

Smarandache \mathcal{N} -subalgebras (resp. filters) of CI -algebras

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Abstract. In this paper, we introduce the notions of \mathcal{N} -subalgebras and \mathcal{N} -filters based on Smarandache CI -algebra and give a number of their properties. The relationship between $\mathcal{N}(Q, f)$ -subalgebras(filters) and \mathcal{N} -subalgebras(filters) are also investigated.

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1 Introduction

Some recent researchers led to generalizations of the notion of fuzzy set that introduced by Zadeh in 1965 [15]. The generalization of the crisp set to fuzzy sets relied on spreading positive information that fit the point $\{1\}$ into the interval $[0, 1]$. In order to provide a mathematical tool to deal with negative information, Jun et. al. introduced \mathcal{N} -structures, based on negative-valued functions [6]. In 1966, Y. Imai and K. Iseki [3] introduced two classes of abstract algebras: BCK -algebras and BCI -algebras. It is known that the class of BCK -algebras is a proper subclass of the class of BCI -algebras. H. S. Kim and Y. H. Kim defined a BE -algebra [5]. Biao Long Meng, defined notion of CI -algebra as a generalization of a BE -algebra [9]. It is known that any BE -algebra is a CI -algebra. Hence, every BE -algebra is

a weaker structure than CI -algebra, thus we can consider in any CI -algebra a weaker structure as BE -algebra. Jun et. al. discussed the notion of \mathcal{N} -structures in $BCH/BCK/BCI$ -algebras and investigated their properties in [6, 7]. They introduced the notions of \mathcal{N} -ideals of subtraction algebras and \mathcal{N} -closed ideals in BCK/BCI -algebras. We introduce the notions of \mathcal{N} -subalgebras and \mathcal{N} -filters in CI -algebras and give a number of their properties and The relationship between \mathcal{N} -subalgebras and \mathcal{N} -filters was discussed in [14]. Also, we discuss on Smarandache CI -algebra and investigated some of their useful properties in [2]. Beside, we introduced the notion of anti fuzzy set and stated the relationship with the \mathcal{N} -function of CI -algebra X . We showed that every anti fuzzy filter is an anti fuzzy subalgebra in [1]. K. J. Lee and Y. B. Jun introduced the notion of \mathcal{N} -subalgebras and \mathcal{N} -ideals based on a sub- BCK -algebra of a BCI -algebras and their relations/properties are investigated in [8].

In the present paper, we continue study of CI -algebras and apply the \mathcal{N} -structures to the filter theory in CI -algebras and Smarandache CI -algebras, also investigate the relationship between \mathcal{N} -subalgebra and \mathcal{N} -filters based on Smarandache CI -algebras. We show that any $\mathcal{N}(Q, f)$ -closed filter is an $\mathcal{N}(Q, f)$ -subalgebra. We give some conditions for \mathcal{N} -subalgebras(filters) to be $\mathcal{N}(Q, \varrho)$ -subalgebras(resp. filters).

2 Preliminaries

In this section we review the basic definitions and some elementary aspects that are necessary for this paper.

Definition 2.1. [9] *An algebra $(X; *, 1)$ of type $(2, 0)$ is called a CI -algebra if it satisfying the following axioms:*

$$(CI1) \quad x * x = 1,$$

$$(CI2) \quad 1 * x = x,$$

$$(CI3) \quad x * (y * z) = y * (x * z), \text{ for all } x, y, z \in X.$$

A CI -algebra X satisfying the condition $x * 1 = 1$ is called a BE -algebra. In any CI -algebra X one can define a binary relation “ \leq ” by $x \leq y$ if and only if $x * y = 1$.

A CI -algebra X has the following properties:

$$(i) \quad y * ((y * x) * x) = 1,$$

$$(ii) \quad (x * 1) * (y * 1) = (x * y) * 1,$$

(iii) if $1 \leq x$, then $x = 1$, for all $x, y \in X$.

A non-empty subset S of a CI -algebra X is called a subalgebra of X if $x * y \in S$ whenever $x, y \in S$. A mapping $f : X \rightarrow Y$ of CI -algebra is called a homomorphism if $f(x * y) = f(x) * f(y)$, for all $x, y \in X$. A non-empty subset F of CI -algebra X is called a filter of X if (1) $1 \in F$, (2) $x \in F$ and $x * y \in F$ implies $y \in F$. A filter F of CI -algebra X is said to be closed if $x \in F$ implies $x * 1 \in F$. A nonempty subset S of a CI -algebra X is called a subalgebra of X if $x * y \in S$, for all $x, y \in S$. For our convenience, the empty set \emptyset is regarded as a subalgebra of X . Denote by $Q(X, [-1, 0])$ the collection of functions from a set X to $[-1, 0]$. We say that an element of $Q(X, [-1, 0])$ is a negative-valued function from X to $[-1, 0]$ (briefly, \mathcal{N} -function on X). By an \mathcal{N} -structure we mean an ordered pair (X, f) of X and an \mathcal{N} -function f on X .

In what follows, let X denote a CI -algebra and f an \mathcal{N} -function on X unless otherwise specified.

Definition 2.2. [14] *By a subalgebra of X based on \mathcal{N} -function f (briefly, \mathcal{N} -subalgebra of X), we mean an \mathcal{N} -structure (X, f) in which f satisfies the following assertion:*

$$f(x * y) \leq \max\{f(x), f(y)\}, \text{ for all } x, y \in X.$$

Definition 2.3. [14] *By a filter of X based on \mathcal{N} -function f (briefly, \mathcal{N} -filter of X), we mean an \mathcal{N} -structure (X, f) in which f satisfies the following conditions:*

$$(i) \quad f(1) \leq f(y),$$

$$(ii) \quad f(y) \leq \max\{f(x * y), f(x)\}, \text{ for all } x, y \in X.$$

Definition 2.4. [2] *A Smarandache CI -algebra X is defined to be a CI -algebra X in which there exists a proper subset Q of X such that satisfies the following conditions:*

$$(S1) \quad 1 \in Q \text{ and } |Q| \geq 2,$$

$$(S2) \quad Q \text{ is a BE-algebra under the operation of } X.$$

Example 2.1. [2] Let $X := \{1, a, b, c, d\}$ be a set with the following table.

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	a	a	d
b	1	1	1	a	d
c	1	1	1	1	d
d	d	d	d	d	1

Then X is a CI -algebra and $Q = \{1, a, b, c\}$ is a BE -algebra.

Definition 2.5. [2] A nonempty subset F of CI -algebra X is called a Smarandache filter of X related to Q (or briefly, Q -Smarandache filter of X) if it satisfies:

- (SF1) $1 \in F$,
- (SF2) $(\forall y \in Q)(\forall x \in F)(x * y \in F \Rightarrow y \in F)$.

Definition 2.6. [11] A fuzzy set $\mu : X \rightarrow [0, 1]$ is called an anti fuzzy subalgebra of X if it satisfy:

$$\mu(x * y) \leq \max\{\mu(x), \mu(y)\}, \text{ for all } x, y \in X.$$

Definition 2.7. [1] A fuzzy set $\mu : X \rightarrow [0, 1]$ is called an anti fuzzy filter of X if it satisfies:

- (AFF1) $\mu(1) \leq \mu(x)$,
- (AFF2) $\mu(y) \leq \max\{\mu(x * y), \mu(x)\}$, for all $x, y \in X$.

3 Smarandache \mathcal{N} -subalgebras

Definition 3.1. Let X be a Q -Smarandache CI -algebra and $\varrho \in [-1, 0]$. An \mathcal{N} -structure (X, f) is called an \mathcal{N} -subalgebra of X based on Q and ϱ (briefly, $\mathcal{N}(Q, \varrho)$ -subalgebra of X) if it is an \mathcal{N} -subalgebra of X such that satisfies the following condition:

- (type 1) $(\forall x \in Q) (\forall y \in X \setminus Q) (f(x) \leq \varrho \leq f(y))$,
- (type 2) $(\forall x \in Q) (\exists y \in X \setminus Q) (f(x) \leq \varrho \leq f(y))$,
- (type 3) $(\exists x \in Q) (\forall y \in X \setminus Q) (f(x) \leq \varrho \leq f(y))$,

- (type 4) $(\exists x \in Q) (\exists y \in X \setminus Q) (f(x) \leq \varrho \leq f(y))$.

Note. If $\varrho := 0$, then $f(y) = 0$, for all $y \in X \setminus Q$. So, (Q, f) is an \mathcal{N} -subalgebra. If $\varrho := -1$, then $f(x) = -1$, for all $x \in Q$. And so $(X, f) = \mathcal{N}(Q, \varrho)$.

Example 3.1. a) In Example 2.1, an \mathcal{N} -structure (X, f) in which f is defined by $f(1) = f(a) = -0.7, f(b) = -0.4, f(c) = -0.6$ and $f(d) = -0.3$ is an $\mathcal{N}(Q, \varrho)$ -subalgebra of all types on X , for $\varrho \in [-0.4, -0.3]$ and $Q = \{1, a, b, c\}$.

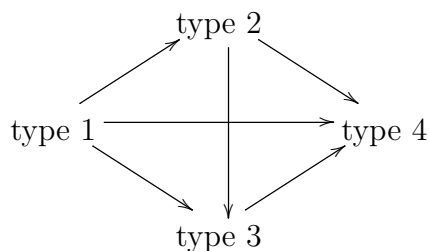
b) In Example 2.1, an \mathcal{N} -structure (X, g) in which g is defined by $g(1) = g(a) = -0.7, g(b) = -0.2, g(c) = -0.6$ and $g(d) = -0.3$ is not an $\mathcal{N}(Q, \varrho)$ -subalgebra of X because $g(d) = -0.3 \not\leq g(b) = -0.2$.

c) In Example 2.1, an \mathcal{N} -structure (X, f) in which f is defined by $f(1) = f(a) = -0.7, f(b) = -0.4, f(c) = -0.5$ and $f(d) = -0.3$ is an $\mathcal{N}(Q, \varrho)$ -subalgebra of type 2, type 3 and type 4 on X , for $\varrho \in [-0.4, -0.3]$ and $Q = \{1, a, b\}$, but it is not of type 1, because $f(c) \not\leq \varrho$.

d) In Example 2.1, an \mathcal{N} -structure (X, f) in which f is defined by $f(1) = f(a) = -0.7, f(b) = -0.2, f(c) = -0.3$ and $f(d) = -0.1$ is an $\mathcal{N}(Q, \varrho)$ -subalgebra of type 3 and type 4 on X , for $\varrho \in [-0.7, -0.3]$ and $Q = \{1, a, b\}$, but it is not of type 1 and type 2 on X , because $f(b) \not\leq \varrho$.

e) In Example 2.1, an \mathcal{N} -structure (X, f) in which f is defined by $f(1) = f(a) = -0.7, f(b) = -0.2, f(c) = -0.5$ and $f(d) = -0.3$ is an $\mathcal{N}(Q, \varrho)$ -subalgebra of type 4 on X , for $\varrho \in [-0.7, -0.3]$ and $Q = \{1, a, b\}$, but it is not of type 1, type 2, type 3 on X .

Now, in the following diagram we summarize the results of this definition. The mark $A \rightarrow B$, means that A implies B .



In this paper, we focus on $\mathcal{N}(Q, \varrho)$ -subalgebra of type 1 and from now on X is a Q -Smarandache CI -algebra.

The following example shows that there exists an \mathcal{N} -structure (X, f) in X such that it satisfies the condition (type 1), but it is not an \mathcal{N} -subalgebra of X .

Example 3.2. In Example 2.1, an \mathcal{N} -structure (X, f) in which f is defined by $f(1) = -0.7, f(a) = -0.2, f(b) = -0.4, f(c) = -0.6$ and $f(d) = -0.3$.

Then (X, f) satisfies the condition (2.1) for $\varrho \in [-0.2, -0.1]$, but it is not an \mathcal{N} -subalgebra. Because

$$f(b * c) = f(a) = -0.2 \not\leq -0.4 = \max\{f(b), f(c)\}.$$

Proposition 3.1. *If an \mathcal{N} -structure (X, f) satisfies the following condition:*

$$(\forall x \in Q)(\forall y \in X \setminus Q)(f(x) \leq f(y)),$$

then (X, f) is an (Q, ϱ) -subalgebra of X , for every $\varrho \in [\bigvee_{x \in Q} f(x), \bigwedge_{y \in X \setminus Q} f(y)]$.

Theorem 3.2. *Let $\varrho \in [-1, 0]$. If (X, f) is an $\mathcal{N}(Q, \varrho)$ -subalgebra of X , then*

- (i) $Q \subseteq C(f; \varrho)$,
- (ii) $(\forall \beta \in [-1, 0]) (\beta < \varrho \Rightarrow C(f; \beta) \text{ is a subalgebra of } Q)$.

Proof. Let (X, f) be a $\mathcal{N}(Q, \varrho)$ -subalgebra of X . Obviously, $Q \subseteq C(f; \varrho)$. If $\beta \in [-1, 0]$ be such that $\beta < \varrho$, then $C(f; \beta) \subseteq Q$. Let $x, y \in C(f; \beta)$. Then $f(x) \leq \beta$ and $f(y) \leq \beta$. Thus $f(x * y) \leq \max\{f(x), f(y)\} \leq \beta$, and so $x * y \in C(f; \beta)$. Thus $C(f; \beta)$ is a subalgebra of Q . \square

In the following theorem we give some conditions for an \mathcal{N} -subalgebra to be an $\mathcal{N}(Q, \varrho)$ -subalgebra.

Theorem 3.3. *Let $\varrho \in [-1, 0]$. If (X, f) is an \mathcal{N} -subalgebra of X satisfies the conditions (i) and (ii) in Theorem 3.2, then (X, f) is an $\mathcal{N}(Q, \varrho)$ -subalgebra of X .*

Proof. Let $x \in Q$ and $y \in X \setminus Q$. Then by Theorem 3.2(i), $x \in C(f; \varrho)$, and so $f(x) \leq \varrho$. Let $f(y) = \beta$. If $\beta < \varrho$, then by Theorem 3.2(ii), $y \in C(f; \beta) \subseteq Q$, which is a contradiction. Hence $f(x) \leq \varrho \leq \beta = f(y)$. Thus (X, f) is an $\mathcal{N}(Q, \varrho)$ -subalgebra of X . \square

4 Smarandache \mathcal{N} -filters

Definition 4.1. *Let X be a Q -Smarandache CI-algebra and $\varrho \in [-1, 0]$. An \mathcal{N} -structure (X, f) is called an \mathcal{N} -filter of X based on Q and ϱ (briefly, $\mathcal{N}(Q, \varrho)$ -filter of X) if it satisfies the following conditions:*

- (i) $(\forall x \in Q) (\forall y \in X \setminus Q) (f(1) \leq f(x) \leq \varrho \leq f(y))$.

$$(ii) \quad (\forall x, y \in Q) \quad (f(y) \leq \max\{f(x * y), f(x)\}).$$

Example 4.1. In Example 2.1, an \mathcal{N} -structure (X, f) in which f is defined by $f(1) = -0.6, f(a) = -0.4, f(b) = -0.5, f(c) = -0.4$ and $f(d) = -0.3$ is an $\mathcal{N}(Q, \varrho)$ -filter of X for $\varrho \in [-0.4, -0.3]$.

Theorem 4.1. Let $\{\mathcal{N}(Q_i, \varrho_i) : i \in \Delta\}$ be a family of $\mathcal{N}(Q_i, \varrho_i)$ -subalgebras (filters) of X where $\Delta \neq \emptyset$ and $\varrho_i \in [-1, 0]$, for all $i \in \Delta$.

Then $\mathcal{N}(\cap Q_i, \min\{\varrho_i\}_{i \in \Delta})$, is a subalgebra (filter) of X , too.

Theorem 4.2. Let $\varrho \in [-1, 0]$. If (X, f) is an $\mathcal{N}(Q, \varrho)$ -filter of X , then

$$(i) \quad Q \subseteq C(f; \varrho),$$

$$(ii) \quad (\forall \beta \in [-1, 0]) \quad (\beta < \varrho \Rightarrow C(f; \beta) \text{ is a filter of } Q).$$

Proof. Let (X, f) be an $\mathcal{N}(Q, \varrho)$ -filter of X . Obviously, $Q \subseteq C(f; \varrho)$. Let $\beta \in [-1, 0]$ be such that $\beta < \varrho$. If $x \in C(f; \beta)$, then $f(x) \leq \beta < \varrho$, and so $x \in Q$. Hence $C(f; \beta) \subseteq Q$. by Definition 4.1(i), $f(1) \leq f(x)$, for all $x \in X$. Hence $f(1) \leq f(x) \leq \beta$ for all $x \in C(f; \beta)$, and so $1 \in C(f; \beta)$. Let $x, y \in Q$ be such that $x * y \in C(f; \beta)$ and $x \in C(f; \beta)$. Then $f(x * y) \leq \beta$ and $f(x) \leq \beta$. If $x, y \in C(f; \beta)$, then $f(x) \leq \beta$. Now by Definition 4.1(ii), $f(y) \leq \max\{f(x * y), f(x)\} \leq \beta$. Thus $y \in C(f; \beta)$. Therefore, $C(f; \beta)$ is a filter of Q . □

For a Q -Smarandache CI -algebra X and $\varrho \in [-1, 0]$, the following example shows that an \mathcal{N} -filter (X, f) of X may not be an $\mathcal{N}(Q, \varrho)$ -filter of X .

Example 4.2. Let $X := \{1, a, b, c\}$ be a set with the following table.

$*$	1	a	b	c
1	1	a	b	c
a	1	1	b	c
b	1	a	1	c
c	c	c	c	1

Then X is a CI -algebra and $Q := \{1, a, b\}$ is a BE -algebra [13]. Define an \mathcal{N} -structure (X, f) in which f is defined by $f(1) = -0.7, f(a) = -0.2, f(b) = -0.4, f(c) = -0.2$. Then (X, f) is an \mathcal{N} -filter of X . But it is not an $\mathcal{N}(Q, \varrho)$ of X for $\varrho \in [-0.7, -0.3]$. Because $f(a) = -0.2 > \varrho$.

In the following theorem we give conditions for an \mathcal{N} -filter to be an $\mathcal{N}(Q, \varrho)$ -filter.

Theorem 4.3. Let $\varrho \in [-1, 0]$ and (X, f) be an \mathcal{N} -filter of X satisfies the conditions (i) and (ii) of Theorem 4.2. Then (X, f) is an $\mathcal{N}(Q, \varrho)$ -filter of X .

Proof. Let $x \in Q$ and $y \in X \setminus Q$. Then by Theorem 4.2(i), $x \in C(f; \varrho)$, and so $f(x) \leq \varrho$. Let $f(y) = \beta$. If $\beta < \varrho$, then by Theorem 4.2(ii), $y \in C(f; \beta) \subseteq Q$, which is a contradiction. Hence $\varrho \leq \beta = f(y)$. Since $f(1) \leq f(x)$ for all $x \in X$, it follows that $f(1) \leq f(x) \leq \varrho \leq \beta = f(y)$ so that condition (i) of Definition 4.1 is valid. Since f is an \mathcal{N} -filter of X , the condition (ii) of Definition 4.1 is obvious. Therefore, (X, f) is an $\mathcal{N}(Q, \varrho)$ -filter of X . \square

The following example shows that an $\mathcal{N}(Q, \varrho)$ -subalgebra may not be an $\mathcal{N}(Q, \varrho)$ -filter.

Example 4.3. Let $X := \{1, a, b, c, d\}$ be a set with the following table.

*	1	a	b	c	d
1	1	a	b	c	d
a	1	1	a	a	d
b	1	1	1	a	d
c	1	1	1	1	d
d	d	d	d	d	1

Then X is a CI -algebra and $Q = \{1, a, b, c\}$ is a BE -algebra. Define an \mathcal{N} -structure (X, f) in which f is defined by $f(1) = -0.4$, $f(a) = -0.4$, $f(b) = -0.3$, $f(c) = -0.2$ and $f(d) = -0.1$. Then (X, f) is an \mathcal{N} -subalgebra, for $\varrho \in [-0.2, 0]$, but it is not an \mathcal{N} -filter because

$$f(c) = -0.2 \not\leq -0.3 = \max\{f(b * c), f(b)\}.$$

Definition 4.2. An \mathcal{N} -function on X is called closed \mathcal{N} -filter if f satisfies:

$$f(x * 1) \leq f(x) \leq \max\{f(y * x), f(y)\}, \text{ for all } x, y \in X.$$

Example 4.4. Let $X := \{1, a, b\}$ be a set with the following table:

*	1	a	b
1	1	a	b
a	a	1	1
b	a	1	1

Then X is a CI -algebra [10]. Define an \mathcal{N} -function $f : X \rightarrow [0, 1]$ by $f(1) = -0.7$, $f(a) = -0.3$ and $f(b) = -0.4$. Then (X, f) is an \mathcal{N} -filter of X . But it is not an \mathcal{N} -closed filter because

$$f(b * 1) = f(a) = -0.3 \not\leq f(b) = -0.4.$$

Example 4.5. In Example 4.4, if define \mathcal{N} -function $f : X \rightarrow [0, 1]$ by $f(1) = -0.7$, $f(a) = -0.4$ and $f(b) = -0.4$. Then (X, f) is an \mathcal{N} -closed filter of X .

Proposition 4.4. *Let (X, f) be an \mathcal{N} -closed filter. Then $f(1) \leq f(x)$, for all $x \in X$.*

Proof. Let $x \in X$. Now, by Definition 4.2, we have

$$f(1) \leq \max\{f(x * 1), f(x)\} \leq \max\{f(x), f(x)\} = f(x).$$

□

Theorem 4.5. *Let (X, f) be an closed \mathcal{N} -filter and $\varrho \in [-1, 0]$. Then every $\mathcal{N}(Q, \varrho)$ -filter is $\mathcal{N}(Q, \varrho)$ -subalgebra of X .*

Proof. Let (X, f) be $\mathcal{N}(Q, \varrho)$ -filter and $x, y \in X$. Then by (CI3) and Definition 4.2, we have

$$\begin{aligned} f(x * y) &\leq \max\{f(y * (x * y)), f(y)\} \\ &= \max\{f(x * (y * y)), f(y)\} \\ &= \max\{f(x * 1), f(y)\} \\ &\leq \max\{f(x), f(y)\}. \end{aligned}$$

Therefore, (X, f) is an \mathcal{N} -subalgebra of X . □

Theorem 4.6. *Let (X, f) and (X, g) be $\mathcal{N}(Q_1, \varrho_1)$ and $\mathcal{N}(Q_2, \varrho_2)$ -subalgebra (filter) of X respectively. Then $(X \times X, f \times g)$ is an $\mathcal{N}(Q_1 \times Q_2, \max\{\varrho_1, \varrho_2\})$ -subalgebra(filter) of $X \times X$.*

Proof. Let $(x, y) \in (Q_1 \times Q_2)$ and $(z, t) \in (X \times X) \setminus (Q_1 \times Q_2)$. Then we have

$$\begin{aligned} (f \times g)(1, 1) = \max\{f(1), g(1)\} &\leq \max\{f(x), g(y)\} \\ &\leq \max\{\varrho_1, \varrho_2\} \\ &\leq \max\{f(z), f(t)\} = (f \times g)(z, t). \end{aligned}$$

Now, let $(x_1, x_2), (y_1, y_2) \in (Q_1 \times Q_2)$. Then

$$\begin{aligned} (f \times g)((x_1, x_2) * (y_1, y_2)) &= (f \times g)((x_1 * y_1), (x_2 * y_2)) \\ &= \max\{f(x_1 * y_1), g(x_2 * y_2)\} \\ &\leq \max\{\max\{f(x_1), f(y_1)\}, \max\{g(x_2), g(y_2)\}\} \\ &= \max\{\max\{f(x_1), g(x_2)\}, \max\{f(y_1), g(y_2)\}\} \\ &= \max\{(f \times g)(x_1, x_2), (f \times g)(y_1, y_2)\}. \end{aligned}$$

Hence $(X \times X, f \times g)$ is an $\mathcal{N}(Q_1 \times Q_2, \max\{\varrho_1, \varrho_2\})$ -subalgebra(resp. filter) of $X \times X$. □

Proposition 4.7. *Let Q_1 and Q_2 be two BE -algebras which are properly contained in X , $Q_1 \subseteq Q_2$ and $\varrho \in [-1, 0]$. Then every $\mathcal{N}(Q_2, \varrho)$ -subalgebra(filter) of X is an $\mathcal{N}(Q_1, \varrho)$ -subalgebra(filter) of X .*

Note. By the following example we show that the converse of above theorem is not correct in general.

Example 4.6. Let $X := \{1, a, b, c\}$ be a set with the following table.

$*$	1	a	b	c
1	1	a	b	c
a	1	1	b	c
b	1	a	1	c
c	c	c	c	1

Then $Q_1 = \{1, a\}$, $Q_2 = \{1, a, b\}$ are BE -algebras which are properly contained in X and $f(1) = -0.7$, $f(a) = -0.4$, $f(b) = -0.2$ and $f(c) = -0.1$. Then (X, f) is an $\mathcal{N}(Q_1, \varrho)$ -subalgebra, for all $\varrho \in [-0.4, 0]$, but it is not an $\mathcal{N}(Q_2, \varrho)$ -subalgebra, because, if $\varrho := -0.3$, then $f(b) = -0.2 \not\leq -0.3$.

5 Conclusion

A Smarandache structure on a set A means a weak structure W on A such that there exist a proper subset B of A which is embedded with a strong structure S . It is that any BE -algebra is a CI -algebra. Hence, every BE -algebra is a weaker structure than CI -algebra, thus we can consider in any CI -algebra a weaker structure as BE -algebra.

In this paper, we have introduced the concept of \mathcal{N} -subalgebra (filter) based on Smarandache CI -algebras and some related properties are investigated. We show that any $\mathcal{N}(Q, f)$ -closed filter is an $\mathcal{N}(Q, f)$ -subalgebra. We give some conditions for an \mathcal{N} -subalgebras (filters) to be $\mathcal{N}(Q, \varrho)$ -subalgebras (filters).

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