THE 57-TH SMARANDACHE'S PROBLEM II *

Liu Huaning
Department of Mathematics, Northwest University, Xi'an, Shaanxi, P.R.China
hnliu@nwu.edu.cn

Gao Jing
School of Science, Xi'an Jiaotong University, Xi'an, Shaanxi, P.R.China

Abstract For any positive integer n, let r be the positive integer such that: the set $\{1, 2, \cdots, r\}$ can be partitioned into n classes such that no class contains integers x, y, z with $x^y = z$. In this paper, we use the elementary methods to give a sharp lower bound estimate for r.

Keywords: Smarandache-type multiplicative functions; Mangoldt function; Hybrid mean value.

§1. Introduction

For any positive integer n, let r be a positive integer such that: the set $\{1, 2, \cdots, r\}$ can be partitioned into n classes such that no class contains integers x, y, z with $x^y = z$. In [1], Schur asks us to find the maximum r. About this problem, Liu Hongyan [2] obtained that $r \geq n^{m+1}$, where m is any integer with $m \leq n + 1$.

In this paper, we use the elementary methods to improve Liu Hongyan's result. That is, we shall prove the following:

Theorem. *For sufficiently large integer n, let r be a positive integer such that: the set $\{1, 2, \cdots, r\}$ can be partitioned into n classes such that no class contains integers x, y, z with $x^y = z$. Then we have*

$$r \geq \left(n^{n!} + 2\right)^{n^{n!}+1} - 1.$$

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§2. Proof of the Theorem

In this section, we complete the proof of the theorem.

Let $r = \left(n^{n!}+2\right)^{n^{n!}+1} - 1$ and partition the set $\{1, 2, \cdots, \left(n^{n!}+2\right)^{n^{n!}+1} - 1\}$ into n classes as follows:

$$
\begin{array}{llllll}
\text{Class 1:} & 1, & n^{n!}+1, & n^{n!}+2, & \cdots, & \left(n^{n!}+2\right)^{n^{n!}+1} - 1. \\
\text{Class 2:} & 2, & n+1, & n+2, & \cdots, & n^2. \\
\vdots & & & & & \\
\text{Class k:} & k, & n^{(k-1)!}+1, & n^{(k-1)!}+2, & \cdots, & n^{k!}. \\
\vdots & & & & & \\
\text{Class n:} & n, & n^{(n-1)!}+1, & n^{(n-1)!}+2, & \cdots, & n^{n!}.
\end{array}
$$

It is obvious that Class $k (k \geq 2)$ contains no integers x, y, z with $x^y = z$. In fact for any integers $x, y, z \in$ Class k, $k = 2, 3, \cdots, n$, we have

$$x^y \geq \left(n^{(k-1)!}+1\right)^k > n^{k!} \geq z.$$

Similarly, Class 1 also contains no integers x, y, z with $x^y = z$.

This completes the proof of the theorem.

Reference

[1] F. Smarandache, Only Problems, Not Solutions, Xiquan Publishing House, Chicago, 1993.

[2] Liu Hongyan and Zhang Wenpeng. A note on the 57-th Smarandache's problem. Smarandache Notions Journal **14** (2004), 164-165.