SMARANDACHE "CHOPPED" N^N AND $N + 1^{N-1}$

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Florentin Smarandache has posed many problems that deal with perfect powers. See [1] for example. Perfect powers of the form N^N are aesthetically pleasing because of their symmetry. But in my opinion they would be more agreeable if their number of decimal digits (their "length" in base-10 representation) were equal to N. In this note we will considere numbers of the form N^N and $N - 1^{N+1}$ that have been "chopped off" to have N decimal digits. We will refer to these numbers as Smarandache Chopped N^N numbers, and Smarandache Chopped $N - 1^{N+1}$ numbers; and we will investigate them to see if 1) they are prime, 2) they are automorphic.

§1 Smarandache Chopped N^N Numbers

There are only three numbers of the form N^N that do not need to be chopped. That is, their decimal length is already equal to N: 11 = 1, 88 = 16777216, and 99 = 387420489. It is easy to see that there will be no more naturally equal to N. For example, 613613 has 1709 digits, 12341234 has 3815 digits; as we progress the decimal lengths continue to increase.

Definition: Smarandache Chopped N^N numbers are numbers formed from the first N digits of N^N. We will call this sequence $SC(n)$:

$$
\begin{aligned}
n &= 1, \quad 2, \quad 3, \quad 4, \quad 5, \quad 6, \quad 7, \quad 8, \quad 9, \\
SC(n) &= 1, \quad x, \quad x, \quad x, \quad x, \quad x, \quad x, \quad 1677216, \quad 387420489, \\
n &= \quad 10, \quad 11, \quad 12, \\
SC(n) &= \quad 1000000000, \quad 28531167061, \quad 891610044825, \\
n &= \quad 13, \cdots \\
SC(n) &= 3028751065922, \cdots
\end{aligned}
$$

For $n = 2$ through 7, $SC(n)$ is not defined, since those values lack one digit of being the proper length. Now we shall consider whether any terms of the

$SC(n)$ sequence are prime, and automorphic. A prime number surely requires no definition here, but perhaps an automorphic number[2] does. The term automorphic is usually applied to squares, but here we broaden the definition a bit. An automorphic number is a positive integer defined by some function, f, whose functional value terminates with the digits of n. For example, if $f(n) = n^2$, then 76 is automorphic because $76^2 = 5776$ ends with 76.

Concerning the question of which Smarandache Chopped N^N numbers are prime, a computer program was written, and $SC(65)$ and $SC(603)$ were discovered and proved to be prime. No more were found up to $n = 3000$. Question: Are there infinitely many SC primes?

Concerning the question of which Smarandache Chopped N^N numbers are automorphic, a computer program was written, and when $n = 1, 9, 66$, and 6051, $SC(n)$ is automorphic. No more were found up to $n = 20000$. Question: Are there infinitely many SC automorphic numbers?

Here is $SC(66)$ to demonstrate that it is automorphic:

$$\begin{aligned} SC(66) &= 1229984803535237425357460579824952453848609953896 8\\ &\quad 2130228631906566 \end{aligned}$$

§2 Smarandache Chopped $N - 1^{N+1}$ Numbers

Numbers formed from the first N digits of $N - 1^{N+1}$ also have an intriguing symmetry. There are only three numbers of the form $N - 1^{N+1}$ that do not need to be chopped: $0^2 = 0$, $6^8 = 1679616$, and $7^9 = 40353607$. It is easy to see that there will be no more that are naturally equal to N. We will call this sequence $SC2(n)$.

$$\begin{aligned} n &= 1,\quad 2,\quad 3,\quad 4,\quad 5,\quad 6,\quad 7,\quad 8,\\ SC2(n) &= 0,\quad x,\quad x,\quad x,\quad x,\quad x,\quad 1679616,\quad 40353607,\\ n &= \quad 9,\quad 10,\quad 11,\cdots\\ SC2(n) &= \quad 107374182,\quad 3138105960,\quad 10000000000,\cdots \end{aligned}$$

Primes: A program was written, and $SC2(44)$, $SC2(64)$, and $SC2(1453)$ were discovered and proved to be prime. No more were found up to $n = 3000$. Question: Are there infinitely many $SC2$ primes?

Automorphics: A program was written, and $SC2(9416)$ was the only term discovered to be automorphic. No more were found up to $n = 20000$. Question: Are there infinitely many $SC2$ automorphic numbers?

§3 Additional Questions

1. Do these sequences, $SC(n)$ and $SC2(n)$, defy basic analysis because of their "chopped" property?
2. What other properties do the $SC(n)$ and $SC2(n)$ sequences have?

References

[1] F. Smarandache, Only Problems, Not Solutions, Xiquan Publishing House, Chicago, 1993.

[2] Eric W. Weinstein, Automorphic Number, From MathWorld - A Wolfram Web Resource. http://mathworld.wolfram.com/AutomorphicNumber.html