

# On the Smarandache pierced chain

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**Abstract** If  $n \geq 1$ , then  $c(n) = 101 \times (10^{4n-4} + 10^{4n-8} + \dots + 10^4 + 1)$  is called as the Smarandache Pierced Chain. Its first few terms are:

101, 1010101, 10101010101, 101010101010101, 10101010101010101, .....

In reference [2], Dr.Kashihara Kenichiro asked whether  $\frac{c(n)}{101}$  is a square-free number for all  $n \geq 2$ ? The main purpose of this paper is using the elementary method to study this problem, and prove that there are infinite positive integers  $n$  such that 9 divides  $\frac{c(n)}{101}$ . That is to say,  $\frac{c(n)}{101}$  is not a square-free number for infinite integers  $n \geq 2$ .

**Keywords** Smarandache Pierced Chain, square-free number, sequence.

## §1. Introduction and results

If  $n \geq 1$ , then  $c(n) = 101 \times (10^{4n-4} + 10^{4n-8} + \dots + 10^4 + 1)$  is defined as the Smarandache Pierced Chain. Its first few terms are:

101, 1010101, 10101010101, 101010101010101, 10101010101010101, .....

In reference [1], F.Smarandache asked the question: how many primes are there in  $\frac{c(n)}{101}$ ? Dr.Kashihara Kenichiro [2] solved this problem completely, and proved that there are no primes in the sequence  $\left\{ \frac{c(n)}{101} \right\}$ . At the same time, Dr. Kashihara Kenichiro [2] also proposed the following problem: Is  $\left\{ \frac{c(n)}{101} \right\}$  a square-free for all  $n \geq 2$ ?

About this problem, it seems that none had studied it yet, at least we have not seen any related papers before. The problem is interesting, because it can help us to know more properties about the sequence  $\left\{ \frac{c(n)}{101} \right\}$ .

The main purpose of this paper is using the elementary method to study this problem, and solved it completely. That is, we shall prove the following :

**Theorem.** For any positive integer  $n$  with  $9 \mid n$ , we have  $9 \mid c(n)$ .

It is clear that  $(101, 9) = 1$ , so 9 divides  $\frac{c(n)}{101}$ . Therefore, from our Theorem we may immediately deduce the following:

**Corollary.** There are infinite positive integers  $n$  such that  $\frac{c(n)}{101}$  is not a square-free number.

## §2. Proof of the theorem

In this section, we shall complete the proof of our Theorem. First we give the definition of the  $k$ -free number: Let  $k \geq 2$  be any fixed integer. For any positive integer  $n > 1$ , we call  $n$  as a  $k$ -free number, if for any prime  $p$  with  $p|n$ , then  $p^k \nmid n$ . We call 2-free number as the square-free number; 3-free number as the cubic-free number. Now we prove our Theorem directly. It is clear that

$$10 \equiv 1 \pmod{9}.$$

From the basic properties of the congruences we know that if  $a \equiv b \pmod{m}$ , then  $a^n \equiv b^n \pmod{m}$  for every positive integer  $n$  (see reference [3] and [4]). So we have

$$10^{4n-4} \equiv 1 \pmod{9},$$

$$10^{4n-8} \equiv 1 \pmod{9},$$

.....

$$10^{4n} \equiv 1 \pmod{9}.$$

Obviously

$$1 \equiv 1 \pmod{9}.$$

Therefore,

$$\frac{c(n)}{101} = 10^{4n-4} + 10^{4n-8} + \cdots + 10^4 + 1 \equiv n \pmod{9}.$$

Now for any positive integer  $n$  with  $9|n$ , from the above congruence we may immediately get

$$\frac{c(n)}{101} \equiv 10^{4n-4} + 10^{4n-8} + \cdots + 10^4 + 1 \equiv n \equiv 0 \pmod{9}.$$

From the definition of the square-free number and the above properties we know that  $\frac{c(n)}{101}$  is not a square-free number if  $9|n$ . This completes the proof of Theorem.

## References

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