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Smarandache Seminormal Subgroupoids

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Abstract: In this paper, we define Smarandache seminormal subgroupoids. We have proved some results for finding the Smarandache seminormal subgroupoids in $Z(n)$ when n is even and n is odd.

Key Words: Groupoids, Smarandache groupoids, Smarandache seminormal subgroupoids

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§1. Introduction

In [5] and [6], W.B.Kandasamy defined new classes of Smarandache groupoids using Z_n. In this paper we define and prove some theorems for construction of Smarandache seminormal subgroupoids according as n is even or odd.

Definition 1.1 *A non-empty set of elements G is said to form a groupoid if in G is defined a binary operation called the product, denoted by $*$ such that $a * b \in G$ $\forall a, b \in G$. We denote groupoids by $(G, *)$.*

Definition 1.2 *Let $(G, *)$ be a groupoid. A proper subset $H \subset G$ is a subgroupoid if $(H, *)$ is itself a groupoid.*

Definition 1.3 *Let S be a non-empty set. S is said to be a semigroup if on S is defined a binary operation $*$ such that*

(1) *for all a, $b \in S$ we have $a * b \in S$;*
(2) *for all a, b, $c \in S$ we have $a * (b * c) = (a * b) * c$.*

$(S, *)$ is a semi-group.

Definition 1.4 *A Smarandache groupoid G is a groupoid which has a proper subset S such that S under the operation of G is a semigroup.*

Definition 1.5 *Let $(G, *)$ be a Smarandache groupoid. A non-empty subgroupoid H of G is said*

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to be a Smarandache subgroupoid if H contains a proper subset K such that K is a semigroup under the operation $*$.

Definition 1.6 *Let G be a Smarandache groupoid. V be a Smarandache subgroupoid of G. We say V is a Smarandache seminormal subgroupoid if $aV = V$ for all $a \in G$ or $Va = V$ for all $a \in G$.*

For example, let $(G, *)$ be groupoid given by the following table:

$*$	a_0	a_1	a_2	a_3	a_4	a_5
a_0	a_0	a_3	a_0	a_3	a_0	a_3
a_1	a_2	a_5	a_2	a_5	a_2	a_5
a_2	a_4	a_1	a_4	a_1	a_4	a_1
a_3	a_0	a_3	a_0	a_3	a_0	a_3
a_4	a_2	a_5	a_2	a_5	a_2	a_5
a_5	a_4	a_1	a_4	a_1	a_4	a_1

It is a Smarandache groupoid as $\{a_3\}$ is a semigroup. $V = \{a_1, a_3, a_5\}$ is a Smarandache subgroupoid,also $aV = V$. Therefore V is Smarandache seminormal subgroupoid in G.

Definition 1.7 *Let $Z_n = \{0, 1, \cdots, n-1\}$, $n \geqslant 3$ and $a, b \in Z_n \backslash \{0\}$. Define a binary operation $*$ on Z_n as follows:*

$a * b = ta + ub \pmod{n}$, where t, u are two distinct elements in $Z_n \backslash \{0\}$ and $(t, u) = 1$. Here $'+'$ is the usual addition of two integers and $'ta'$ means the product of the two integers t and a.

Elements of Z_n form a groupoid with respect to the binary operation $*$. We denote these groupoid by $\{Z_n(t, u), *\}$ or $Z_n(t, u)$ for fixed integer n and varying $t, u \in Z_n \backslash \{0\}$ such that $(t, u) = 1$. Thus we define a collection of groupoids $Z(n)$ as follows
$Z(n) = \{\{Z_n(t, u), *\}|$ for integers $t, u \in Z_n \backslash \{0\}$ such that $(t, u) = 1\}$.

§2. Smarandache Seminormal Subgroupoids When $n \equiv 0 (\text{mod} 2)$

When n is even we are interested in finding Smarandache seminormal subgroupoid in $Z_n(t, t+1)$.

Theorem 2.1 *Let $Z_n(t, t+1) \in Z(n)$, n is even, $n > 3$ and $t = 1, \cdots, n-2$. Then $Z_n(t, t+1)$ is Smarandache groupoid.*

Proof Let $x = \dfrac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + x(t+1) = 2xt + x \\ &= (2t+1)x \equiv x \bmod n \end{aligned}$$

Consequently, $\{x\}$ is a semigroup in $Z_n(t, t+1)$. Thus $Z_n(t, t+1)$ is a Smarandache groupoid when n is even. □

Remark In the above theorem we can also show that beside $\{n/2\}$ the other semigroup is $\{0, n/2\}$ in $Z_n(t, t+1) \in Z(n)$.

Proof If t is even, $0*t+\frac{n}{2}*(t+1) \equiv \frac{n}{2} \bmod n$, $\frac{n}{2}*t+0*(t+1) \equiv 0 \bmod n$, $\frac{n}{2}*t+\frac{n}{2}*(t+1) \equiv \frac{n}{2} \bmod n$ and $0*t+0*(t+1) \equiv 0 \bmod n$. So $\{0, \frac{n}{2}\}$ is semigroup in $Z_n(t, t+1)$. If t is odd, $0*t+\frac{n}{2}*(t+1) \equiv 0 \bmod n$, $\frac{n}{2}*t+0*(t+1) \equiv \frac{n}{2} \bmod n$, $\frac{n}{2}*t+\frac{n}{2}*(t+1) \equiv \frac{n}{2} \bmod n$ and $0*t+0*(t+1) \equiv 0 \bmod n$. So $\{0, \frac{n}{2}\}$ is a semigroup in $Z_n(t, t+1)$. □

Theorem 2.2 *Let $n > 3$ be even and $t = 1, \cdots, n-2$,*

(1) *If $\frac{n}{2}$ is even then $A_0 = \{0, 2, \cdots, n-2\} \subseteq Z_n$ is Smarandache subgroupoid in $Z_n(t, t+1) \in Z(n)$.*

(2) *If $\frac{n}{2}$ is odd then $A_1 = \{1, 3, \cdots, n-1\} \subseteq Z_n$ is Smarandache subgroupoid in $Z_n(t, t+1) \in Z(n)$.*

Proof (1) Let $\frac{n}{2}$ is even.$\Rightarrow \frac{n}{2} \in A_0$. We will show that A_0 is subgroupoid.

Let $x_i, x_j \in A_0$ and $x_i \neq x_j$. Then

$$\begin{aligned} x_i * x_j &= x_i t + x_j(t+1) \\ &= (x_i + x_j)t + x_j \equiv x_k \bmod n \end{aligned}$$

for some $x_k \in A_0$ as $(x_i + x_j)t + x_j$ is even. So $x_i * x_j \in A_0$. Thus A_0 is subgroupoid in $Z_n(t, t+1)$.

Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + x(t+1) \\ &= (2t+1)x \equiv x \bmod n. \end{aligned}$$

Therefore, $\{x\}$ is a semigroup in A_0. Thus A_0 is a subgroupoid in $Z_n(t, t+1)$.

(2) Let $\frac{n}{2}$ is odd. $\Rightarrow \frac{n}{2} \in A_1$. We show that A_1 is subgroupoid.

Let $x_i, x_j \in A_1$ and $x_i \neq x_j$. Then

$$\begin{aligned} x_i * x_j &= x_i t + x_j(t+1) \\ &= (x_i + x_j)t + x_j \equiv x_k \bmod n \end{aligned}$$

for some $x_k \in A_1$ as $(x_i + x_j)t + x_j$ is odd. Therefore, $x_i * x_j \in A_1$. Thus A_1 is subgroupoid in $Z_n(t, t+1)$.

Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + x(t+1) \\ &= (2t+1)x \equiv x \bmod n. \end{aligned}$$

So $\{x\}$ is a semigroup in A_1. Thus A_1 is a Smarandache subgroupoid in $Z_n(t, t+1)$. □

Theorem 2.3 *Let $n > 3$ be even and $t = 1, \cdots, n-2$,*

(1) *If $\frac{n}{2}$ is even then $A_0 = \{0, 2, \cdots, n-2\} \subseteq Z_n$ is Smarandache seminormal subgroupoid of $Z_n(t, t+1) \in Z(n)$.*

(2) *If $\frac{n}{2}$ is odd then $A_1 = \{1, 3, \cdots, n-1\} \subseteq Z_n$ is Smarandache seminormal subgroupoid of $Z_n(t, t+1) \in Z(n)$.*

Proof By Theorem 2.1, $Z_n(t, t+1)$ is a Smarandache groupoid.

(1) Let $\frac{n}{2}$ is even. Then by Theorem 2.2, $A_0 = \{0, 2, \cdots, n-2\}$ is Smarandache subgroupoid of $Z_n(t, t+1)$. Now we show that either $aA_0 = A_0$ or $A_0 a = A_0$ $\forall$ $a \in Z_n = \{0, 1, 2, \cdots, n-1\}$.

Case 1 t is even.

Let $a_i \in A_0$ and $a \in Z_n = \{0, 1, 2, \cdots, n-1\}$. Then

$$\begin{aligned} a * a_i &= at + a_i(t+1) \\ &\equiv a_j \bmod n \end{aligned}$$

for some $a_j \in A_0$ as $at + a_i(t+1)$ is even. Therefore, $a * a_i \in A_0$ $\forall$ $a_i \in A_0$, $aA_0 = A_0$. Thus, A_0 is a Smarandache seminormal subgroupoid in $Z_n(t, t+1)$.

Case 2 t is odd.

Let $a_i \in A_0$ and $a \in Z_n = \{0, 1, 2, \cdots, n-1\}$. Then

$$\begin{aligned} a_i * a &= a_i t + a(t+1) \\ &\equiv a_j \text{ mod n} \end{aligned}$$

for some $a_j \in A_0$ as $a_i t + a(t+1)$ is even. Therefore, $a_i * a \in A_0$ $\forall$ $a_i \in A_0$, $A_0 a = A_0$. Thus A_0 is a Smarandache seminormal subgroupoid in $Z_n(t, t+1)$.

(2) Let $\frac{n}{2}$ is odd. Then by Theorem 2.2, $A_1 = \{1, 3, 5, \cdots, n-1\}$ is Smarandache subgroupoid of $Z_n(t, t+1)$. Now we show that either $aA_1 = A_1$ or $A_1 a = A_1$ $\forall$ $a \in Z_n = \{0, 1, 2, \cdots, n-1\}$.

Case 1 t is even.

Let $a_i \in A_1$ and $a \in Z_n = \{0, 1, 2, \cdots, n-1\}$. Then

$$\begin{aligned} a * a_i &= at + a_i(t+1) \\ &= (a + a_i)t + a_i \\ &\equiv a_j \bmod n \end{aligned}$$

for some $a_j \in A_1$ as $(a + a_i)t + a_i$ is odd. Therefore, $a * a_i \in A_1$ $\forall$ $a_i \in A_1$, $\therefore aA_1 = A_1$. Thus A_1 is Smarandache seminormal subgroupoid in $Z_n(t, t+1)$.

Case 2 t is odd.

Let $a_i \in A_1$ and $a \in Z_n = \{0, 1, 2, \cdots, n-1\}$. Then

$$\begin{aligned} a_i * a &= a_i t + a(t+1) \\ &\equiv a_j \text{ mod n} \end{aligned}$$

for some $a_j \in A_1$ as $a_i t + a(t+1)$ is odd. Therefore, $a_i * a \in A_1\ \forall\ \ a_i \in A_1,\ A_1 a = A_1$. Thus A_1 is Smarandache seminormal subgroupoid in $Z_n(t, t+1)$. □

By the above theorem we can determine the Smarandache seminormal subgroupoid in $Z_n(t, t+1)$ of $Z(n)$ when n is even and $n > 3$.

n	$n/2$	t	$Z_n(t,t+1)$	Smarandache seminormal subgroupoid in $Z_n(t,t+1)$
4	2	1	$Z_4(1,2)$	$\{0,2\}$
		2	$Z_4(2,3)$	
6	3	1	$Z_6(1,2)$	$\{1,3,5\}$
		2	$Z_6(2,3)$	
		3	$Z_6(3,4)$	
		4	$Z_6(4,5)$	
8	4	1	$Z_8(1,2)$	$\{0,2,4,6\}$
		2	$Z_8(2,3)$	
		3	$Z_8(3,4)$	
		4	$Z_8(4,5)$	
		5	$Z_8(5,6)$	
		6	$Z_8(6,7)$	
10	5	1	$Z_{10}(1,2)$	$\{1,3,5,7,9\}$
		2	$Z_{10}(2,3)$	
		3	$Z_{10}(3,4)$	
		4	$Z_{10}(4,5)$	
		5	$Z_{10}(5,6)$	
		6	$Z_{10}(6,7)$	
		7	$Z_{10}(7,8)$	
		8	$Z_{10}(8,9)$	
12	6	1	$Z_{12}(1,2)$	$\{0,2,4,6,8\}$
		2	$Z_{12}(2,3)$	
		3	$Z_{12}(3,4)$	
		4	$Z_{12}(4,5)$	
		5	$Z_{12}(5,6)$	
		6	$Z_{12}(6,7)$	
		7	$Z_{12}(7,8)$	
		8	$Z_{12}(8,9)$	
		9	$Z_{12}(9,10)$	
		10	$Z_{12}(10,11)$	

§3. Smarandache Seminormal Subgroupoids Depend on t, u when $n \equiv 0(\text{mod}2)$

When n is even we are interested in finding Smarandache seminormal subgroupoid in $Z_n(t,u) \in Z(n)$ when t is even and u is odd or when t is odd and u is even.

Theorem 3.1 *Let $Z_n(t,u) \in Z(n)$,if n is even,$n > 3$ and for each $t, u \in Z_n$,if one is even and other is odd then $Z_n(t,u)$ is Smarandache groupoid.*

Proof Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + xu \\ &= (t+u)x \equiv x \bmod n. \end{aligned}$$

So $\{x\}$ is a semigroup in $Z_n(t,u)$. Thus $Z_n(t,u)$ is a Smarandache groupoid when n is even. □

Remark In the above theorem we can also show that beside $\{n/2\}$ the other semigroup is $\{0, n/2\}$ in $Z_n(t,u) \in Z(n)$.

Proof If t is even and u is odd, $0 * t + \frac{n}{2} * u \equiv \frac{n}{2} \bmod n$, $\frac{n}{2} * t + 0 * u \equiv 0 \bmod n$, $\frac{n}{2} * t + \frac{n}{2} * u \equiv \frac{n}{2} \bmod n$ and $0 * t + 0 * u \equiv 0 \bmod n$. So $\{0, \frac{n}{2}\}$ is semigroup in $Z_n(t,u)$. If t is odd and u is even, $0 * t + \frac{n}{2} * u \equiv 0 \bmod n$, $\frac{n}{2} * t + 0 * u \equiv \frac{n}{2} \bmod n$, $\frac{n}{2} * t + \frac{n}{2} * u \equiv \frac{n}{2} \bmod n$ and $0 * t + 0 * u \equiv 0 \bmod n$. So $\{0, \frac{n}{2}\}$ is semigroup in $Z_n(t,u)$. □

Theorem 3.2 *Let $n > 3$ be even and $t, u \in Z_n$.*

(1) *If $\frac{n}{2}$ is even then $A_0 = \{0, 2, \cdots, n-2\} \subseteq Z_n$ is Smarandache subgroupoid of $Z_n(t,u) \in Z(n)$ when one of t and u is odd and other is even.*

(2) *If $\frac{n}{2}$ is odd then $A_1 = \{1, 3, \cdots, n-1\} \subseteq Z_n$ is Smarandache subgroupoid of $Z_n(t,u) \in Z(n)$ when one of t and u is odd and other is even.*

Proof (1) Let $\frac{n}{2}$ be even. $\Rightarrow \frac{n}{2} \in A_0$. We show that A_0 is subgroupoid.
Let $x_i, x_j \in A_0$ and $x_i \neq x_j$. Then

$$x_i * x_j = x_i t + x_j u \equiv x_k \bmod n$$

for some $x_k \in A_0$ as $x_i t + x_j u$ is even. So $x_i * x_j \in A_0$. Thus A_0 is a subgroupoid in $Z_n(t,u)$.
Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + xu \\ &= x(t+u) \equiv x \bmod n. \end{aligned}$$

Whence, $\{x\}$ is a semigroup in A_0. Thus, A_0 is a Smarandache subgroupoid in $Z_n(t,u)$.

(2) Let $\frac{n}{2}$ be odd. $\Rightarrow \frac{n}{2} \in A_1$. We show that A_1 is subgroupoid.
Let $x_i, x_j \in A_1$ and $x_i \neq x_j$. Then

$$x_i * x_j = x_i t + x_j u \equiv x_k \bmod n$$

for some $x_k \in A_1$ as $x_i + x_j u$ is odd. So $x_i * x_j \in A_1$. Consequently, A_1 is subgroupoid in $Z_n(t,u)$.

Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + xu \\ &= x(t+u) \equiv x \bmod n. \end{aligned}$$

So $\{x\}$ is a semigroup in A_1. Thus A_1 is a Smarandache subgroupoid in $Z_n(t,u)$. □

Theorem 3.3 *Let $n > 3$ be even and $t = 1, \cdots, n-2$.*

(1) If $\frac{n}{2}$ is even then $A_0 = \{0, 2, \cdots, n-2\} \subseteq Z_n$ is Smarandache seminormal subgroupoid of $Z_n(t,u) \in Z(n)$ when one of t and u is odd and other is even;

(2) If $\frac{n}{2}$ is odd then $A_1 = \{1, 3, \cdots, n-1\} \subseteq Z_n$ is Smarandache seminormal subgroupoid of $Z_n(t,u) \in Z(n)$ when one of t and u is odd and other is even.

Proof By Theorem 3.1, $Z_n(t,u)$ is a Smarandache groupoid.

(1) Let $\frac{n}{2}$ is even. Then by Theorem 3.2, $A_0 = \{0, 2, \cdots, n-2\}$ is Smarandache subgroupoid of $Z_n(t,u)$. Now we show that either $aA_0 = A_0$ or $A_0 a = A_0$ $\forall$ $a \in Z_n = \{0, 1, 2, \cdots, n-1\}$.

Case 1 t is even and u is odd.

Let $a_i \in A_0$ and $a \in Z_n = \{0, 1, 2, \cdots, n-1\}$. Then

$$\begin{aligned} a * a_i &= at + a_i u \\ &\equiv a_j \bmod n \end{aligned}$$

for some $a_j \in A_0$ as $at + a_i u$ is even. Whence, $a * a_i \in A_0$ $\forall$ $a_i \in A_0$, $aA_0 = A_0$. Thus, A_0 is a Smarandache seminormal subgroupoid in $Z_n(t,u)$.

Case 2 t is odd and u is even.

Let $a_i \in A_0$ and $a \in Z_n = \{0, 1, 2, \cdots, n-1\}$. Then

$$\begin{aligned} a_i * a &= a_i t + au \\ &\equiv a_j \bmod \mathrm{n} \end{aligned}$$

for some $a_j \in A_0$ as $a_i t + au$ is even. Therefore, $a_i * a \in A_0$ $\forall$ $a_i \in A_0$, $A_0 a = A_0$. Thus, A_0 is Smarandache seminormal subgroupoid in $Z_n(t,u)$.

(2) Let $\frac{n}{2}$ is odd then by Theorem 3.2 is $A_1 = \{1, 3, 5, \cdots, n-1\}$ is Smarandache subgroupoid of $Z_n(t,u)$. We show that either $aA_1 = A_1$ or $A_1 a = A_1$ $\forall$ $a \in Z_n = \{0, 1, 2, \cdots, n-1\}$.

Case 1 t is even and u is odd.

Let $a_i \in A_1$ and $a \in Z_n = \{0, 1, 2, \cdots, n-1\}$. Then

$$\begin{aligned} a * a_i &= at + a_i u \\ &\equiv a_j \bmod n \end{aligned}$$

for some $a_j \in A_1$ as $at + a_i u$ is odd. So, $a * a_i \in A_1 \ \forall \ a_i \in A_1$, $\therefore \ aA_1 = A_1$. Thus, A_1 is a Smarandache seminormal subgroupoid in $Z_n(t,u)$.

Case 2 t is odd and u is even.

Let $a_i \in A_1$ and $a \in Z_n = \{0, 1, 2, \cdots, n-1\}$.

$$\begin{aligned} a_i * a &= a_i t + au \\ &\equiv a_j \bmod \text{n} \end{aligned}$$

for some $a_j \in A_1$ as $a_i t + au$ is odd. Therefore, $a_i * a \in A_1 \ \forall \ a_i \in A_1$, $A_1 a = A_1$. Thus, A_1 is a Smarandache seminormal subgroupoid in $Z_n(t,u)$. □

By the above theorem we can determine Smarandache seminormal subgroupoid in $Z_n(t,u) \in Z(n)$ for $n > 3$, when n is even and when one of t and u is odd and other is even.

n	$n/2$	t	$Z_n(t,u)$	Smarandache seminormal subgroupoid
4	2	1	$Z_4(1,2)$	$\{0,2\}$
		2	$Z_4(2,3)$	
6	3	1	$Z_6(1,2), Z_6(1,4)$	$\{1,3,5\}$
		2	$Z_6(2,1), Z_6(2,3), Z_6(2,5)$	
		3	$Z_6(3,2), Z_6(3,4)$	
		4	$Z_6(4,1), Z_6(4,3), Z_6(4,5)$	
		5	$Z_6(5,2), Z_6(5,4)$	
8	4	1	$Z_8(1,2), Z_8(1,4), Z_8(1,6)$	$\{0,2,4,6\}$
		2	$Z_8(2,1), Z_8(2,3), Z_8(2,5), Z_8(2,7)$	
		3	$Z_8(3,2), Z_8(3,4)$	
		4	$Z_8(4,1), Z_8(4,3), Z_8(4,5), Z_8(4,7)$	
		5	$Z_8(5,2), Z_8(5,4), Z_8(5,6)$	
		6	$Z_8(6,1), Z_8(6,5), Z_8(6,7),$	
		7	$Z_8(7,2), Z_8(7,4), Z_8(7,6),$	

n	$n/2$	t	$Z_n(t,u)$	Smarandache seminormal subgroupoid
10	5	1	$Z_{10}(1,2), Z_{10}(1,4), Z_{10}(1,6), Z_{10}(1,8)$	$\{1,3,5,7,9\}$
		2	$Z_{10}(2,1), Z_{10}(2,3), Z_{10}(2,5), Z_{10}(2,7), Z_{10}(2,9)$	
		3	$Z_{10}(3,2), Z_{10}(3,4), Z_{10}(3,8),$	
		4	$Z_{10}(4,1), Z_{10}(4,3), Z_{10}(4,5), Z_{10}(4,7), Z_{10}(4,9)$	
		5	$Z_{10}(5,2), Z_{10}(5,4), Z_{10}(5,6), Z_{10}(5,8)$	
		6	$Z_{10}(6,1), Z_{10}(6,5), Z_{10}(6,7),$	
		7	$Z_{10}(7,2), Z_{10}(7,4), Z_{10}(7,6), Z_{10}(7,8)$	
		8	$Z_{10}(8,1), Z_{10}(8,3), Z_{10}(8,5), Z_{10}(8,7), Z_{10}(8,9)$	
		9	$Z_{10}(9,2), Z_{10}(9,4), Z_{10}(9,8)$	
12	6	1	$Z_{12}(1,2), Z_{12}(1,4), Z_{12}(1,6), Z_{12}(1,8), Z_{12}(1,10)$	$\{0,2,4,6,8,10\}$
		2	$Z_{12}(2,1), Z_{12}(2,3), Z_{12}(2,5), Z_{12}(2,7), Z_{12}(2,9), Z_{12}(2,11)$	
		3	$Z_{12}(3,2), Z_{12}(3,4), Z_{12}(3,8), Z_{12}(3,10)$	
		4	$Z_{12}(4,1), Z_{12}(4,3), Z_{12}(4,5), Z_{12}(4,7), Z_{12}(4,9), Z_{12}(4,11)$	
		5	$Z_{12}(5,2), Z_{12}(5,4), Z_{12}(5,6), Z_{12}(5,8)$	
		6	$Z_{12}(6,1), Z_{12}(6,3), Z_{12}(6,5), Z_{12}(6,7), Z_{12}(6,11)$	
		7	$Z_{12}(7,2), Z_{12}(7,4), Z_{12}(7,6), Z_{12}(7,8), Z_{12}(7,10)$	
		8	$Z_{12}(8,1), Z_{12}(8,3), Z_{12}(8,5), Z_{12}(8,7), Z_{12}(8,9), Z_{12}(8,11)$	
		9	$Z_{12}(9,2), Z_{12}(9,4), Z_{12}(9,8), Z_{12}(9,10)$	
		10	$Z_{12}(10,1), Z_{12}(10,3), Z_{12}(10,7), Z_{12}(10,9), Z_{12}(10,11)$	
		11	$Z_{12}(11,2), Z_{12}(11,4), Z_{12}(11,6), Z_{12}(11,8), Z_{12}(11,10)$	

§4. Smarandache Seminormal Subgroupoids When $n \equiv 1(\text{mod}2)$

When n is odd we are interested in finding Smarandache seminormal subgroupoid in $Z_n(t,u) \in Z(n)$.We have proved the similiar result in [4].

Theorem 4.1 *Let $Z_n(t,u) \in Z(n)$. If n is odd, $n > 4$ and for each $t = 2, \cdots, \frac{n-1}{2}$ and $u = n-(t-1)(t,u) = 1$, then $Z_n(t,u)$ is a Smarandache groupoid.*

Proof Let $x \in \{0, \cdots, n-1\}$. Then

$$x * x = xt + xu = (n+1)x \equiv x \bmod n.$$

So $\{x\}$ is semigroup in Z_n. Thus $Z_n(t,u)$ is a Smarandache groupoid in $Z(n)$. □

Remark We note that all $\{x\}$ where $x \in \{1, \cdots, n-1\}$ are proper subsets which are semigroups in $Z_n(t,u)$.

Theorem 4.2 *Let $n > 4$ be odd and $t = 2, \cdots, \frac{n-1}{2}$ and $u = n-(t-1)$ such that $(t,u) = 1$ if $s = (n,t)$ or $s = (n,u)$ then $A_k = \{k, k+s, \cdots, k+(r-1)s\}$ for $k = 0, 1, \cdots, s-1$ where $r = \frac{n}{s}$ is a Smarandache subgroupoid in $Z_n(t,u) \in Z(n)$.*

Proof Let $x_p, x_q \in A_k$. Then

$$x_p \neq x_q \Rightarrow \left.\begin{array}{l} x_p = k+ps \\ x_q = k+qs \end{array}\right\} p,\ q\ \in \{0, 1, \cdots, r-1\}.$$

Also,

$$\begin{aligned} x_p * x_q &= x_pt + x_qu \\ &= (k+ps)t + (k+qs)(n-(t-1)) \\ &= k(n+1) + ((p-q)t + q(n+1))s \\ &\equiv (k+ls) \bmod n \\ &\equiv x_l \bmod n \end{aligned}$$

$x_l \in A_k$ as $x_l = k+ls$ for some $l \in \{0, 1, \cdots, r-1\}$. Whence, $x_p * x_q \in A_k$. Consequently, A_k is a subgroupoid in $Z_n(t,u)$. By the above remark all singleton sets are semigroup. Thus,A_k is a Smarandache subgroupoid. □

Theorem 4.3 *Let $n > 4$ be odd and $t = 2, \cdots, \frac{n-1}{2}$ and $u = n-(t-1)$ such that $(t,u) = 1$ if $s = (n,t)$ or $s = (n,u)$ then $A_k = \{k, k+s, \cdots, k+(r-1)s\}$ for $k = 0, 1, \cdots, s-1$ where $r = \frac{n}{s}$ is a Smarandache seminormal subgroupoid in $Z_n(t,u) \in Z(n)$.*

Proof By Theorem 4.1, $Z_n(t,u)$ is a Smarandache groupoid. Also by Theorem 4.2, $A_k = \{k, k+s, \cdots, k+(r-1)s\}$ for $k = 0, 1, \cdots, s-1$ is Smarandache subgroupoid of $Z_n(t,u)$.

If $s = (n, t)$, let $x_p \in A_k$ and $a \in Z_n = \{0, 1, 2, \cdots, n-1\}$. Then

$$\begin{aligned} a * x_p &= at + x_p u \\ &= at + (k + ps)(n - t + 1) \\ &= k(n+1) + [(a-k)v_1 + (pn - pt + p)]s \text{ where } t = v_1 s \\ &\equiv k + ls \bmod n \end{aligned}$$

$x_l \in A_k$ as $x_l = k + ls$ for some $l \in \{0, 1, \cdots, r-1\}$. So, $a * x_p \in A_k$, $a * A_k = A_k$. Thus, A_k is a Smarandache seminormal subgroupoid in $Z_n(t, u)$.

If $s = (n, u)$, let $x_p \in A_k$ and $a \in Z_n = \{0, 1, 2, \cdots, n-1\}$. Then

$$\begin{aligned} x_p * a &= x_p t + au \\ &= (k + ps)(n - u + 1) + au \\ &= k(n+1) + [(a-k)v_2 + (pn - pu + p)]s \text{ where } t = v_2 s \\ &\equiv (k + ls) \bmod n \end{aligned}$$

$x_l \in A_k$ as $x_l = k + ls$ for some $l \in \{0, 1, \cdots, r-1\}$. Therefore, $a * x_p \in A_k$, $a * A_k = A_k$. Thus A_k is a Smarandache seminormal subgroupoid in $Z_n(t, u)$. □

By the above theorem we can determine Smarandache seminormal subgroupoid in $Z_n(t, u)$ when n is odd and $n > 4$.

n	t	u	$Z_n(t,u)$	$s = (n,u)$ or $s = (n,t)$	$r = n/s$	Smarandache seminormal subgroupoid in $Z_n(t,u)$
9	3	7	$Z_9(3,7)$	3 = (9,3)	3	$A_0 = \{0,3,6\}$
						$A_1 = \{1,4,7\}$
						$A_2 = \{2,5,8\}$
15	3	13	$Z_{15}(3,13)$	3 = (15,3)	5	$A_0 = \{0,3,6,9,12\}$
						$A_1 = \{1,4,7,10,13\}$
						$A_2 = \{2,5,8,11,14\}$
	5	11	$Z_{15}(5,11)$	5 = (15, 5)	3	$A_0 = \{0,5,10\}$
						$A_1 = \{1,6,11\}$
						$A_2 = \{2,7,12\}$
						$A_3 = \{3,8,13\}$
						$A_4 = \{4,9,14\}$
	7	9	$Z_{15}(7,9)$	3 = (15,9)	5	$A_0 = \{0,3,6,9,12\}$
						$A_1 = \{1,4,7,10,13\}$
						$A_2 = \{2,5,8,11,14\}$

n	t	u	$Z_n(t,u)$	$s=(n,u)$ or $s=(n,t)$	$r=n/s$	Smarandache seminormal subgroupoid in $Z_n(t,u)$
21	3	19	$Z_{21}(3,19)$	$3=$ (21,3)	7	$A_0=\{0,3,6,9,12,15,18\}$
						$A_1=\{1,4,7,10,13,16,19\}$
						$A_2=\{2,5,8,11,14,17,20\}$
	7	15	$Z_{21}(7,15)$	$7=$ (21,7)	3	$A_0=\{0,7,14\}$
						$A_1=\{1,8,15\}$
						$A_2=\{2,9,16\}$
						$A_3=\{3,10,17\}$
						$A_4=\{4,11,18\}$
						$A_5=\{5,12,19\}$
						$A_6=\{6,13,14\}$
				$3=$ (21,15)	7	$A_0=\{0,3,6,9,12,15,18\}$
						$A_1=\{1,4,7,10,13,16,19\}$
						$A_2=\{2,5,8,11,14,17,20\}$
	9	13	$Z_{21}(9,13)$	$3=$ (21,9)	7	$A_0=\{0,3,6,9,12,15,18\}$
						$A_1=\{1,4,7,10,13,16,19\}$
						$A_2=\{2,5,8,11,14,17,20\}$

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