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The t-Pebbling Number of Jahangir Graph

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Abstract: Given a configuration of pebbles on the vertices of a connected graph G, a pebbling move (or pebbling step) is defined as the removal of two pebbles from a vertex and placing one pebble on an adjacent vertex. The t-pebbling number, $f_t(G)$ of a graph G is the least number m such that, however m pebbles are placed on the vertices of G, we can move t pebbles to any vertex by a sequence of pebbling moves. In this paper, we determine $f_t(G)$ for Jahangir graph $J_{2,m}$.

Key Words: Smarandachely d-pebbling move, Smarandachely d-pebbling number, pebbling move, t-pebbling number, Jahangir graph.

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§1. Introduction

One recent development in graph theory, suggested by Lagarias and Saks, called pebbling, has been the subject of much research. It was first introduced into the literature by Chung [1], and has been developed by many others including Hulbert, who published a survey of pebbling results in [2]. There have been many developments since Hulbert's survey appeared.

Given a graph G, distribute k pebbles (indistinguishable markers) on its vertices in some configuration C. Specifically, a configuration on a graph G is a function from $V(G)$ to $N \cup \{0\}$ representing an arrangement of pebbles on G. For our purposes, we will always assume that G is connected. A *Smarandachely d-pebbling move* (Smarandachely d-pebbling step) is defined as the removal of two pebbles from some vertex and the replacement of one of these pebbles on such a vertex with distance d to the initial vertex with pebbles and the Smarandachely (t, d)-pebbling number $f_t^d(G)$, is defined to be the minimum number of pebbles such that regardless of their initial configuration, it is possible to move to any root vertex v, t pebbles by a sequence

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of Smarandachely d-pebbling moves. Particularly, if $d = 1$, such a Smarandachely 1-pebbling move is called a pebbling move (or pebbling step)and The Smarandache $(t, 1)$-pebbling number $f_t^d(G)$ is abbreviated to $f_t(G)$, i.e., it is possible to move to any root vertex v, t pebbles by a sequence of pebbling moves. Implicit in this definition is the fact that if after moving to vertex v one desires to move to another root vertex, the pebbles reset to their original configuration. There are certain results regarding the t-pebbling graphs that are investigated in [3-6,9].

Definition 1.1 *Jahangir graph $J_{n,m}$ for $m \geq 3$ is a graph on $nm + 1$ vertices, that is, a graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to C_{nm}.*

Example 1.2 Fig.1 shows Jahangir graph $J_{2,8}$. The graph $J_{2,8}$ appears on Jahangir's tomb in his mausoleum. It lies in 5 kilometer north- west of Lahore, Pakistan, across the River Ravi.

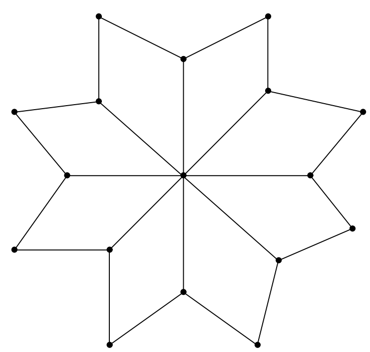

Fig.1 $J_{2,8}$

Remark 1.3 *Let v_{2m+1} be the label of the center vertex and $v_1, v_2, \cdots, v_{2m}$ be the label of the vertices that are incident clockwise on cycle C_{2m} so that $deg(v_1) = 3$.*

In Section 2, we determine the t-pebbling number for Jahangir graph $J_{2,m}$. For that we use the following theorems.

Theorem 1.4([7]) *For the Jahangir graph $J_{2,3}, f(J_{2,3}) = 8$.*

Theorem 1.5([7]) *For the Jahangir graph $J_{2,4}, f(J_{2,4}) = 16$.*

Theorem 1.6([7]) *For the Jahangir graph $J_{2,5}, f(J_{2,5}) = 18$.*

Theorem 1.7([7]) *For the Jahangir graph $J_{2,6}, f(J_{2,6}) = 21$.*

Theorem 1.8([7]) *For the Jahangir graph $J_{2,7}, f(J_{2,7}) = 23$.*

Theorem 1.9([8]) *For the Jahangir graph $J_{2,m} (m \geq 8), f(J_{2,m}) = 2m + 10$.*

We now proceed to find the t-pebbling number for $J_{2,m}$.

§2. The t-Pebbling Number for Jahangir Graph $J_{2,m}, m \geq 3$

Theorem 2.1 *For the Jahangir graph* $J_{2,3}$, $f_t(J_{2,3}) = 8t$.

Proof Consider the Jahangir graph $J_{2,3}$. We prove this theorem by induction on t. By Theorem 1.4, the result is true for $t = 1$. For $t > 1$, $J_{2,3}$ contains at least 16 pebbles. Using at most 8 pebbles, we can put a pebble on any desired vertex, say $v_i(1 \leq \ i \leq 7)$, by Theorem 1.4. Then, the remaining number of pebbles on the vertices of $J_{2,3}$ is at least $8t - 8$. By induction we can put $t - 1$ additional pebbles on the desired vertex $v_i(1 \leq i \leq 7)$. So, the result is true for all t. Thus, $f_t(J_{2,3}) \leq 8t$.

Now, consider the following configuration C such that $C(v_4) = 8t-1$, and $C(x) = 0$, where $x \in V \setminus \{v_4\}$, then we cannot move t pebbles to the vertex v_1. Thus, $f_t(J_{2,3}) \geq 8t$. Therefore, $f_t(J_{2,3}) = 8t$. □

Theorem 2.2 *For the Jahangir graph* $J_{2,4}$, $f_t(J_{2,4}) = 16t$.

Proof Consider the Jahangir graph $J_{2,4}$. We prove this theorem by induction on t. By Theorem 1.5, the result is true for $t = 1$. For $t > 1$, $J_{2,4}$ contains at least 32 pebbles. By Theorem 1.5, using at most 16 pebbles, we can put a pebble on any desired vertex, say $v_i(1 \leq i \leq 9)$. Then, the remaining number of pebbles on the vertices of $J_{2,4}$ is at least $16t - 16$. By induction, we can put $t - 1$ additional pebbles on the desired vertex $v_i(1 \leq i \leq 9)$. So, the result is true for all t. Thus, $f_t(J_{2,4}) \leq 16t$.

Now, consider the following configuration C such that $C(v_6) = 16t - 1$, and $C(x) = 0$, where $x \in V \setminus \{v_6\}$, then we cannot move t pebbles to the vertex v_2. Thus, $f_t(J_{2,4}) \geq 16t$. Therefore, $f_t(J_{2,4}) = 16t$. □

Theorem 2.3 *For the Jahangir graph* $J_{2,5}$, $f_t(J_{2,5}) = 16t + 2$.

Proof Consider the Jahangir graph $J_{2,5}$. We prove this theorem by induction on t. By Theorem 1.6, the result is true for $t = 1$. For $t > 1$, $J_{2,5}$ contains at least 34 pebbles. Using at most 16 pebbles, we can put a pebble on any desired vertex, say $v_i(1 \leq \ i \leq 11)$. Then, the remaining number of pebbles on the vertices of the graph $J_{2,5}$ is at least $16t-14$. By induction, we can put $t - 1$ additional pebbles on the desired vertex $v_i(1 \leq i \leq 11)$. So, the result is true for all t. Thus, $f_t(J_{2,5}) \leq 16t + 2$.

Now, consider the following distribution C such that $C(v_6) = 16t-1$, $C(v_8) = 1$, $C(v_{10}) = 1$ and $C(x) = 0$, where $x \in V \setminus \{v_6, v_8, v_{10}\}$. Then we cannot move t pebbles to the vertex v_2. Thus, $f_t(J_{2,5}) \geq 16t + 2$. Therefore, $f_t(J_{2,5}) = 16t + 2$. □

Theorem 2.4 *For the Jahangir graph* $J_{2,m}(m \geq 6)$, $f_t(J_{2,m}) = 16(t - 1) + f(J_{2,m})$.

Proof Consider the Jahangir graph $J_{2,m}$, where $m > 5$. We prove this theorem by induction on t. By Theorems $1.7 - 1.9$, the result is true for $t = 1$. For $t > 1$, $J_{2,m}$ contains at least $16 + f(J_{2,m}) = 16 + \begin{cases} 2m+9 & m = 6,7 \\ 2m+10 & m \geq 8. \end{cases}$ pebbles. Using at most 16 pebbles, we can put a pebble on any desired vertex, say $v_i(1 \le i \le 2m+1)$. Then, the remaining number of pebbles on the vertices of the graph $J_{2,m}$ is at least $16t + f(J_{2,m}) - 32$. By induction, we can put $t-1$ additional pebbles on the desired vertex $v_i(1 \le i \le 2m+1)$. So, the result is true for all t. Thus, $f_t(J_{2,m}) \le 16(t-1) + f(J_{2,m})$.

Now, consider the following distributions on the vertices of $J_{2,m}$.

For $m = 6$, consider the following distribution C such that $C(v_6) = 16(t-1)+15$, $C(v_{10}) = 3$, $C(v_8) = 1$, $C(v_{12}) = 1$ and $C(x) = 0$, where $x \in V \setminus \{v_6, v_8, v_{10}, v_{12}\}$.

For $m = 7$, consider the following distribution C such that $C(v_6) = 16(t-1)+15$, $C(v_{10}) = 3$, $C(v_8) = C(v_{12}) = C(v_{13}) = C(v_{14}) = 1$ and $C(x) = 0$, where $x \in V \setminus \{v_6, v_8, v_{10}, v_{12}, v_{13}, v_{14}\}$.

For $m \ge 8$, if m is even, consider the following distribution C_1 such that $C_1(v_{m+2}) = 16(t-1)+15$, $C_1(v_{m-2}) = 3$, $C_1(v_{m+6}) = 3$, $C_1(x) = 1$, where $x \in \{N[v_2], N[v_{m+2}], N[v_{m-2}], N[v_{m+6}]\}$ and $C_1(y) = 0$, where $y \in \{N[v_2], N(v_{m+2}), N(v_{m-2}), N(v_{m+6})\}$.

If m is odd, then consider the following configuration C_2 such that $C_2(v_{m+1}) = 16(t-1)+15$, $C_2(v_{m-3}) = 3$, $C_2(v_{m+5}) = 3$, $C_2(x) = 1$, where $x \in \{N[v_2], N[v_{m+1}], N[v_{m-3}], N[v_{m+5}]\}$ and $C_2(y) = 0$, where $y \in \{N[v_2], N(v_{m+1}), N(v_{m-3}), N(v_{m+5})\}$. Then, we cannot move t pebbles to the vertex v_2 of $J_{2,m}$ for all $m \ge 6$. Thus, $f_t(J_{2,m}) \ge 16(t-1) + f(J_{2,m})$. Therefore, $f_t(J_{2,m}) = 16(t-1) + f(J_{2,m})$. □

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