SMARANDACHE SEQUENCE OF UNHAPPY NUMBERS

Muneer Jebreel Karama
SS-Math-Hebron (Hebron Education Office / UNRWA), Field Education Officer, Jerusalem, Box 19149, Israel

Abstract The main purpose of this paper is to introduce new concepts of Smarandache numbers, namely Smarandache Sequence of Unhappy Numbers, and give definition, theorem, and ask open problems.

Keywords: Happy Number, Unhappy Number, Smarandache Sequence of Unhappy Numbers, Reversed Smarandache Sequence of Unhappy Numbers.

1.1 Definition. Iterating the process of summing the squares of the decimal digits of a number and if the process terminates in 4, then the original number is called Unhappy Number (UN).

Examples:

1) $2 \rightarrow 4$.

2) $3 \rightarrow 9 \rightarrow 8165 \rightarrow 61 \rightarrow 35 \rightarrow 34 \rightarrow 25 \rightarrow 29 \rightarrow 85 \rightarrow 89 \rightarrow 145 \rightarrow 42 \rightarrow 20 \rightarrow 4$.

3) $99 \rightarrow 162 \rightarrow 41 \rightarrow 17 \rightarrow 50 \rightarrow 25 \rightarrow 29 \rightarrow 95 \rightarrow 106 \rightarrow 37 \rightarrow 58 \rightarrow 89 \rightarrow 145 \rightarrow 42 \rightarrow 20 \rightarrow 4$. Hence, 2, 3, and 99 are unhappy numbers.

1.3 The sequence of Unhappy Numbers (UN). The proposed sequence of the UN is;

$$UN = 2, 3, 4, 5, 6, 8, 9, 11, 12, 14, 15, 16, 17, 18, 20, 21, \cdots.$$

Note that UN is a counterpart of the sequence of Happy Numbers (HN), for more details about HN see [1].

1.4 Theorem. HN UN = N, where HN, the set of Happy Numbers, and UN, the set of Unhappy Numbers, and N, the set of Natural numbers.

That's to say that the natural numbers may be classified to happy or unhappy, there are no other choice.

proof of the theorem. Consider the order subsets HN, and UN of N (i.e. UN N, and HN N). Where

$$UN = 2, 3, 4, 5, 6, 8, 9, 11, 12, 14, 15, 16, 17, 18, 20, 21, \cdots,$$

$$HN = 1, 7, 10, 13, 19, 23, 28, 31, 32, 44, 49, 68, 70, 79, 82, 86, 91, 94, \cdots,$$

and

$$N = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \cdots.$$

Now UN and HN are well- ordered sets, since 2, 1 are the first elements respectively of UN, and HN.

Now

$$HN \bigcup UN = 1, 7, 10, 13, 19, 23, 28, 31, 32, \cdots; 2, 3, 4, 5, 6, 8, 9, 11, 12, \cdots$$

i.e. the union ordered from left to right, and it is well-ordered.

Thus, HN $\bigcup$ UN = N (since N is well-ordered set).

1.5 Interesting Note: Iterating the process of summing the squares of the decimal digits of a number in both sequence UN, and HN then the process terminates in 4, and in 1 respectively, i.e. 4 and 1 are two squares, so $4 - 1 = 3, 4 + 1 = 5$, and $3^2 + 4^2 = 5^2$, hence the first Pythagorean triples may have relationship with happy and unhappy numbers.

1.6 Smarandache Unhappy Sequence (SUS). SUS is the sequence formed from concatenation of numbers in UN sequence, i.e. $SUS = \{2, 23, 234, 2345, 23456, 234568, 2345689 \cdots\}$. Problems:

1)$2, 23$ are prime numbers; how many terms of SUS are primes?

2)23 is a happy number; how many terms of SUS are happy numbers?

3)234 is an unhappy number; how many terms of SUS are unhappy numbers?

4)$2, 3, 4, 5, 6$ are consecutive unhappy numbers; how many consecutive terms of SUS are unhappy numbers?

1.7 Reversed Smarandache Unhappy Sequence (RSUS).

$$RSUS = 2, 32, 432, 5432, 65432, 865432, 9865432, \cdots.$$

It is obvious that there are no such prime numbers (excluding 2) Problems:

1)32 is a happy number; how many terms of RSUS are happy numbers?

2)432 is an unhappy number; how many terms of RSUS are unhappy numbers?

Reference

[1] Sloane, N. J. A., Sequence A007770 in "The online version of the Encyclopedia of integer's sequence",

http:// www.research.att.com/ njas/sequence