Single Valued Neutrosophic Kruskal-Wallis and Mann Whitney Tests

Mohamed Bisher Zeina
University of Aleppo

Neutrosophic Sets and Systems, Vol. 51, 2022
University of New Mexico

Single Valued Neutrosophic Kruskal-Wallis and Mann Whitney Tests

Mahmoud Miari [1], Mohamad Taher Anan [2] and Mohamed Bisher Zeina [3]*

[1] Department of Mathematical Statistics, Faculty of Science, University of Aleppo, Aleppo, Syria; mahmoudmiari1994@gmail.com
[2] Department of Mathematical Statistics, Faculty of Science, University of Aleppo, Aleppo, Syria; mtanan200988@gmail.com
[3] Department of Mathematical Statistics, Faculty of Science, University of Aleppo, Aleppo, Syria; bisher.zeina@gmail.com

* Correspondence: bisher.zeina@gmail.com

Abstract: In this paper, Kruskal-Wallis test is extended to deal with neutrosophic data in single valued form using score, accuracy and certainty functions to calculate ranks of SVNNs, also Mann-Whitney test is extended to deal with same data type which makes it possible to do a post-hoc test after rejecting null hypothesis using Neutrosophic Statistics Kruskal-Wallis test. Numerical examples were successfully solved showing the power of this new idea to deal with SVNNs and make statistical decisions on them.

Keywords: Kruskal-Wallis; Test Statistic; Chi Square Distribution; Hypothesis Testing; Significance Level; Single Valued Neutrosophic Number.

1. Introduction

F. Smarandache presented neutrosophic logic as an extension to fuzzy logic [1] and intuitionistic fuzzy logic [2] to deal with indeterminacy, ambiguity, uncertainty, contradiction, unsureness, nihilness, vagueness and emptiness [3], this new extension make decisions more flexible and reliable [4] [5] and has been applied in many scientific fields including abstract algebra, mathematical modelling, probability theory, statistics, operations research, artificial intelligence, machine learning, etc. [6] [5] [7] [8] [9] [10] [11] [12] [13] [14] [15] [16] [17] [18]. He also introduced the Neutrosophic Statistics as an extension of the Interval Statistics, since the neutroosphic statistics may deal with all types of indeterminacies (with respect to the data, inferential procedures, probability distributions, graphical representations, etc.), it allows the reduction of indeterminacy, and it uses the neutrosophic probability that is more general than imprecise and classical probabilities, and has more detailed corresponding probability density functions - while Interval Statistics only deals with indeterminacy that can be represented by intervals. [27].

In statistics, M. Aslam presented many neutrosophic statistical tests to deal with indeterminacy in data considering that observations are classical neutrosophic numbers of the form $N = D + I$ where D is the determinant part of the number and I is its indeterminant part [19] [20] [21] [22].

Comparing population means is one of the most important statistical tests to test whether several drawn samples are from one population (then we say that means are equal) or from different populations (here we say that means are not equal). This procedure is done using hypothesis testing with respect to a test statistic having a previously known probability distribution comparing its value with acceptance region and rejection region.

The problem arises when dealing with neutrosophic number or judges, e.g., if a doctor says that a patient is 70% infected with COVID-19 with 20% indeterminacy because of similar flu syndromes and with 50% chance to be wrong diagnosis, here we cannot deal with this data type using classical statistical tests neither with previously studied neutrosophic statistical tests.

A mathematical solve for this problem in lattice theory and abstract algebra was presented in [23] where ranking of observations was done and presented in [24] to compare between judges. also, previous work was generalized in [25] [26].

In this paper we are going to solve this problem from statistical point of view where we are dealing with samples data derived from different populations to make generalize decisions made based on samples to population extending Kruskal-Wallis test to deal with (T, I, F) data sets which is the well-known single valued neutrosophic numbers and make it possible to compare several samples and take decision if those samples are drawn from same population or from different populations, then we will extend Mann-Whitney test to make a multiple comparison between each two groups.

2. Preliminaries

We recall here some basic definitions of single valued neutrosophic sets and single valued neutrosophic numbers and some operations on them.

2.1 Single Valued Neutrosophic Sets:

Suppose that Ω is the universe and let A be a subset of Ω then A is said to be Single Valued Neutrosophic Set (SVNS) with truth, indeterminacy and falsity memberships and denoted as follows:

$$A = \{(x|T_A(x), I_A(x), F_A(x))\}$$

Where:

$$T_A: \Omega \rightarrow [0,1]$$
$$I_A: \Omega \rightarrow [0,1]$$
$$F_A: \Omega \rightarrow [0,1]$$

And:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

2.2 Single Valued Neutrosophic Numbers:

Single Valued Neutrosophic Number (SVNN) takes the form (T, I, F) where T reflects truth, I reflects indeterminacy and F reflects falsity where $0 \leq T, I, F \leq 1$ and $0 \leq T + I + F \leq 3$.

2.3 Operations on Single Valued Neutrosophic Numbers:

Suppose that $A = (t_1, i_1, f_1), B = (t_2, i_2, f_2)$ are two SVNNs then operations on A, B are defined as follows:

$$A \oplus B = (t_1 + t_2 - t_1 t_2, i_1 i_2, f_1 f_2)$$
$$A \otimes B = (t_1 t_2, i_1 + i_2 - i_1 i_2, f_1 + f_2 - f_1 f_2)$$
$$A \ominus B = \left(\frac{t_1 - t_2}{1 - t_2}, \frac{i_1}{i_2}, \frac{f_1}{f_2}\right); t_2 \neq 1; i_2 \neq 0; f_2 \neq 0$$
$$\frac{A}{B} = \left(\frac{t_1}{t_2}, \frac{i_1 - i_2}{1 - i_2}, \frac{f_1 - f_2}{1 - f_2}\right); t_2 \neq 0; i_2 \neq 1; f_2 \neq 1$$
$$\lambda A = (1 - (1 - t_1)^\lambda, {i_1}^\lambda, {f_1}^\lambda); \lambda > 0$$
$$A^\lambda = ({t_1}^\lambda, 1 - (1 - i_1)^\lambda, 1 - (1 - f_1)^\lambda); \lambda > 0$$

2.4 Ranking of Single Valued Neutrosophic Numbers

Let $A(T, I, F)$ be a SVNN, the score function $s(A)$, accuracy function $a(A)$ and certainty function $c(A)$ are defined as follows:

$$s(A) = \frac{2 + T - I - F}{3}$$
$$a(A) = T - F$$
$$c(A) = T$$

We can rank A, B using the following algorithm:

1) If $s(A) > s(B)$ $then$ $A > B$.
2) If $s(A) = s(B)$ and $a(A) > a(B)$ $then$ $A > B$.
3) If $s(A) = s(B)$ and $a(A) = a(B)$ and $c(A) > c(B)$ $then$ $A > B$.
4) If $s(A) = s(B)$ and $a(A) = a(B)$ and $c(A) = c(B)$ $then$ $A = B$

3. Classical Kruskal-Wallis and Mann Whitney Tests

Kruskal-Wallis Test (H Test) one of the nonparametric tests that based on ranks used to compare the means of c independent random samples of sizes $n_1, \dots, n_c$ drawn from c univariate populations with unknown cumulative distribution functions $F_1, \dots, F_C$.

The technique of (H Test) performed by ranking all observation and defined as follows:

Formally, letting the distribution function of X over the group i be of the form $F_i(x) = F(y - \theta_i)$, we'd like to test

$$H_0: \theta_1 = \theta_2 = \cdots = \theta_C \quad \text{against} \quad H_1: \theta_i \neq \theta_j \; for \; some \; i, j$$

The test is based on $\chi^2(c-1)$ distribution using test statistic:

$$H = \frac{12}{N(N+1)} \sum_{i=1}^{C} \frac{R_i^2}{n_i} - 3(N+1)$$

Where:

c number of samples

n_i number of observations in the i[th] group

$N = \sum n_i$ number of observations in all samples

R_i sum of ranks for the i[th] group

Notice that H test tells us whether the samples are drawn from same population (when accepting H_0) or those sample are drawn from different populations.

If we reject H_0 then we must determine the true differences location, i.e. we must do a post hoc test, and one of the famous used tests is Mann Whitney test that tests the following hypothesis:

$$H_0: \theta_i = \theta_j$$
$$H_1: \theta_i \neq \theta_j$$

Using test statistic:

$$Z = \frac{U - \overline{U}}{std_U}$$

Where:

$$\overline{U} = \frac{n_i n_j}{2}$$

$$std_U = \sqrt{\frac{n_i n_j (n_i + n_j + 1)}{12}}$$

$$U = \min\left(n_i n_j + \frac{n_i(n_j + 1)}{2} - R_i, n_i n_j + \frac{n_j(n_j + 1)}{2} - R_j\right)$$

4. Single Valued Neutrosophic Kruskal Wallis and Mann Whitney Tests

Suppose that we have c random samples as follows:

Table 1. Neutrosophic Observations.

Sample 1	Sample 2	…	Sample c
S_{11}	S_{21}		S_{c1}
S_{12}	S_{22}	⋱	S_{c2}
⋮	⋮		⋮

S_{1n_1}	S_{2n_2}	S_{cn_c}

Where $S_{11}, S_{12}, \dots, S_{cn_c}$ are SVNNs, e.g., judgments, sentiments, point of views, considerations, … etc. and we would like to check whether these judgments are consistent. Kruskal Wallis test can answer our question but the problem that arises is how to calculate the ranks of these judges since it is base on neutrosophic numbers. We will present the following algorithm to solve this problem:

1. Merge all the observation from different samples and deal with it as one sample.
2. Calculate score, accuracy and certainty of each observation.
3. Compare and rank these observations based on its score, accuracy and certainty.
4. Give the ranked observations ranks from 1 to N and if we have two equal observation the we average its ranks.
5. Compute Kruskal Wallis test statistic using the formula:
$$H_N = \frac{12}{N(N+1)} \sum_{i=1}^{C} \frac{(R_i^2)_N}{n_i} - 3(N+1)$$
where $(R_i^2)_N$ is sum of ith sample neutrosophic rank, hence H_N is neutrosophic test statistic.
6. Compare the test statistic with $\chi^2_{1-\alpha}(c-1)$ critical values, if $H_N < \chi^2_{1-\alpha}(c-1)$ then samples are drawn from same population, i.e., judgments are consistent and here test is done. elsewhere judgments are inconsistent and we must go to step 7.
7. Compute Mann Whitney test statistic pairwise based on ranked data using steps 1-4 using the formula:
$$Z_N = \frac{U_N - \overline{U}_N}{std_{U_N}}$$
Where:
$$\overline{U_N} = \frac{n_i n_j}{2}$$
$$std_{U_N} = \sqrt{\frac{n_i n_j (n_i + n_j + 1)}{12}}$$
$$U_N = \min\left(n_i n_j + \frac{n_i(n_j+1)}{2} - (R_i)_N, n_i n_j + \frac{n_j(n_j+1)}{2} - (R_j)_N\right)$$
8. if $|Z_N| < Z_{1-\frac{\alpha}{2}}$ then two compared samples are drawn from same population and otherwise samples are drawn from different populations.

Example 4.1

We would like to compare judgments of 3 independent doctors on infecting with COVID-19 for 10 sick people, each doctor is confident T% and unsure I% and may be giving wrong judgment F%.

Table 2. Neutrosophic judgments of infecting with COVID-19.

A			B			C		
T	I	F	T	I	F	T	I	F
0.207	0.922	0.550	0.905	0.808	0.657	0.949	0.034	0.000
0.879	0.968	0.419	0.555	0.238	0.571	0.057	0.842	0.398
0.200	0.825	0.208	0.726	0.552	0.689	0.845	0.042	0.662
0.824	0.378	0.011	0.230	0.046	0.825	0.858	0.622	0.833
0.859	0.988	0.654	0.779	0.470	0.897	0.853	0.055	0.383
0.874	0.347	0.499	0.599	0.293	0.607	0.416	0.092	0.972
0.842	0.772	0.402	0.007	0.013	0.371	0.407	0.330	0.140
0.855	0.999	0.378	0.688	0.027	0.571	0.978	0.257	0.495
0.368	0.458	0.078	0.940	0.628	0.441	0.048	0.109	0.983

0.698	0.220	0.712	0.614	0.003	0.628	0.110	0.509	0.063

First, we calculate score, accuracy and certainty of the previous data as follows:

Table 3. Score, accuracy and certainty of judgments.

S(A)	S(B)	S(C)	A(A)	A(B)	A(C)	C(A)	C(B)	C(C)
0.245	0.480	0.972	-0.343	0.248	0.949	0.207	0.905	0.949
0.497	0.582	0.272	0.460	-0.016	-0.341	0.879	0.555	0.057
0.389	0.495	0.714	-0.008	0.037	0.183	0.200	0.726	0.845
0.812	0.453	0.468	0.813	-0.595	0.025	0.824	0.230	0.858
0.406	0.471	0.805	0.205	-0.118	0.470	0.859	0.779	0.853
0.676	0.566	0.451	0.375	-0.008	-0.556	0.874	0.599	0.416
0.556	0.541	0.646	0.440	-0.364	0.267	0.842	0.007	0.407
0.493	0.697	0.742	0.477	0.117	0.483	0.855	0.688	0.978
0.611	0.624	0.319	0.290	0.499	-0.935	0.368	0.940	0.048
0.589	0.661	0.513	-0.014	-0.014	0.047	0.698	0.614	0.110

Then we rank our neutrosophic numbers based on its score, accuracy and certainty as follows:

Table 4. Ranks of judgments.

Doctor	Score	Accuracy	Certainty	Rank
A	0.139	-0.598	0.074	1
A	0.33	-0.148	0.754	5
A	0.383	-0.535	0.31	9
A	0.426	0.003	0.638	14
A	0.44	-0.047	0.803	15
A	0.507	0.379	0.733	21
A	0.56	0.06	0.746	23
A	0.568	-0.115	0.723	24
A	0.665	0.206	0.442	28
A	0.822	0.584	0.642	30
B	0.206	-0.449	0.023	3
B	0.288	-0.434	0.541	4
B	0.352	-0.085	0.569	6
B	0.37	0.03	0.906	8
B	0.385	-0.658	0.23	11
B	0.406	0.194	0.342	12
B	0.424	-0.382	0.545	13
B	0.559	-0.2	0.614	22
B	0.594	-0.058	0.343	25
B	0.624	0.125	0.826	26
C	0.181	-0.921	0.022	2
C	0.362	-0.353	0.231	7
C	0.383	-0.037	0.472	10
C	0.468	0.363	0.446	16
C	0.474	-0.217	0.393	17

C	0.489	-0.158	0.737	18
C	0.499	-0.238	0.755	19
C	0.504	0.017	0.854	20
C	0.66	0.107	0.653	27
C	0.705	0.399	0.709	29

Now we rearrange samples and calculate sum of each sample neutrosophic ranks and we get:

$$(R_A)_N = 170, (R_B)_N = 130, (R_C)_N = 165$$

And test statistic is:

$$H_N = \frac{12}{30(30+1)}\left(\frac{170^2 + 130^2 + 165^2}{10}\right) - 3(30+1) = 1.2258$$

Comparing with critical value say at 0.05 significance level we find that $H_N = 1.2258 < \chi^2(2) = 5.9915$ so we accept the null hypothesis and we say that all judgments are consistent.

Example 4.2

3 samples of students were drawn to test whether there is a significant difference between nervous before exam where 3 sets of students were following three strategies of learning, data is shown in Table 5:

Table 5. Nervous Before Exam.

A			B			C		
T	I	F	T	I	F	T	I	F
0.399	0.056	0.457	0.127	0.4545	0.3855	0.152	0.622	0.292
0.4155	0.0705	0.373	0.0025	0.0735	0.083	0.498	0.143	0.748
0.037	0.5	0.206	0.0095	0.171	0.4055	0.357	0.831	0.625
0.4635	0.137	0.3055	0.442	0.2785	0.4225	0.464	0.761	0.551
0.0755	0.029	0.171	0.003	0.4755	0.3055			
0.3335	0.2995	0.207	0.0615	0.072	0.184			

First, we calculate score, accuracy and certainty of the previous data as follows:

Table 6. Score, accuracy and certainty of nervous.

S(A)	S(B)	S(C)	A(A)	A(B)	A(C)	C(A)	C(B)	C(C)
0.629	0.429	0.413	-0.058	-0.259	-0.140	0.399	0.127	0.152
0.657	0.615	0.536	0.043	-0.081	-0.250	0.416	0.003	0.498
0.444	0.478	0.300	-0.169	-0.396	-0.268	0.037	0.010	0.357
0.674	0.580	0.384	0.158	0.020	-0.087	0.464	0.442	0.464
0.625	0.407		-0.096	-0.303		0.076	0.003	
0.609	0.602		0.127	-0.123		0.334	0.062	

Then we rank our neutrosophic numbers based on its score, accuracy and certainty as follows:

Table 7. Ranks of nervous.

Learning Strategy	Score	Accuracy	Certainty	Rank
A	0.674	0.158	0.4635	16
A	0.657	0.0425	0.4155	15
A	0.629	-0.058	0.399	14
A	0.625	-0.0955	0.0755	13
A	0.609	0.1265	0.3335	11

A	0.444	-0.169	0.037	6
B	0.615	-0.0805	0.0025	12
B	0.602	-0.1225	0.0615	10
B	0.580	0.0195	0.442	9
B	0.478	-0.396	0.0095	7
B	0.429	-0.2585	0.127	5
B	0.407	-0.3025	0.003	3
C	0.536	-0.25	0.498	8
C	0.413	-0.14	0.152	4
C	0.384	-0.087	0.464	2
C	0.300	-0.268	0.357	1

Now we rearrange samples and calculate sum of each sample neutrosophic ranks and we get:

$$(R_A)_N = 75, (R_B)_N = 46, (R_C)_N = 15$$

And test statistic is:

$$H_N = \frac{12}{16(16+1)}\left(\frac{75^2}{6} + \frac{46^2}{6} + \frac{15^2}{4}\right) - 3(16+1) = 8.4007$$

Comparing with critical value, say at 0.05 significance level, we find that $H_N = 8.4007 > \chi^2(2) = 5.9915$ so we reject the null hypothesis and we say that level of nervous are not equal, so we must perform Neutrosophic Mann Whitney Test and we have three cases:

Case 1 between A, B:

$$U_N = \min\left(n_1 n_2 + \frac{n_1(n_1+1)}{2} - (R_A)_N, n_1 n_2 + \frac{n_2(n_2+1)}{2} - (R_B)_N\right) = \min(5,31) = 5$$

$$\overline{U_N} = \frac{n_1 n_2}{2} = 18$$

$$std_{U_N} = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}} = 6.244998$$

$$Z_N = \frac{U_N - \overline{U}_N}{std_{U_N}} = -2.08167$$

So $|Z_N| > Z_{0.975} = 1.96$ and hence we reject the null hypothesis and take alternative hypothesis and methods A, B making different nervous level, since $\overline{R_A} = \frac{75}{6} = 12.5 > \overline{R_B} = 7.667$ then nervous level of group A is higher than nervous level of group B.

Case 2 between B, C:

Following same steps, we see that $|Z_N| = |-1.7056|$ <1.96 so there is no difference in nervous level between group B and C.

Case 3 between A, C:

Following same steps, we see that $|Z_N| = |-2.3452|$ >1.96 so there is a significant difference in nervous level between group A and C and nervous level of group A is higher than nervous level of group C because $\overline{R_A} = \frac{75}{6} = 12.5 > \overline{R_C} = \frac{15}{4} = 3.75$.

5. Conclusions

In this paper we have solved the problem of making statistical tests on single valued neutrosophic number-based problems which wasn't solved before. An algorithm to perform Kruskal-Wallis test and Mann Whitney test when dealing with SVNNs is presented and numerical examples were solved successfully in two fields of real-life problems, medical field and educational field. In future we are looking forward to extend other statistical tests which are important in decision making problems.

Funding: "This research received no external funding"

Acknowledgments: Authors are very grateful to the editor in chief and reviewers for their comments and suggestions, which is helpful in improving the paper.

Conflicts of Interest: "The Authors declare no conflict of interest."

References

[1] L. Zadeh, "Fuzzy Sets," *Information and Control,* vol. 8, no. 3, pp. 338-353, 1965.

[2] K. Atanassov, "Intuitionistic Fuzzy Sets," *Fuzzy Sets and Systems,* vol. 20, no. 1, pp. 87-96, 1986.

[3] F. Smarandache, Symbolic Neutrosophic Theory, Belgium: EuropaNova, 2015.

[4] H. Hashim, L. Abdullah, A. Al-Quran and A. Awang, "Entropy Measures for Interval Neutrosophic Vague Sets and Their Application in Decision Making," *Neutrosophic Sets and Systems,* vol. 45, pp. 74-95, 2021.

[5] A. A. Abd El-Khalek, A. T. Khalil, M. A. Abo El-Soud and I. Yasser, "A Robust Machine Learning Algorithm for Cosmic Galaxy Images Classification Using Neutrosophic Score Features," *Neutrosophic Sets and Systems,* vol. 42, pp. 79-101, 2021.

[6] A. AL-Nafee, S. Broumi and F. Smarandache, "Neutrosophic Soft Bitopological Spaces," *International Journal of Neutrosophic Science,* vol. 14, no. 1, pp. 47-56, 2021.

[7] M. Abobala, "Neutrosophic Real Inner Product Spaces," *Neutrosophic Sets and Systems,* vol. 43, pp. 225-246, 2021.

[8] M. Abobala and A. Hatip, "An Algebraic Approach to Neutrosophic Euclidean Geometry," *Neutrosophic Sets and Systems,* vol. 43, pp. 114-123, 2021.

[9] M. Abobala and M. Ibrahim, "An Introduction to Refined Neutrosophic Number Theory," *Neutrosophic Sets and Systems,* vol. 45, pp. 40-53, 2021.

[10] M. Abobala and M. Ibrahim, "An Introduction to Refined Neutrosophic Number Theory," *Neutrosophic Sets and Systems,* vol. 45, pp. 40-53, 2021.

[11] K. Alhasan and F. Smarandache, "Neutrosophic Weibull distribution and Neutrosophic Family Weibull Distribution," *Neutrosophic Sets and Systems,* vol. 28, pp. 191-199, 2019.

[12] R. Ali, "A Short Note On The Solution Of n-Refined Neutrosophic Linear Diophantine Equations," *International Journal of Neutrosophic Science,* vol. 15, no. 1, pp. 43-51, 2021.

[13] I. Shahzadi, M. Aslam and H. Aslam, "Neutrosophic Statistical Analysis of Income of YouTube Channels," *Neutrosophic Sets and Systems,* vol. 39, pp. 101-106, 2021.

[14] M. B. Zeina and A. Hatip, "Neutrosophic Random Variables," *Neutrosophic Sets and Systems,* vol. 39, pp. 44-52, 2021.

[15] M. B. Zeina, "Neutrosophic Event-Based Queueing Model," *International Journal of Neutrosophic Science,* vol. 6, no. 1, pp. 48-55, 2020.

[16] M. B. Zeina, "Erlang Service Queueing Model with Neutrosophic Parameters," *International Journal of Neutrosophic Science,* vol. 6, no. 2, pp. 106-112, 2020.

[17] F. Smarandache, "Indeterminacy in Neutrosophic Theories and their Applications," *International Journal of Neutrosophic Science,* vol. 15, no. 2, pp. 89-97, 2021.

[18] A. Mohammad, "On the Characterization of Maximal and Minimal Odeals in Several Neutrosophic Rings," *Neutrosophic Sets and Systems,* vol. 45, pp. 62-73, 2021.

[19] M. Aslam, "Neutrosophic analysis of variance: application to university students," *Complex Intell. Syst,* vol. 5, pp. 403-407, 2019.

[20] M. Aslam and M. Albassam, "Presenting post hoc multiple comparison tests under neutrosophic statistics," *Journal of King Saud University,* vol. 32, no. 6, pp. 2728-2732, 2020.

[21] R. Sherwani, H. Shakeel, W. Awan, M. Faheem and M. Aslam , "Analysis of COVID-19 data using neutrosophic Kruskal Wallis H test," *BMC Medical Research Methodology,* vol. 21, 2021.

[22] M. Albassam, N. Khan and M. Aslam, "The W/S Test for Data Having Neutrosophic Numbers: An Application to USA Village Population," *Complexity,* p. 8, 2020.

[23] P. K. Singh, "Three-Way n-Valued Neutrosophic Concept Lattice at Different," *International Journal of Machine Learning and Cybernetics,* vol. 9, no. 11, pp. 1839-1855, 2018.

[24] P. K. Singh, "Three-Way Fuzzy Concept Lattice Representation using Neutrosophic Set," *International Journal of Machine Learning and Cybernetics,* vol. 8, no. 1, pp. 69-79, 2017.

[25] P. K. Singh, "Turiyam Set a Fourth Dimension Data Representation," *Journal of Applied Mathematics and Physics,* vol. 9, no. 7, pp. 1821-1828, 2021.

[26] F. Smarandache, Neutrosophic Statistics is an extension of Interval Statistics, while Plithogenic Statistics is the most general form of statistics (second version), International Journal of Neutrosophic Science, United States, Volume 19 , Issue 1, PP: 148-165 , 2022, https://www.americaspg.com/articleinfo/21/show/1263

[26] P. K. Singh, "Data with Turiyam Set for Fourth Dimension Quantum Information Processing," *Journal of Neutrosophic and Fuzzy Systems*, vol. 1, no. 1, pp. 9-23, 2021.

[27] F. Smarandache, Neutrosophic Statistics is an extension of Interval Statistics, while Plithogenic Statistics is the most general form of statistics (second version), *International Journal of Neutrosophic Science*, United States, Volume 19 , Issue 1, PP: 148-165 , 2022, https://www.americaspg.com/articleinfo/21/show/1263

Received: June 6, 2022. Accepted: Sep 25, 2022