NSS

Neutrosophic Sets and Systems, Vol. 53, 2023

University of New Mexico

On Superhyper BCK-Algebras

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Abstract. BCK-algebras are algebraic structures in universal algebra such that are based on logical axioms and have some applications. This paper introduces the concept of super hyper BCK-algebras as a generalization of BCK-algebras and investigates some properties of this novel concept.

Keywords: BCK-algebra, hyper BCK-algebra, super hyper BCK-algebra, generalized operation.

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1. **Introduction**

Smarandache introduced a new concept in neutrosophy branches as neutro-algebra as a generalization of partial algebra. A neutro algebra is an algebra which has at least one neutro-operation (an operation that is partially well-defined, partially indeterminate, and partially outer-defined) or one neutro-axiom (axiom that is true for some elements, indeterminate for other elements, and false for the other elements). A partial algebra is an algebra that has at least one partial operation, and all its axioms are classical (i.e. axioms true for all elements). Through a theorem he proved that Neutro-algebra is a generalization of partial algebra, and he gave examples of neutro-algebras that are not partial algebras. He also introduced the neutro-function (and neutro-operation). Recently, Smarandache, introduced a new concept as a generalization of hypergraphs to n-super hypergraph, plithogenic n-super hypergraph {with super-vertices (that are groups of vertices) and hyper-edges {defined on power-set of power-set...} that is the most general form of graph as today}, and n-ary hyperalgebra, n-ary neutro hyperalgebra, n-ary anti hyperalgebra respectively, which have several properties and are connected with the real world [2,8]. Recently in the scope of neutro logical (hyper) algebra, Hamidi, et al. introduced the concept of neutro BCK-subalgebras [4], neutro d-subalgebras [3] and single-valued neutro hyper BCK-subalgebras [5] as a generalization of BCK-algebras and hyper BCK-subalgebras, respectively and presented the main results in this regard. Also

Smarandache a novel concept as super hyperalgebra with its super hyperoperations and super hyperaxioms, then is introduced some concepts such as super hypertopology and especially the super hyperfunction and neutrosophic super hyperfunction [10, 11].

Regarding these points, we try to develop the notation of BCK-algebras to the concept of super hyper BCK-algebras and so we want to seek the connection between BCK-algebras and super hyper BCK-algebras.

2. Preliminaries

In this section, we recall some concepts that need to our work.

Definition 2.1. [6] Let $X \neq \emptyset$. Then a universal algebra $(X, \vartheta, 0)$ of type $(2, 0)$ is called a *BCK-algebra*, if $\forall\ x, y, z \in X$:
(BCI-1) $((x\vartheta\ y)\vartheta\ (x\vartheta\ z))\vartheta\ (z\vartheta\ y) = 0$,
(BCI-2) $(x\vartheta\ (x\vartheta\ y))\vartheta\ y = 0$,
(BCI-3) $x\vartheta\ x = 0$,
(BCI-4) $x\vartheta\ y = 0$ and $y\vartheta\ x = 0$ imply $x = y$,
(BCK-5) $0\vartheta\ x = 0$,
where $\vartheta(x, y)$ is denoted by $x\vartheta\ y$.

Definition 2.2. [1,7] Let $X \neq \emptyset$ and $P^*(X) = \{Y \mid \emptyset \neq Y \subseteq X\}$. Then for a map $\varrho : X^2 \to P^*(X)$ a hyperalgebraic system $(X, \varrho, 0)$ is called a *hyper BCK-algebra*, if $\forall\ x, y, z \in X$:
($H1$) $(x\ \varrho\ z)\ \varrho\ (y\ \varrho\ z) \ll x\ \varrho\ y$,
($H2$) $(x\ \varrho\ y)\ \varrho\ z = (x\ \varrho\ z)\ \varrho\ y$,
($H3$) $x\ \varrho\ X \ll x$,
($H4$) $x \ll y$ and $y \ll x$ imply $x = y$,
where $x \ll y$ is defined by $0 \in x\ \varrho\ y$, $\forall\ W, Z \subseteq X$, $W \ll Z \Leftrightarrow\ \forall\ a \in W\ \exists\ b \in Z$ s.t $a \ll b$, $(W\ \varrho\ Z) = \bigcup\limits_{a\in W, b\in Z} (a\ \varrho\ b)$ and $\varrho(x, y)$ is denoted by $x\varrho\ y$.

We will call X is a *weak commutative* hyper BCK-algebra if, $\forall\ x, y \in X, (x\ \varrho\ (x\ \varrho\ y)) \cap (y\ \varrho\ (y\ \varrho\ x)) \neq \emptyset$.

Theorem 2.3. [7] *Let $(X, \varrho, 0)$ be a hyper BCK-algebra. Then $\forall\ x, y, z \in X$ and $W, Z \subseteq X$,*

(i) $(0\ \varrho\ 0) = 0, 0 \ll x, (0\ \varrho\ x) = 0$, $x \in (x\ \varrho\ 0)$ *and* $(W \ll 0 \Rightarrow W = 0)$,
(ii) $x \ll x$, $x\ \varrho\ y \ll x$ *and* $(y \ll z \Rightarrow x\ \varrho\ z \ll x\ \varrho\ y)$,
(iii) $W\ \varrho\ Z \ll W$, $W \ll W$ *and* $(W \subseteq Z \Rightarrow W \ll Z)$.

Definition 2.4. [10,11] Let X be a nonempty set and $0 \in X$. Then $(X, \circ^*_{(m,n)})$ is called an (m,n)-super hyperalgebra, where $\circ^*_{(m,n)} : X^m \to P^n_*(X)$ is called an (m,n)-super hyperoperation, $P^n_*(X)$ is the n^{th} powerset of the set $X, \emptyset \notin P^n_*(X)$, for any $A \in P^n_*(X)$, we identify $\{A\}$ with $A, m, \geq 2, n \geq 0, X^m = \underbrace{X \times X \times \ldots X}_{m-times}$ and $P^0_*(X) = X$.

3. Superhyper BCK-subalgebra

In this section, we make the concept of superhyper BCK-subalgebras as an extension of BCK-subalgebras and seek some of their properties.

Proposition 3.1. *Let $(X, \vartheta, 0)$ be a BCK-algebra. Then for all $x, y, z \in X$,*

(i) $\vartheta(\vartheta(x,y), \vartheta(x,z)) = \vartheta(\vartheta(\vartheta(x,y), \vartheta(x,z)), 0)$.
(ii) $\vartheta(\vartheta(x), \vartheta(x,y)) = \vartheta(\vartheta(\vartheta(x), \vartheta(x,y)), 0)$.

Proof. Since for all $x \in X, \vartheta(x,0) = x$, results are clear. □

By Proposition 3.1, we define the concept of (m,n)-super hyper BCK-subalgebras.

Definition 3.2. Let X be a nonempty set and $0 \in X$ and $\alpha = \underbrace{0, 0, \ldots 0}_{(m-1)-times}$. Then $(X, \circ^*_{(m,n)})$ is called an (m,n)-super hyper BCK-subalgebra, if

(i) $0 \in \circ^*_{(m,n)}\big(\circ^*_{(m,n)}(\circ^*_{(m,n)}(x^1_1, x^1_2, \ldots, x^1_m), \ldots, \circ^*_{(m,n)}(x^1_1, x^m_2, \ldots, x^m_m)), \alpha, \circ^*_{(m,n)}(x^m_m, x^{m-1}_m, \ldots, x^1_m)\big)$,
(ii) $0 \in \circ^*_{(m,n)}\big(\circ^*_{(m,n)}(x^1_1, \underbrace{0, 0, \ldots 0}_{(m-2)-times}, \circ^*_{(m,n)}(x^1_1, x^1_2, \ldots, x^1_m)), \underbrace{0, 0, \ldots 0}_{(m-1)-times}, x^1_m)\big)$,
(iii) $0 \in \circ^*_{(m,n)}\big(x, x, \ldots, x\big)$,
(iv) if $0 \in \circ^*_{(m,n)}\big(x_1, x_2, \ldots, x_m\big)$ and $0 \in \circ^*_{(m,n)}\big(x_m, x_{m-1}, \ldots, x_1\big)$, then $x_i = x_j$, where $i + j = m + 1$,
(v) $0 \in \circ^*_{(m,n)}\big(0, 0, \ldots, x\big)$,

Example 3.3. (i) Let $(X, \circ^*_{(m,n)})$ be a (m,n)-super hyper BCK-subalgebra. Then $(X, \circ^*_{(2,0)})$ is a BCK-subalgebra.

(ii) Let $(X, \circ^*_{(m,n)})$ be a (m,n)-super hyper BCK-subalgebra. Then $(X, \circ^*_{(2,1)})$ is a hyper BCK-subalgebra.

Example 3.4. Let $X = \{0, a\}$.

(i) Then $(X, \circ^*)$ is a $(3,3)$-super hyper BCK-subalgebra as follows:

$$\circ^*_{(3,3)}(x,y,z) = \begin{cases} P^3_*(\{0,x,z\}) & \text{if } x = z \\ P^3_*(\{0,z\}) & \text{if } x = y = 0 \\ P^3_*(\{a\}) & o.w \end{cases},$$

where

$$P_*(\{a\}) = P_*^2(\{a\}) = P_*^3(\{a\}) = \{a\}, P_*(\{0,a\}) = \{0,a,\{0,a\}\},$$
$$P_*^2(\{0,a\}) = \{0,a,\{0,a\},\{0,\{0,a\}\},\{a,\{0,a\}\}\},$$
$$P_*^3(\{0,a\}) = \big\{0,a,\{0,a\},\{0,\{0,a\}\},\{a,\{0,a\}\},\{0,\{0,\{0,a\}\},\{0,\{a,\{0,a\}\},\{a,\{0,\{0,a\}\},$$
$$\{a,\{a,\{0,a\}\},\{\{0,a\},\{0,\{0,a\}\}\},\{\{0,a\},\{a,\{0,a\}\},\{\{0,\{0,a\}\},\{a,\{0,a\}\}\}\big\}.$$

(i) By definition, $\circ^*_{(3,3)}\big(\circ^*_{(3,3)}(\circ^*_{(3,3)}(x,y,z),\circ^*_{(3,3)}(x',y',z'),\circ^*_{(3,3)}(x'',y'',z'')),0,\circ^*_{(3,3)}(z'',z',z)\big) \subseteq \{0,a\}$. (ii) It is similar to item (i).

(iii) By definition, $\circ^*_{(3,3)}(a,a,a) = \{0,a\}$.

(iv) By definition, if $0 \in \circ^*_{(3,3)}(x,y,z)$ and $0 \in \circ^*_{(3,3)}(z,y,x)$, then $x = z$ and so $(x,y,z) = (z,y,x)$.

(v) By definition, $\circ^*_{(3,3)}(0,0,a) = \{0,a\}$.

(ii) Then $(X,\circ^*)$ is a $(3,0)$-super hyper BCK-subalgebra as follows:

$$\circ^*_{(3,1)}(x,y,z) = \begin{cases} 0 & \text{if } x = y = z \\ x & o.w \end{cases},$$

Theorem 3.5. *Let $(X,\circ^*_{(m,n)})$ be an (m,n)-super hyper BCK-subalgebra. Then for any $k \geq n$, $(X,\circ^*_{(m,n)})$ is an (m,k)-super hyper BCK-subalgebra.*

Proof. Let $(X,\circ^*_{(m,n)})$ be an (m,n)-super hyper BCK-subalgebra and $k \geq n$. Since $P_*^n(X) \subseteq P_*^k(X)$, for any $x_1,x_2,\ldots,x_m \in X$, $\circ^*_{(m,n)}(x_1,x_2,\ldots,x_m) \subseteq \circ^*_{(m,k)}(x_1,x_2,\ldots,x_m)$. Thus $0 \in \circ^*_{(m,n)}(x_1,x_2,\ldots,x_m)$ implies that $0 \in \circ^*_{(m,k)}(x_1,x_2,\ldots,x_m)$ and all axioms are valid. □

Example 3.6. Let $X = \{0,a\}$. Then for any $n \geq 3$, by Theorem 3.5, $(X,\circ^*)$ is a $(3,n)$-super hyper BCK-subalgebra as follows:

$$\circ^*_{(3,3)}(x,y,z) = \begin{cases} P_*^n(\{0,x,z\}) & \text{if } x = z \\ P_*^n(\{0,z\}) & \text{if } x = y = 0 \\ P_*^n(\{a\}) & o.w \end{cases}.$$

Let m be an even and $(X,\circ^*_{(m,n)})$ be an (m,n)-super hyper BCK-subalgebra. For any given $x_1,x_2,\ldots,x_m \in X$, define $(x_1,x_2,\ldots,x_{\frac{m}{2}}) \leq (x_{\frac{m}{2}+1},x_{\frac{m}{2}+2},\ldots,x_m)$ if and only if $0 \in \circ^*_{(m,n)}(x_1,x_2,\ldots,x_m)$.

Theorem 3.7. *Let m be an even and $x_1,x_2,\ldots,x_m \in X$. Then $(X,\circ^*_{(m,n)})$ is an (m,n)-super hyper BCK-subalgebra if and only if*

(i) $\circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1^1,x_2^1,\ldots,x_m^1),\ldots,\circ^*_{(m,n)}(x_1^1,x_2^m,\ldots,x_m^m))) \leq \circ^*_{(m,n)}(x_m^m,x_m^{m-1},\ldots,x_m^1)$,

(ii) $\circ^*_{(m,n)}(x^1_1, \underbrace{0,0,\ldots 0}_{(m-2)-times}, \circ^*_{(m,n)}(x^1_1, x^1_2, \ldots, x^1_m)) \leq \circ^*_{(m,n)}(\underbrace{0,0,\ldots 0}_{(m-1)-times}, x^1_m)),$

(iii) $(\underbrace{x, x, \ldots, x}_{(\frac{m}{2})-times}) \leq (\underbrace{x, x, \ldots, x}_{(\frac{m}{2})-times}),$

(iv) *if* $(x_1, x_2, \ldots, x_{\frac{m}{2}}) \leq (x_{\frac{m}{2}+1}, x_{\frac{m}{2}+2}, \ldots, x_m)$ *and* $(x_{\frac{m}{2}+1}, x_{\frac{m}{2}+2}, \ldots, x_m) \leq (x_1, x_2, \ldots, x_{\frac{m}{2}})$, *then* $x_i = x_j$, *where* $|i-j| = 2$,

(v) $(\underbrace{0,0,\ldots,0}_{(\frac{m}{2})-times}) \leq (x_{\frac{m}{2}+1}, x_{\frac{m}{2}+2}, \ldots, x_m),$

(vi) $(x_1, x_2, \ldots, x_{\frac{m}{2}}) \leq (x_{\frac{m}{2}+1}, x_{\frac{m}{2}+2}, \ldots, x_m)$ *if and only if* $0 \in \circ^*_{(m,n)}(x_1, x_2, \ldots, x_m)$.

Proof. Immediate by definition. □

Theorem 3.8. *Let m be an even and $(X, \circ^*_{(m,n)})$ be an (m,n)-super hyper BCK-subalgebra and $x_1, x_2, \ldots, x_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}}, z_1, z_2, \ldots, z_{\frac{m}{2}} \in X$. If $0 \in \circ^*_{(m,n)}(x_1, x_2, \ldots, x_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}})$, then $0 \in \circ^*_{(m,n)}\big(\circ^*_{(m,n)}(z_1, z_2, \ldots, z_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}}), \underbrace{0,0,\ldots 0}_{(m-2)-times}, \circ^*_{(m,n)}(z_1, z_2, \ldots, z_{\frac{m}{2}}, x_1, x_2, \ldots, x_{\frac{m}{2}})\big)$.*

Proof. Let $x_1, x_2, \ldots, x_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}}, z_1, z_2, \ldots, z_{\frac{m}{2}} \in X$. Clearly,

$$\circ^*_{(m,n)}(\circ^*_{(m,n)}(z_1, z_2, \ldots, z_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}}), \underbrace{0,0,\ldots 0}_{(m-2)-times}, \circ^*_{(m,n)}(z_1, z_2, \ldots, z_{\frac{m}{2}}, x_1, x_2, \ldots, x_{\frac{m}{2}})))$$
$$\leq \circ^*_{(m,n)}(x_1, x_2, \ldots, x_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}}).$$

Since $0 \in \circ^*_{(m,n)}(x_1, x_2, \ldots, x_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}})$, we get that

$$0 \in \circ^*_{(m,n)}\big(\circ^*_{(m,n)}(z_1, z_2, \ldots, z_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}}), \underbrace{0,0,\ldots 0}_{(m-2)-times}, \circ^*_{(m,n)}(z_1, z_2, \ldots, z_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}})\big).$$

□

Theorem 3.9. *Let m be an even and $(X, \circ^*_{(m,n)})$ be an (m,n)-super hyper BCK-subalgebra and $x_1, x_2, \ldots, x_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}}, z_1, z_2, \ldots, z_{\frac{m}{2}} \in X$. If*

$$0 \in \circ^*_{(m,n)}(x_1, x_2, \ldots, x_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}}) \cap \circ^*_{(m,n)}(y_1, y_2, \ldots, y_{\frac{m}{2}}, z_1, z_2, \ldots, z_{\frac{m}{2}}),$$

then $0 \in \circ^*_{(m,n)}(x_1, x_2, \ldots, x_{\frac{m}{2}}, z_1, z_2, \ldots, z_{\frac{m}{2}})$.

Proof. Let $x_1, x_2, \ldots, x_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}}, z_1, z_2, \ldots, z_{\frac{m}{2}} \in X$. Since

$$0 \in \circ^*_{(m,n)}(x_1, x_2, \ldots, x_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}}) \cap \circ^*_{(m,n)}(y_1, y_2, \ldots, y_{\frac{m}{2}}, z_1, z_2, \ldots, z_{\frac{m}{2}}),$$

by Theorem 3.8, we get that

$$0 \in \circ^*_{(m,n)}\big(\circ^*_{(m,n)}(z_1, z_2, \ldots, z_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}}), \underbrace{0,0,\ldots 0}_{(m-2)-times}, \circ^*_{(m,n)}(z_1, z_2, \ldots, z_{\frac{m}{2}}, x_1, x_2, \ldots, x_{\frac{m}{2}})\big)$$

and

$$0\in \circ^*_{(m,n)}\big(\circ^*_{(m,n)}(x_1,x_2,\ldots,x_{\frac{m}{2}},z_1,z_2,\ldots,z_{\frac{m}{2}}),\ \underbrace{0,0,\ldots 0}_{(m-2)-times},\circ^*_{(m,n)}(x_1,x_2,\ldots,x_{\frac{m}{2}},y_1,y_2,\ldots,y_{\frac{m}{2}})\big).$$

It follows that $0\in \circ^*_{(m,n)}(x_1,x_2,\ldots,x_{\frac{m}{2}},z_1,z_2,\ldots,z_{\frac{m}{2}})$. □

Let $(X,\circ^*_{(m,n)})$ be an (m,n)-super hyper BCK-subalgebra and $A,B\subseteq X$. If $\circ^*_{(m,n)}(A)\cap \circ^*_{(m,n)}(B)\neq\emptyset$, will denote it by $A\approx B$.

Theorem 3.10. *Let m be an even and $(X,\circ^*_{(m,n)})$ be an (m,n)-super hyper BCK-subalgebra and $x_1,x_2,\ldots,x_{\frac{m}{2}},y_1,y_2,\ldots,y_{\frac{m}{2}},z_1,z_2,\ldots,z_{\frac{m}{2}}\in X$. If $\alpha=\underbrace{0,\ldots,0}_{(\frac{m}{2}-1)-times}$, then*

$$\circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1,\ldots,x_{\frac{m}{2}},y_1,\ldots,y_{\frac{m}{2}}),\alpha,z_1,\ldots,z_{\frac{m}{2}})\approx \circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1,\ldots,x_{\frac{m}{2}},z_1,\ldots,z_{\frac{m}{2}}),\alpha,y_1,\ldots,y_{\frac{m}{2}}).$$

Proof. Let $x_1,x_2,\ldots,x_{\frac{m}{2}},y_1,y_2,\ldots,y_{\frac{m}{2}},z_1,z_2,\ldots,z_{\frac{m}{2}}\in X$. Since $0\in \circ^*_{(m,n)}\big(\circ^*_{(m,n)}(x_1,x_2,\ldots,x_{\frac{m}{2}},(\circ^*_{(m,n)}(x_1,x_2,\ldots,x_{\frac{m}{2}},z_1,z_2,\ldots,z_{\frac{m}{2}})),z_1,z_2,\ldots,z_{\frac{m}{2}})\big)$, we get that

$$\circ^*_{(m,n)}\big(\circ^*_{(m,n)}(x_1,x_2,\ldots,x_{\frac{m}{2}},y_1,y_2,\ldots,y_{\frac{m}{2}}),\ \underbrace{0,0,\ldots 0}_{(m-2)-times},z_1,z_2,\ldots,z_{\frac{m}{2}})\big)$$
$$\leq \circ^*_{(m,n)}\big(\circ^*_{(m,n)}(x_1,x_2,\ldots,x_{\frac{m}{2}},z_1,z_2,\ldots,z_{\frac{m}{2}}),\ \underbrace{0,0,\ldots 0}_{(m-2)-times},y_1,y_2,\ldots,y_{\frac{m}{2}})\big)$$

and in similar to

$$\circ^*_{(m,n)}\big(\circ^*_{(m,n)}(x_1,x_2,\ldots,x_{\frac{m}{2}},z_1,z_2,\ldots,z_{\frac{m}{2}}),\ \underbrace{0,0,\ldots 0}_{(m-2)-times},y_1,y_2,\ldots,y_{\frac{m}{2}})\big)$$
$$\leq \circ^*_{(m,n)}\big(\circ^*_{(m,n)}(x_1,x_2,\ldots,x_{\frac{m}{2}},y_1,y_2,\ldots,y_{\frac{m}{2}}),\ \underbrace{0,0,\ldots 0}_{(m-2)-times},z_1,z_2,\ldots,z_{\frac{m}{2}})\big).$$

It follows that

$$\circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1,\ldots,x_{\frac{m}{2}},y_1,\ldots,y_{\frac{m}{2}}),\alpha,z_1,\ldots,z_{\frac{m}{2}})\approx \circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1,\ldots,x_{\frac{m}{2}},z_1,\ldots,z_{\frac{m}{2}}),\alpha,y_1,\ldots,y_{\frac{m}{2}}).$$

□

Corollary 3.11. *Let m be an even and $(X,\circ^*_{(m,n)})$ be an (m,n)-super hyper BCK-subalgebra and $x_1,x_2,\ldots,x_{\frac{m}{2}},y_1,y_2,\ldots,y_{\frac{m}{2}},z_1,z_2,\ldots,z_{\frac{m}{2}}\in X$. If*

$$0\approx \circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1,\ldots,x_{\frac{m}{2}},y_1,\ldots,y_{\frac{m}{2}}),\ \underbrace{0,\ldots 0}_{(\frac{m}{2}-1)-times},z_1,\ldots,z_{\frac{m}{2}})$$

then

$$0\approx \circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1,\ldots,x_{\frac{m}{2}},z_1,\ldots,z_{\frac{m}{2}}),\ \underbrace{0,\ldots 0}_{(\frac{m}{2}-1)-times},y_1,\ldots,y_{\frac{m}{2}}).$$

Example 3.12. Consider the $(3,3)$-super hyper BCK-subalgebra in Example 3.4. Clearly

$$\begin{aligned}\circ^*_{(3,3)}(\circ^*_{(3,3)}(0,a,0),0,a) &= \circ^*_{(3,3)}(P^3_*(\{0\}),0,a) = \circ^*_{(3,3)}(0,0,a) = P^3_*(\{0,a\})\\ &= \circ^*_{(3,3)}(a,0,a) = \circ^*_{(3,3)}(P^3_*(\{a\}),0,a) = \circ^*_{(3,3)}(\circ^*_{(3,3)}(0,a,a),0,a).\end{aligned}$$

Thus $\circ^*_{(3,3)}(\circ^*_{(3,3)}(0,a,0),0,a) = \circ^*_{(3,3)}(\circ^*_{(3,3)}(0,a,a),0,a)$, while m is an odd. It follows that the converse of Theorem 3.10, is not necessarily true.

Theorem 3.13. *Let m be an even and $(X,\circ^*_{(m,n)})$ be an (m,n)-super hyper BCK-subalgebra and $x_1,x_2,\dots,x_m,y_1,y_2,\dots,y_m \in X$. If $\alpha = \underbrace{0,\dots,0}_{(\frac{m}{2}-1)-times}$, then*

(i)

$$\circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1,\dots,x_{\frac{m}{2}},x_1,\dots,x_{\frac{m}{2}}),\alpha,y_1,\dots,y_{\frac{m}{2}}) \approx \circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1,\dots,x_{\frac{m}{2}},y_1,\dots,y_{\frac{m}{2}}),\alpha,x_1,\dots,x_{\frac{m}{2}}).$$

(ii)

$$\circ^*_{(m,n)}(\underbrace{0,\dots0}_{(\frac{m}{2})-times},y_1,\dots,y_{\frac{m}{2}}) \approx \circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1,\dots,x_{\frac{m}{2}},y_1,\dots,y_{\frac{m}{2}}),\underbrace{0,\dots,0}_{(\frac{m}{2}-1)-times},x_1,\dots,x_{\frac{m}{2}}).$$

(iii)

$$\circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1,\dots,x_{\frac{m}{2}},x_1,\dots,x_{\frac{m}{2}}),\underbrace{0,\dots0}_{(\frac{m}{2}-1)-times},y_1,\dots,y_{\frac{m}{2}}) \approx \circ^*_{(m,n)}(\underbrace{0,\dots,0}_{(\frac{m}{2})-times},y_1,\dots,y_{\frac{m}{2}}).$$

Proof. (i), (ii), (iii) Let $x_1,x_2,\dots,x_{\frac{m}{2}},y_1,y_2,\dots,y_{\frac{m}{2}},z_1,z_2,\dots,z_{\frac{m}{2}} \in X$. Using Corollary 3.11, we get that

$$0 \approx \circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1,\dots,x_{\frac{m}{2}},y_1,\dots,y_{\frac{m}{2}}),\underbrace{0,\dots0}_{(\frac{m}{2}-1)-times},x_1,\dots,x_{\frac{m}{2}})$$

and

$$0 \approx \circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1,\dots,x_{\frac{m}{2}},x_1,\dots,x_{\frac{m}{2}}),\underbrace{0,\dots0}_{(\frac{m}{2}-1)-times},y_1,\dots,y_{\frac{m}{2}}).$$

In addition, by definition we get that $0 \approx \circ^*_{(m,n)}(\underbrace{0,\dots0}_{(\frac{m}{2})-times},y_1,\dots,y_{\frac{m}{2}})$, hence the proof is completed. □

Corollary 3.14. *Let m be an even and $(X,\circ^*_{(m,n)})$ be an (m,n)-super hyper BCK-subalgebra and $x_1,x_2,\dots,x_{m-1},y_1,y_2,\dots,y_{m-1} \in X$. Then*

(i)

$$\circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1,\dots,x_{\frac{m}{2}},x_1,\dots,x_{\frac{m}{2}}),y_1,\dots,y_{m-1}) \approx \circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1,\dots,x_{\frac{m}{2}},y_1,\dots,y_{\frac{m}{2}}),x_1,\dots,x_{m-1}).$$

(ii)

$$\circ^*_{(m,n)}(0, y_1, \ldots, y_{m-1}) \approx \circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1, \ldots, x_{\frac{m}{2}}, y_1, \ldots, y_{\frac{m}{2}}), x_1, \ldots, x_{m-1}).$$

(iii)

$$\circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1, \ldots, x_{\frac{m}{2}}, x_1, \ldots, x_{\frac{m}{2}}), y_1, \ldots, y_{m-1}) \approx \circ^*_{(m,n)}(0, y_1, \ldots, y_{m-1}).$$

Theorem 3.15. *Let m be an even and $(X, \circ^*_{(m,n)})$ be an (m,n)-super hyper BCK-subalgebra and $x_1, x_2, \ldots, x_{m-1} \in X$. Then $(x_1, \ldots, x_{m-1}) \approx \circ^*_{(m,n)}(x_1, \ldots, x_{m-1}, 0)$.*

Proof. Let $x_1, x_2, \ldots, x_m \in X$. Then $0 \approx \circ^*_{(m,n)}(x_1, x_2, \ldots, x_{m-1}, \circ^*_{(m,n)}(x_1, x_2, \ldots, x_{m-1}, 0))$. Moreover by Theorem 3.13, we have $0 \approx \circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1, \ldots, x_{m-1}, 0), x_1, \ldots, x_{m-1})$. Thus we conclude that $(x_1, \ldots, x_{m-1}) \approx \circ^*_{(m,n)}(x_1, \ldots, x_{m-1}, 0)$. □

4. Conclusion

The concept of super hyper BCK-algebras as a generalization of BCK-algebras is introduced in this paper such that for special cases, we can obtain the concepts of BCK-algebras and hyper BCK-algebras. We wish this research is important for the next studies in logical super hyperalgebras. In our future studies, we hope to obtain more results regarding single-valued neutrosophic super(hyper)BCK-subalgebras and their applications in handing information regarding various aspects of uncertainty, non-classical mathematics (fuzzy mathematics or great extension and development of classical mathematics) that are considered to be a more powerful technique than classical mathematics.

Conflicts of Interest: "The authors declare no conflict of interest."

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Received: Sep 20, 2022. Accepted: Dec 18, 2022