consider X = {0, 1, 2, 3} with binary operation $''*''$ defined by the following table:

*	0	1	2	3
0	0	0	0	0
1	1	0	1	2
2	2	2	0	1
3	3	2	1	0

where Q = {0, 1} is a BCK-algebra. The Smarandache filter F = {0, 1, 3} is a Smarandache closed filter of X.

Proposition 3.3. Let X be a Smarandache BH-algebra . Then every closed filter of X is a Smarandache closed filter of X.

Proof. Let Directly by Definition 2.6 and Proposition2.11.

Remark 3.4. The convers of proposition 3.4 is not correct in general as in the following example.

Example 3.5. Consider X = {0, 1, 2, 3, 4} with binary operation $''*''$ define by the following table:

*****	**0**	**1**	**2**	**3**	**4**
0	0	0	0	0	0
1	1	0	1	0	2
2	2	2	0	2	0
3	3	1	3	0	3
4	4	4	4	4	0

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Where Q = {0, 2}. The subset F = {0, 1, 2} is a Smarandache closed filter of X but it is not a filter. Since x = 0, y = 3 0 ∗ 3 = 0 , but 3 ∉ F

Definition 3.6. Smarandache filter of a Smarandache BH-algebra X is called a Smarandache completely closed filter of X if: x ∗ y ∈ F, for all x, y ∈ F.

Example 3.7. Consider X = {0, 1, 2, 3} in Example 3.2, where Q = {0, 1} is a BCK-algebra. F = {0, 1, 2} is a Smarandache completely closed filter

Proposition 3.8. Let X be a Smarandache BH-algebra . Then every completely closed filter of X is a Smarandache completely closed filter of X.

Proof. Directly by Proposition 2.11 and by Definition 2.7 .

Example 3.9. Consider X = {0, 1, 2, 3} in Example 3.2, where Q = {0, 1} is a BCK-algebra. F = {0, 1, 2} is a Smarandache completely closed filter of X, but it is not completely closed filter of X. Because F is not filter of X

Proposition 3.10. Let X be a Smarandache BH-algebra, and F be a Smarandche completely closed filter of X. Then 0 ∈ F.

Proof. Let F be a Smarandache completely closed filter of X, and let x ∈ F we get x*x ∈ F[By Definition3.6]. Therefore, 0 ∈ F [By Definition2.2(i)].

Proposition 3.11. Let X be a a Smarandache BH-algebra. Then every a Smarandache completely closed filter of X is a Smarandache closed filter BH-algebra of X.

Proof. Let F be a Smarandache completely closed filter of X, it follows that F is a Smarandache filter of X [By Definition2.7]

Now, let x ∈ F imply x ∗ y ∈ F [Since F is a Smarandache completely closed filter of X, By Definition 2.7] choose x = 0 ∈ F [By Proposition 3.10] thus 0 ∗ y ∈ F: Therefore, F is Smarandache closed filter of X [By Definition 3.1]

Example 3.12. Consider X = {0, 1, 2, 3} in Example 3.2, where Q ={0, 1} is a BCK-algebra. F = {0, 1, 3} is a Smarandache closed filter but is not a Smarandache completely closed filter since x = 1, y = 3 ∈ F but 1 ∗ 3 ∉ F

Proposition 3.13. Let X be a Smarandache BH-algebra, and let F be a Smarandache completely closed filter of X . Then F is BH-algebra with the same binary operation on X and the constant 0.

Proof. Let F be a Smarandache completely closed filter of X. (i) Let x ∈ F imply x ∈ X; it follows that x ∗ x = 0 [By Definition2.3(i)], (ii) Let x ∈ F

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we get $x \in X$, then $x * 0 = x$ [By Definition2.3(iii)](iii) Let $x, y \in F$, $x * y = 0$ and $y * x = 0$, imply $x = y$ [By Definition2.3(ii)] Therefore, F is a BH-algebra.

Proposition 3.14. Let X be a Smarandache BH-algebra and let $\{F_i, i \in \lambda\}$ be a family of Smarandache closed filter of X. Then $\bigcap_{i\in\lambda} F_i$ is a Smarandache closed filter of X.

Proof. Since F_i is a Smarandache closed filter of X $\forall i \in \lambda$, Since Fi is a Smarandache filter $\forall i \in \lambda$ [By Definition3.1]imply $\bigcap_{i\in\lambda} F_i$, is a Smarandache filter of X.[By Proposition2.14].Now, let $x \in \bigcap_{i\in\lambda} F_i$ since $x \in F_i$ $\forall i \in \lambda$. then $0 * x \in F_i$ $\forall i \in \lambda$ [By Definition3.1]imply $0 * x \in \bigcap_{i\in\lambda} F_i$. Therefore, $\bigcap_{i\in\lambda} F_i$ is a Smarandache closed filter of X.

Proposition 3.15. Let X be a Smarandache BH-algebra and let $\{F_i, i \in \lambda\}$ be a family of Smarandache completely closed filter of X. Then $\bigcap_{i\in\lambda} F_i$ is a Smarandache completely closed filter of X.

Proof. Since F_i is a Smarandache completely closed filter of X $\forall i \in \lambda$, then F_i is a Smarandache filter $\forall i \in \lambda$ [By Definition3.6]imply $\bigcap_{i\in\lambda} F_i$ is a Smarandache filter of X.[By Proposition2.14].Now, let $x \in \bigcap_{i\in\lambda} F_i$ since $x \in F_i$ $\forall i \in \lambda$. then $x*y \in F_i$ $\forall i \in \lambda$ [By Definition3.6] imply $x * y \in \bigcap_{i\in\lambda} F_i$. Therefore, $\bigcap_{i\in\lambda} F_i$ is a Smarandache completely closed filter of X.

Proposition 3.16. Let X be a Smarandache filter and let $\{F_i, i \in \}$ be a chain of Smarandache closed filter of X. Then $\bigcup_{i\in\lambda} F_i$ is a Smarandache closed filter of X.

Proof. Let $\{F_i, i \in \}$ be a chain of Smarandache closed filter of X then F_i is a Smarandache filter $\forall i \in \lambda$ [By Definition3.1]then $\bigcup_{i\in\lambda} F_i$ is a Smarandache filter [By Proposition2.15]. Now let $x \in \bigcup_{i\in\lambda} F_i$ it follows that $i \in \lambda$ such that $x \in F_i$ then $0 * x \in F_i$ $\forall i \in$ [By Definition3.1]imply $0 * x \in \bigcup_{i\in\lambda} F_i$. Therefore, $\bigcup_{i\in\lambda} F_i$ is a Smarandache closed filter of X.

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Proposition 3.17. Let X be a Smarandache filter and let {F_i; i $\in\lambda$} be a chain of Smarandache completely closed filter of X. Then $\bigcup_{i\in\lambda} F_i$ is a Smarandache completely closed filter of X.

Proof. Let {F_i, i $\in \lambda$} be a chain of Smarandache completely closed filter of X then F_i is a Smarandache filter $\forall$i $\in$ λ [By Definition3.6] then $\bigcup_{i\in\lambda} F_i$ is a Smarandache filter [By Proposition2.15]. Now let x, y $\in \bigcup_{i\in\lambda} F_i$ F_i $\exists$i, j $\in$ such that

x $\in F_i$ and y $\in F_j$, since {F_i, i $\in \lambda$ } is a chain ether $F_i \subseteq F_j$ or $F_j \subseteq F_i$

Suppose $F_i \subseteq F_j$ imply x $*$ y $\in F_i$ $\forall$ i $\in \lambda$ [By Definition 3.6]imply x $*$ y $\in \bigcup_{i\in\lambda} F_i$

Therefore, $\bigcup_{i\in\lambda} F_i$ is a Smarandache closed filter of X.

Proposition 3.18. Let X be a Smarandache BH-algebra and F be a Smarandache closed filter of X such that x $*$ y $\neq 0$ $\forall$ y$\notin$ F and x $\in$ F Then F is a closed filter of X.

Proof. let F be a Smarandache closed filter of X and let y $\in$ X, x $\in$ F, imply F be a Smarandache filter of X [By Definition 3.1] F is a filter of X [By Theorem2.12]. Now let x $\in$ F then 0 $*$ x $\in$ F [Since F is Smarandache closed filter of X, By Definition 3.1]. Therefore, F is a closed filter of X

Proposition 3.19. Let X be a Smarandache BH-algebra and F be a Smarandache closed filter of X such that x $*$ y $\neq 0$ $\forall$ y $\notin$ F and x $\in$ F. Then F is a completely closed filter of X.

Proof. let F be a Smarandache completely closed filter of X and let y $\in$ X, x $\in$ F, imply F be a Smarandache filter of X [By Definition 3.6]F is a filter of X [By Theorem2.12]. Now let x, y $\in$ F we get x $*$ y $\in$ F[Since F is Smarandache closed filter of X, By Definition 3.6]. Therefore, F is a completely closed filter of X.

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