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ON THE SECOND SMARANDACHE'S PROBLEM

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The second problem from [1] (see also 16-th problem from [2]) is the following:

Smarandache circular sequence:

$$\underbrace{1}_{1}, \underbrace{12, 21}_{2}, \underbrace{123, 231, 312}_{3}, \underbrace{1234, 2341, 3412, 4123}_{4},$$

$$\underbrace{12345, 23451, 34512, 45123, 51234}_{5}, \underbrace{123456, 234561, 345612, 456123, 561234, 612345}_{6}, \ldots$$

Let $]x[$ be the largest natural number strongly smaller than real (positive) number x. For example, $]7.1[= 7$, but $]7[= 6$.

Let $f(n)$ is the n-th member of the above sequence. We shall prove the following

Theorem: For every natural number n:

$$f(n) = \overline{s(s+1)...k12...(s-1)}, \tag{1}$$

where

$$k \equiv k(n) =]\frac{\sqrt{8n+1}-1}{2}[\tag{2}$$

and

$$s \equiv s(n) = n - \frac{k(k+1)}{2}. \tag{3}$$

Proof: When $n = 1$, then from (1) and (2) it follows that $k = 0$, $s = 1$ and from (3) – that $f(1) = 1$. Let us assume that the assertion is valid for some natural number n. Then for $n+1$ we have the following two possibilities:

1. $k(n+1) = k(n)$, i.e., k is the same as above. Then

$$s(n+1) = n+1 - \frac{k(n+1)(k(n+1)+1)}{2} = n+1 - \frac{k(n)(k(n)+1)}{2} = s(n)+1,$$

i.e.,

$$f(n+1) = \overline{(s+1)...k12...s}.$$

2. $k(n+1) = k(n) + 1$. Then

$$s(n+1) = n+1 - \frac{k(n+1)(k(n+1)+1)}{2}. \tag{4}$$

On the other hand, it is seen directly, that in (2) number $\frac{\sqrt{8n+1}-1}{2}$ is an integer if and only if $n = \frac{m(m+1)}{2}$. Also, for every natural numbers n and $m \geq 1$ such that

$$\frac{(m-1)m}{2} < n < \frac{m(m+1)}{2} \tag{5}$$

it will be valid that

$$]\frac{\sqrt{8n+1}-1}{2}[=]\frac{\sqrt{\frac{m(m+1)}{2}+1}-1}{2}[= m.$$

Therefore, when $k(n+1) = k(n) + 1$, then

$$n = \frac{m(m+1)}{2} + 1$$

and for it from (4) we obtain:

$$s(n+1) = 1,$$

i.e.,

$$f(n+1) = \overline{12...(n+1)}.$$

Therefore, the assertion is valid.

Let

$$S(n) = \sum_{i=1}^{n} f(i).$$

Then, we shall use again formulae (2) and (3). Therefore,

$$S(n) = \sum_{i=1}^{p} f(i) + \sum_{i=p+1}^{n} f(i),$$

where

$$p = \frac{m(m+1)}{2}.$$

It can be seen directly, that

$$\sum_{i=1}^{p} f(i) = \sum_{i=1}^{m} \overline{12...i} + \overline{23...i1} + \overline{i12...(i-1)} = \sum_{i=1}^{m} \frac{i(i+1)}{2}.\underbrace{11...1}_{i}$$

On the other hand, if $s = n - p$, then

$$\sum_{i=p+1}^{n} f(i) = \overline{12...(m+1)} + \overline{23...(m+1)1} + \overline{s(s+1)...m(m+1)12...(s-1)}$$

$$= \sum_{i=0}^{m+1} (\frac{(s+i)(s+i+1)}{2} - \frac{i(i+1)}{2}).10^{m-i}.$$

REFERENCES:

[1] C. Dumitrescu, V. Seleacu, Some Sotions and Questions in Number Theory, Erhus Univ. Press, Glendale, 1994.

[2] F. Smarandache. Only Problems, Not Solutions!. Xiquan Publ. House, Chicago, 1993.