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Smarandache hyper BCC-algebra

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ABSTRACT

In this paper, we define the Smarandache hyper BCC-algebra, and Smarandache hyper BCC-ideals of type 1, 2, 3 and 4. We state and prove some theorems in Smarandache hyper BCC-algebras, and then we determine the relationships between these hyper ideals.

1. Introduction

A Smarandache structure on a set A means a weak structure W on A such that there exists a proper subset B of A which is embedded with a strong structure S. In [1], Vasantha Kandasamy studied the concept of Smarandache groupoids, subgroupoids, ideal of groupoids and strong Bol groupoids and obtained many interesting results about them. Smarandache semigroups are very important for the study of congruences, and it was studied by Padilla [2]. It will be very interesting to study the Smarandache structure in this algebraic structures. Borumand Saeid et al. defined the Smarandache structure in BL-algebras [3].

It is clear that any hyper BCK-algebra is a hyper BCC-algebra. A hyper BCC-algebra is a weaker structure than hyper BCK-algebra, and then we can consider in any hyper BCC-algebra a stronger structure as hyper BCK-algebra.

In this paper, we introduce the notion of Smarandache hyper BCC-algebra and we deal with Smarandache hyper BCC-ideal structures in Smarandache BCC-algebra, and then we obtain some related results which have been mentioned in the abstract.

2. Preliminaries

Definition 2.1 (*[4–6]*). A BCC-algebra is defined as a nonempty set X endowed with a binary operation "$*$" and a constant "0" satisfying the following axioms:

(a_1) $((x * y) * (z * y)) * (x * z) = 0$,
(a_2) $0 * x = 0$,
(a_3) $x * 0 = x$,
(a_4) $x * y = 0$ and $y * x = 0$ imply $x = y$,

for all $x, y, z \in X$.

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A *BCC*-algebra with the condition

$(a_5)(x * (x * y)) * y = 0$

is called a *BCK*-algebra [7,8]. Note that every *BCK*-algebra is a *BCC*-algebra, but the converse is not true. A *BCC*-algebra which is not a *BCK*-algebra is called a proper *BCC*-algebra. The smallest proper *BCC*-algebra has four elements, and for every $n \geq 4$, there exists at least one proper *BCC*-algebra [9].

Definition 2.2 (*[9]*)**.** A Smarandache *BCC*-algebra (briefly, *S*-*BCC*-algebra) is defined to be a *BCC*-algebra X in which there exists a proper subset Q of X such that

(i) $0 \in Q$ and $|Q| \geq 4$,
(ii) Q is a *BCK*-algebra with respect to the same operation on X.

Note that any proper *BCC*-algebra X with four elements cannot be a *S*-*BCC*-algebra. Hence, if X is a *S*-*BCC*-algebra, then $|X| \geq 5$ [9].

Definition 2.3 (*[10]*)**.** A hyper *BCC*-algebra is defined as a nonempty set H endowed with hyper operation "*o*" and a constant "0" satisfying the following axioms:

(HC_1) $(x \circ z) \circ (y \circ z) \ll x \circ y$,
(HC_2) $0 \circ x = \{0\}$,
(HC_3) $x \circ 0 = \{x\}$,
(HC_4) $x \ll y$ and $y \ll x$ imply $x = y$,

for all $x, y, z \in H$, where $x \ll y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A \ll B$ is defined for all $a \in A$, there exists $b \in B$ such that $a \ll b$. In such case "$\ll$" is called the hyper order in H.

Note that if $A, B \subseteq H$, then by $A \circ B$ we mean the subset $\bigcup_{a \in A, b \in B} a \circ b$ of H.

Definition 2.4 (*[11]*)**.** A hyper *BCK*-algebra is defined as a nonempty set H endowed with hyper operation "$\circ$" and a constant "0" satisfying the following axioms:

(HK_1) $(x \circ z) \circ (y \circ z) \ll x \circ y$,
(HK_2) $(x \circ y) \circ z = (x \circ z) \circ y$,
(HK_3) $x \circ H \ll \{x\}$,
(HK_4) $x \ll y$ and $y \ll x$ imply $x = y$,

for all $x, y, z \in H$, where $x \ll y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A \ll B$ is defined by for all $a \in A$, there exists $b \in B$ such that $a \ll b$. In such case "$\ll$" is called the hyper order in H.

Proposition 2.5 (*[11]*)**.** *In any hyper BCK-algebra H, for all $x, y, z \in H$, the following holds:*

(a) $0 \circ 0 = \{0\}$,
(b) $0 \circ x = \{0\}$,
(c) $x \circ 0 = \{x\}$.

Definition 2.6 (*[11]*)**.** Let I be a nonempty subset of a hyper *BCK*-algebra H and $0 \in I$. Then I is said to be a weak hyper *BCK*-ideal of H if $x \circ y \subseteq I$ and $y \in I$ imply $x \in I$ for all $x, y \in H$, hyper *BCK*-ideal of H if $x \circ y \ll I$ and $y \in I$ imply $x \in I$ for all $x, y \in H$, strong hyper *BCK*-ideal of H if $(x \circ y) \cap I \neq \emptyset$ and $y \in I$ imply $x \in I$ for all $x, y \in H$, hyper subalgebra of H if $x \circ y \subseteq I$, for all $x, y \in H$.

Theorem 2.7 (*[10]*)**.** *Any hyper BCK-algebra is a hyper BCC-algebra.*

The converse of Theorem 2.7 is not true in general.

Example 2.8 (*[10]*)**.** Let $H = \{0, 1, 2\}$ in the following table.

$\circ$	0	1	2
0	{0}	{0}	{0}
1	{1}	{0}	{0}
2	{2}	{2}	{0, 1}

then H is a hyper *BCC*-algebra, but it is not a hyper *BCK*-algebra, because $(2 \circ 1) \circ 2 \neq (2 \circ 2) \circ 1$.

Theorem 2.9 (*[10]*)**.** *Let H be a hyper BCC-algebra. Then H is a hyper BCK-algebra if and only if*

$(x \circ y) \circ z = (x \circ z) \circ y$

for all $x, y, z \in H$.

Definition 2.10 ([10]). Hyper BCC-algebra H is called a proper hyper BCC-algebra if H is not a hyper BCK-algebra.

Corollary 2.11 ([10]). *For $n \geq 3$, there exists at least one proper hyper BCC-algebra of order n.*

Definition 2.12 ([10]). A nonempty subset I of a hyper BCC-algebra X satisfies the closed condition if $x \ll y$ and $y \in I$ imply $x \in I$.

Definition 2.13 ([10]). A nonempty subset I of X such that $0 \in I$ is called:

(i) a hyper BCC-ideal of type 1, if

$$((x \circ y) \circ z \ll I, y \in I) \Rightarrow x \circ z \subseteq I,$$

(ii) a hyper BCC-ideal of type 2, if

$$((x \circ y) \circ z \subseteq I, y \in I) \Rightarrow x \circ z \subseteq I,$$

(iii) a hyper BCC-ideal of type 3, if

$$((x \circ y) \circ z \ll I, y \in I) \Rightarrow x \circ z \ll I,$$

(iv) a hyper BCC-ideal of type 4, if

$$((x \circ y) \circ z \subseteq I, y \in I) \Rightarrow x \circ z \ll I.$$

Theorem 2.14 ([10]). *In any hyper BCC-algebra, any hyper BCC-ideal of type* (1), (2) *and* (3) *is a hyper BCC-ideal of type* (4).

3. Smarandache hyper *BCC*-algebra and Smarandache hyper *BCC*-ideals

Definition 3.1. A Smarandache hyper BCC-algebra (briefly, S-H-BCC-algebra) is defined to be a hyper BCC-algebra X in which there exists a proper subset Q of X such that

(S_1) $0 \in Q$ and $|Q| \geq 3$,
(S_2) Q is a hyper BCK-algebra with respect to the same operation on X.

Note that any proper hyper BCC-algebra X with three elements cannot be a S-H-BCC-algebra. Hence, if X is a S-H-BCC-algebra, then $|X| \geq 4$ [10].

Example 3.2. Consider $X = \{0, 1, 2, 3\}$ in the following table.

$\circ$	0	1	2	3
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0\}$	$\{0\}$	$\{0\}$
2	$\{2\}$	$\{2\}$	$\{0, 1\}$	$\{2\}$
3	$\{3\}$	$\{1, 3\}$	$\{0, 1, 3\}$	$\{0, 1, 3\}$

Then X is a hyper BCC-algebra. If we consider $Q_1 = \{0, 1, 2\}$, then we can see that Q_1 is not a hyper BCK-algebra since $(2 \circ 1) \circ 2 \neq (2 \circ 2) \circ 1$ also $Q_2 = \{0, 1, 3\}$ is not a hyper BCK-algebra since $(2 \circ 2) \circ 3 \neq (2 \circ 3) \circ 2$. Therefore X is not a S-H-BCC-algebra.

Example 3.3. (i) Let $X = \{0, 1, 2, 3\}$ in the following tables.

$\circ$	0	1	2	3
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0, 1\}$	$\{0\}$	$\{0\}$
2	$\{2\}$	$\{2\}$	$\{0\}$	$\{0\}$
3	$\{3\}$	$\{2\}$	$\{2\}$	$\{0, 1\}$

$*$	0	1	2	3
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0, 1\}$	$\{0\}$	$\{0\}$
2	$\{2\}$	$\{2\}$	$\{0, 2\}$	$\{0, 2\}$
3	$\{3\}$	$\{2\}$	$\{1, 2\}$	$\{0, 1, 2\}$

$\star$	0	1	2	3
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0, 1\}$	$\{0, 1\}$	$\{1\}$
2	$\{2\}$	$\{1, 2\}$	$\{0, 2\}$	$\{2\}$
3	$\{3\}$	$\{0\}$	$\{0\}$	$\{0, 3\}$

Note that $Q = \{0, 1, 2\}$ is a hyper BCK-algebra with each of the above operations and is properly contained in X. Then $(X, \circ, 0)$, $(X, \star, 0)$ and $(X, *, 0)$ are S-H-BCC-algebra.

(ii) Let $\{X, \circ_1, 0\}$ be a finite hyper BCK-algebra containing at least three elements and $c \notin X$, $Y = X \cup \{c\}$. Define the hyper operation "$\circ$" on H as follows:

$$x \circ y = \begin{cases} \{c\} & \text{if } x = c, y = 0, \\ \{x\} & \text{if } x \in X, y = c, \\ \{0, c\} & \text{if } x = y = c, \\ \{0\} & \text{if } x = c, y \in X - \{0\}, \\ x \circ_1 y & \text{if } x, y \in X \end{cases}$$

for all $x, y \in Y$, then $(Y, \circ, 0)$ is a hyper BCC-algebra; therefore Y is a S-H-BCC-algebra.

Theorem 3.4. *Any S-BCC-algebra is a S-H-BCC-algebra.*

Proof. Straightforward. □

The converse of Theorem 3.4 is not true in general, since any hyper BCC-algebra is not necessary a BCC-algebra.

Theorem 3.5. *Let X be a S-H-BCC-algebra and $|X| \geq 5$. Then the set*

$$S(X) = \{x \in X : x \circ x = \{0\}\}$$

is a S-BCC-algebra.

Proof. Let X be a S-H-BCC-algebra and $S(X) = \{x \in X : x \circ x = \{0\}\}$. We claim that for all $y, z \in S(X)$, $|y \circ z| = 1$. Let there exist $y, z \in S(X)$ such that $|y \circ z| \geq 1$. Hence there exist $a, b \in y \circ z$ such that $a \neq b$. By (HC_1) and hypothesis

$$a \circ b, b \circ a \in (y \circ z) \circ (y \circ z) \ll y \circ y = \{0\}.$$

Then $a \circ b \ll \{0\}$ and $b \circ a \ll \{0\}$ and so $a \ll b$ and $b \ll a$. Hence by (HC_4), $a = b$ which is a contradiction. Therefore, for all $y, z \in S(X)$, $y \circ z$ is a singleton set and so $S(X)$ is a S-BCC-algebra. □

If $S(X) = X$, then the S-H-BCC-algebra become a S-BCC-algebra, which shows that S-H-BCC-algebra is a generalization of S-BCC-algebra.

In what follows, let X and Q denote a S-H-BCC-algebra and a nontrivial hyper BCK-algebra which is properly contained in X, respectively, unless otherwise specified.

Definition 3.6. A nonempty subset I of X such that $0 \in I$ is called

(i) a Smarandache hyper BCC-ideal of type 1 of X related to Q, if

$$((\forall x, z \in Q)(x \circ y) \circ z \ll I, y \in I) \Rightarrow x \circ z \subseteq I,$$

(ii) a Smarandache hyper BCC-ideal of type 2 of X related to Q, if

$$((\forall x, z \in Q)(x \circ y) \circ z \subseteq I, y \in I) \Rightarrow x \circ z \subseteq I,$$

(iii) a Smarandache hyper BCC-ideal of type 3 of X related to Q, if

$$((\forall x, z \in Q)(x \circ y) \circ z \ll I, y \in I) \Rightarrow x \circ z \ll I,$$

(iv) a Smarandache hyper BCC-ideal of type 4 of X related to Q, if

$$((\forall x, z \in Q)(x \circ y) \circ z \subseteq I, y \in I) \Rightarrow x \circ z \ll I.$$

Example 3.7. Consider $X = \{0, 1, 2, 3\}$ in the following table.

$\circ$	0	1	2	3
0	{0}	{0}	{0}	{0}
1	{1}	{0, 1}	{0}	{0}
2	{2}	{2}	{0}	{0}
3	{3}	{2}	{2}	{0, 1}

X is a S-H-BCC-algebra where $Q = \{0, 1, 2\}$ is hyper BCK-algebra. We can see

- $I_1 = \{0\}$, $I_2 = \{0, 1\}$, $I_3 = \{0, 1, 2\}$, and $I_4 = \{0, 1, 2, 3\}$ are Smarandache hyper BCC-ideals of types (1)–(4) of X related to Q.
- $I_5 = \{0, 1, 3\}$ and $I_6 = \{0, 2, 3\}$ are not Smarandache hyper BCC-ideals of type (1) of X related to Q. (Since $(2 \circ 3) \circ 0 \ll I_5$, $3 \in I_5$, but $(2 \circ 0) = 2 \not\subseteq I_5$ and $(1 \circ 3) \circ 0 \ll I_6$, $3 \in I_6$, but $(1 \circ 0) = 1 \not\subseteq I_6$.)

- I_5 and I_6 are not Smarandache hyper BCC-ideals of type (2) of X related to Q. (Since $(2 \circ 3) \circ 0 \subseteq I_5, 3 \in I_5$, but $(2 \circ 0) = 2 \nsubseteq I_5$ and $(1 \circ 2) \circ 0 \subseteq I_6, 2 \in I_6$, but $(1 \circ 0) = 1 \nsubseteq I_6$.)
- $I_7 = \{0, 2\}$ and $I_8 = \{0, 3\}$ are not Smarandache hyper BCC-ideals of type (1) of X related to Q. (Since $(1 \circ 0) \circ 0 \ll I_7, I_8$ and $0 \in I_7, I_8$, but $(1 \circ 0) = 1 \nsubseteq I_7, I_8$.)
- I_7 and I_8 are not Smarandache hyper BCC-ideals of type (2) of X related to Q. (Since $(1 \circ 2) \circ 0 \subseteq I_7, 2 \in I_7$, but $(1 \circ 0) = 1 \nsubseteq I_7$ and $(1 \circ 3) \circ 0 \subseteq I_8, 3 \in I_8$, but $(1 \circ 0) = 1 \nsubseteq I_8$.)
- I_7 and I_8 are Smarandache hyper BCC-ideals of types (3) and (4) of X related to Q.

Theorem 3.8. *In any S-H-BCC-algebra, the following statements are valid.*

(i) *Any Smarandache hyper BCC-ideal of type* (1) *of X related to Q is a Smarandache hyper BCC-ideal of types* (2) *and* (3) *of X related to Q.*
(ii) *Any Smarandache hyper BCC-ideal of type* (2) *of X related to Q is a Smarandache hyper BCC-ideal of type* (4) *of X related to Q.*
(iii) *Any Smarandache hyper BCC-ideal of type* (3) *of X related to Q is a Smarandache hyper BCC-ideal of type* (4) *of X related to Q.*

Proof. The proof is straightforward. □

The converse of Theorem 3.8 is not true in general.

Example 3.9. Consider $X = \{0, 1, 2, 3\}$ in the following table.

$\circ$	0	1	2	3
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0, 1\}$	$\{0\}$	$\{0\}$
2	$\{2\}$	$\{2\}$	$\{0, 2\}$	$\{0, 2\}$
3	$\{3\}$	$\{2\}$	$\{1, 2\}$	$\{0, 1, 2\}$

X is a S-H-BCC-algebra where $Q = \{0, 1, 2\}$ is hyper BCK-algebra. $I = \{0, 1, 3\}$ is a Smarandache hyper BCC-ideal of type (2) of X related to Q, but it is not a Smarandache hyper BCC-ideal of type (1) of X related to Q. (Since $(2 \circ 1) \circ 1 \ll I, 1 \in I$, but $(2 \circ 1) = 2 \nsubseteq I$.)

Example 3.10. In Example 3.7, I_7 and I_8 are Smarandache hyper BCC-ideals of types (3) and (4) of X related to Q but are not Smarandache hyper BCC-ideals of types (1) and (2) of X related to Q.

Example 3.11. Consider $X = \{0, 1, 2, 3\}$ in the following table.

$\circ$	0	1	2	3
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0\}$	$\{0\}$	$\{0\}$
2	$\{2\}$	$\{1\}$	$\{0\}$	$\{0\}$
3	$\{3\}$	$\{2\}$	$\{1, 2\}$	$\{0, 1\}$

X is a S-H-BCC-algebra where $Q = \{0, 1, 2\}$ is hyper BCK-algebra. $I_1 = \{0, 1\}$ is not a Smarandache hyper BCC-ideal of type (4) of X related to Q, (since $(2 \circ 1) \circ 0 \subseteq I, 1 \in I$, but $(2 \circ 0) = 2 \not\ll I$). Therefore by Theorem 3.8, I is not Smarandache hyper BCC-ideal of types (3),(2) and (1) of X related to Q.

Theorem 3.12. *In any S-H-BCC-algebra the following statements are valid.*

(i) *I is a Smarandache hyper BCC-ideal of type* (1) *of X related to Q if and only if I is a hyper BCK-ideal of Q,*
(ii) *I is a Smarandache hyper BCC-ideal of type* (2) *of X related to Q if and only if I is a weak hyper BCK-ideal of Q.*

Proof. (i) Let I be a Smarandache hyper BCC-ideal of type (1) of X related to Q, $x \circ y \ll I$ and $y \in I$, for all $x, y \in Q$. Hence by Proposition 2.5(c), we obtain $(x \circ y) \circ 0 = (x \circ y) \ll I, y \in I$, so applying the hypothesis and Proposition 2.5(c) we get that $\{x\} = x \circ 0 \subseteq I$. This shows that I is a hyper BCK-ideal of Q.

Conversely, let I be a hyper BCK-ideal of Q, $(x \circ y) \circ z \ll I$ and $y \in I$, for all $x, y \in Q$. Since $y \in I \subseteq Q$; therefore $y \in Q$, by $(HK_2)(x \circ z) \circ y = (x \circ y) \circ z \ll I$ and so for each $a \in x \circ z, a \circ y \ll I$ since $y \in I$ and I is a hyper BCK-ideal of Q, then $a \in I$ and so $x \circ y \subseteq I$, hence I is a hyper BCC-ideal of type (1) of X related to Q.

(ii) The proof is similar to the proof of (i). □

In the following diagram we show the relationship between all types of Smarandache hyper BCC-ideals in Smarandache hyper BCC-algebras, and also the relationship with hyper BCK-ideals.

$$
\begin{array}{cccccccc}
 & & & & & (3) & & \\
 & & & \not\swarrow & \nearrow & & \searrow & \\
\text{hyper } BCK\text{-ideals} & \Longleftrightarrow & (1) & & & \not\downarrow & & (4) \\
\downarrow & & & \not\nwarrow & \searrow & & \nearrow & \not\swarrow \\
\downarrow & & & & & (2) & & \\
\downarrow & & & & & \Updownarrow & & \\
 & \longrightarrow & \longrightarrow & \longrightarrow & \text{weak hyper } BCK\text{-ideals} & & &
\end{array}
$$

(1)–(4) denote the Smarandache hyper BCC-ideal of types 1, 2, 3 and 4 of X related to Q, respectively.

Proposition 3.13. *Let I be a Smarandache hyper BCC-ideal of type 2 of X related to Q and $A \subseteq Q$. If $A \circ B \subseteq I$ and $B \subseteq I$, then $A \subseteq I$.*

Proof. For all $a \in A$, $b \in B$ we have $a \circ b \subseteq A \circ B \subseteq I$, then $a \circ b = (a \circ b) \circ 0 \subseteq I$. Since I is a Smarandache hyper BCC-ideal of type 2 of X related to Q and $b \in I$ we conclude that $a = a \circ 0 \subseteq I$, thus $A \subseteq I$. $\square$

Proposition 3.14. *Let I be a Smarandache hyper BCC-ideal of type 3 of X related to Q and $A \subseteq Q$. If $A \circ B \ll I$ and $B \subseteq I$, then $A \ll I$.*

Proof. We have $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$ and $A \circ B \ll I$. Thus there exists $t \in a \circ b$ for some $a \in A$, $b \in B$ and $s \in I$ such that $t \ll s$. Hence $a \circ b \ll I$, then $a \circ b = (a \circ b) \circ 0 \ll I$. Since I is a Smarandache hyper BCC-ideal of type 3 of X related to Q and $b \in I$ we conclude that $a = a \circ 0 \ll I$, thus $A \ll I$. $\square$

Proposition 3.15. *Let I be a Smarandache hyper BCC-ideal of type 4 of X related to Q and $A \subseteq Q$. If $A \circ B \subseteq I$ and $B \subseteq I$, then $A \ll I$.*

Proof. For all $a \in A$, $b \in B$, we have $a \circ b \subseteq A \circ B \subseteq I$, then $a \circ b = (a \circ b) \circ 0 \subseteq I$. Since I is a Smarandache hyper BCC-ideal of type 4 of X related to Q and $b \in I$ we conclude that $a = a \circ 0 \ll I$, thus $A \ll I$. $\square$

Example 3.16. If I_0 is a Smarandache hyper BCC-ideal of type i of X related to Q, for $1 \le i \le 4$ of X and $I_0 \subseteq I_1$, then I_1 is not a Smarandache hyper BCC-ideal of type i of X related to Q, for $1 \le i \le 4$ of X. In Example 3.9, $I_0 = \{0\}$ is a Smarandache hyper BCC-ideal of type i of X related to Q, for $1 \le i \le 4$ of X and consider $I_1 = \{0, 1\}$, then $I_0 \subseteq I_1$ but I_1 is not a Smarandache hyper BCC-ideal of type i of X related to Q, for $1 \le i \le 4$ of X. Thus "extension property" does not hold for Smarandache hyper BCC-ideal of type i of X related to Q, for $1 \le i \le 4$ of X.

Theorem 3.17. *A nonempty subset I of a S-H-BCC-algebra X satisfying the closed condition is a Smarandache hyper BCC-ideal of type i of X related to Q, for $1 \le i \le 4$ if and only if I is a Smarandache hyper BCC-ideal of type j of X related to Q, for $1 \le j \le 4$, $i \ne j$.*

Proof. Let I satisfy the closed condition. It is easy to prove that for any subset A of X if $A \ll I$, then $A \subseteq I$. Hence the proof is clear. $\square$

Proposition 3.18. *Every Smarandache hyper BCC-ideal of type 1 of X related to Q satisfies the following*

(i′) $(\forall x \in Q)(\forall a \in I)(x \circ a \ll I \Rightarrow x \subseteq I)$.

Proof. Taking $z = 0$ and $y = a$ in Definition 3.6(i) and using Proposition 2.5(c) induce the desired implication. $\square$

Theorem 3.19. *If I is a subset of Q and $0 \in I$ that satisfies condition (i′), then I is a Smarandache hyper BCC-ideal of type 1 of X related to Q.*

Proof. Let $x, z \in Q$ and $a \in I$ be such that $(x \circ a) \circ z \ll I$. Since $a \in I \subseteq Q$ and Q is a hyper BCK-algebra, it follows that $(x \circ z) \circ a = (x \circ a) \circ z \ll I$, from (i′) we conclude that $x \circ z \subseteq I$. Hence I is a Smarandache hyper BCC-ideal of type 1 of X related to Q. $\square$

Remark 3.20. Similarly we can prove the above theorem for the other types of Smarandache hyper BCC-ideals of X related to Q.

The following example shows that the condition (i′) is necessary in the above theorem.

Example 3.21. In Example 3.11, $I_1 = \{0, 1\}, I_1 \subseteq Q$ but is not satisfying the condition (i') (since $2 \in Q, 1 \in I$ and $2 \circ 1 = 1 \ll I$ but $\{2\} \nsubseteq I$) and I is not Smarandache hyper BCC-ideal of type 1 of X related to Q; therefore condition (i') is necessary in the above theorem.

Remark 3.22. Every hyper BCC-ideal of type i of X, for $1 \le i \le 4$, of X is a Smarandache hyper BCC-ideal of the same type of X related to Q.

The converse of Remark 3.22 is not true in general.

Example 3.23. In Example 3.7, I_3 is a Smarandache hyper BCC-ideal of type i of X related to Q, for $1 \le i \le 4$ but is not a hyper BCC-ideal of the same type of X (Since $(3 \circ 1) \circ 0 \subseteq I_3$, $1 \in I_3$, but $(3 \circ 0) = 3 \not\ll I_3$ hence is not hyper BCC-ideal of type 4 of X related to Q; therefore by Theorem 2.14 is not a hyper BCC-ideal of type i of X for $1 \le i \le 4$.)

Theorem 3.24. *Let Q_1, Q_2 be hyper BCK-algebras which are properly contained in X and $Q_1 \subset Q_2$. Then every Smarandache hyper BCC-ideal of type i, for $1 \le i \le 4$ of X related to Q_2 is Smarandache hyper BCC-ideal (the same type) of X related to Q_1.*

Proof. Straightforward. □

In the following example, we show that the converse of Theorem 3.24 is not true.

Example 3.25. Consider $X = \{0, 1, 2, 3, 4, 5\}$ in the following table.

$\circ$	0	1	2	3	4	5
0	{0}	{0}	{0}	{0}	{0}	{0}
1	{1}	{0}	{0}	{0}	{0}	{1}
2	{2}	{1}	{0}	{0}	{0}	{2}
3	{3}	{1}	{1}	{0}	{1}	{3}
4	{4}	{1}	{1}	{1}	{0}	{4}
5	{5}	{0}	{0}	{0}	{0}	{0, 5}

X is a S-H-BCC-algebra related to Q_1 and Q_2, where $Q_1 = \{0, 1, 2, 3\}$ and $Q_2 = \{0, 1, 2, 3, 4\}$ are hyper BCK-algebra. $I = \{0, 1, 2, 3\}$ is a Smarandache hyper BCC-ideal of type i, for $1 \le i \le 4$ of X related to Q_1 but is not a Smarandache hyper BCC-ideal of the same type of X related to Q_2. (Since $(4 \circ 2) \circ 0 \subseteq I, 2 \in I$, but $(4 \circ 0) = 4 \not\ll I$. Therefore by Theorem 3.8, I is not a Smarandache hyper BCC-ideal of type (3), (2) and (1) of X related to Q_2.)

Proposition 3.26. *Let X be a S-H-BCC-algebra. Then $\{0\}$ is a Smarandache hyper BCC-ideal of type i, for $1 \le i \le 4$ of X related to Q.*

Proof. For all $x, z \in Q$, let $(x \circ y) \circ z \ll \{0\}$, $y \in \{0\}$ hence $x \circ z = (x \circ 0) \circ z \ll \{0\}$ and we have $\{0\} \ll x \circ z$; therefore by HC_4, $x \circ z = \{0\}$, hence $\{0\}$ is a Smarandache hyper BCC-ideal of type 1, then by Theorem 3.8, $\{0\}$ is a Smarandache hyper BCC-ideal of type i, for $1 \le i \le 4$ of X related to Q. □

Proposition 3.27. *Let X be a S-H-BCC-algebra. Q and X are Smarandache hyper BCC-ideals of type i of X related to Q, for $1 \le i \le 4$.*

Proof. Straightforward. □

4. Conclusion

A Smarandache structure is a structure S which has a proper subset with a stronger structure, or a proper subset with a weaker structure, or both (two proper subsets, one with a stronger structure, and another with a weaker structure). In the present paper, by using this notion we have introduced the concept of Smarandache hyper BCC-algebras and investigated some of their useful properties. In our opinion, these definitions and main results can be similarly extended to some other algebraic systems such as lattices and Lie algebras. It is our hope that this work will laid other foundations for further study of the theory of hyper BCC-algebra and hyper BCK-algebra.

In our future study of Smarandache structure of hyper BCC-algebras, the following topics may be considered.

(1) To get more results in Smarandache hyper BCC-algebras and application.
(2) To get more connection between hyper BCC-algebra and Smarandache hyper BCC-algebra.
(3) To define another Smarandache structure.
(4) To define fuzzy structure of Smarandache hyper BCC-algebras.

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