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Solutions of Some Kandasamy-Smarandache Open Problems About the Algebraic Structure of Neutrosophic Complex Finite Numbers

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Abstract: The aim of this paper is to study the neutrosophic complex finite rings $C(Z_n)$ and $C(< Z_n \cup I >)$, and to give a classification theorem of these rings. Also, this work introduces full solutions for 12 Kandasamy-Smarandache open problems concerning these structures of generalized rings modulo integers. Also, a necessary and sufficient condition of invertibility in $C(Z_n)$ and $C(< Z_n \cup I >)$ is presented as a partial solution of the famous group of units problem.

Keywords: Neutrosophic complex number, neutrosophic finite complex number, maximal ideal, minimal ideal

Introduction.

Neutrosophy as a new kind of generalized logic deals with indeterminacy in nature, reality, and ideas found its way into algebraic studies. A lot of neutrosophic algebraic structures were defined and studied in a wide range. See [1-11].

In the literature, many generalizations appeared such as refined neutrosophic rings, n-refined neutrosophic rings, n-refined neutrosophic groups, and n-refined neutrosophic vector spaces and modules. Recently, algebraic equations and Diophantine linear equations were solved in neutrosophic rings and refined neutrosophic rings. See [5-18].

In [20], Smarandache and Kandasamy introduced the neutrosophic complex numbers modulo integers as an interesting generalized structure. Their work suggests a new approach to the concept of classical complex numbers, and they proposed 150 open problems concerning substructures and factorization properties in these complex neutrosophic structures modulo integers (some of these problems were solved in [17]). In this paper, we aim to continue their efforts and to suggest a classification of neutrosophic complex finite rings modulo integers. Also, we suggest solutions for 12 problems of Kandasamy-Smarandache problems introduced in [20].

Main results

We start our discussion by some easy Kandasamy-Smarandache problems about finite neutrosophic complex rings.

Problem (56): Does every $C(Z_n)$ contain a zero divisor?.

The answer is no. If n is a prime and there are $a, b \in Z_n;\ a^2 + b^2 \equiv 0(mod\ n)$, then $C(Z_n)$ is a field according to Theorem , and then it has no zero divisors.

Problem (58): Is every element in C(Z_7) invertible?.

The answer is yes, since C(Z_7) is a field, thus all elements different from zero are invertible.

Problem (57): Can every C(Z_n) be a field?.

The answer is no, since C(Z_5) is just a ring but not a field.

Problem (53): Find a subring S in C(Z_n) so that S is not an ideal.

We take $S = Z_n$ which is a subring of C(Z_n) , but it is not an ideal, that is because 1∈ Z_n and $i_F \in C(Z_n)$, where1. $i_F = i_F$, which is not in S. Thus S is not an ideal.

Problem (26): Can C(⟨Z_{12} ∪ I⟩) be a S-ring? Justify.

The answer is yes. That is because the set $M = \{0,9,3\}$ is a field under multiplication with 9 acts as the identity.

Problem (25): Prove C(⟨Z_{25} ∪ I⟩) can only be a ring.

It is sufficient to prove that C(⟨Z_{25} ∪ I⟩) has zero divisors. We take $5 + 5I \in C(\langle Z_{25} \cup I\rangle)$, and

$(5 + 5I).(5 + 5I) = 25(1 + I)(1 + I) = 0.$

Definition:

(a) Let R be any commutative ring, m be any element (not from R) which is a root of a polynomial $p(x) \in R[x]$. Then if there is no root of $p(x)$ in R, we call R(m) an algebraic extension. For example the ring Z(i) is an algebraic extension of the ring Z, since i is a root of the polynomial $p(x) = x^2 + 1 \in Z[x]$, and p(x) has no roots in Z. (The concept of classical algebraic extension).

(b) Let R be any commutative ring, m be any element (not from R) which is a root of a polynomial $p(x) \in R[x]$. Then if there is a root of $p(x)$ in R, we call R(m) a logical extension.

For example the neutrosophic ring Z(I) is a logical extension of the ring Z, since I is a root of the polynomial $p(x) = x^2 - x \in Z[x]$, and p(x) has roots {0,1} in Z.

The following theorem realizes the algebraic structure of $C(Z_n)$.

Theorem:

Let $C(Z_n)$ be the ring of complex numbers modulo n, we have the following:

(a) If n=p is a prime and $p(x) = x^2 + 1$ is irreducible over Z_p, then $C(Z_p)$ is an algebraic extension field of the field Z_p with degree two.

(b) If n=p is a prime and $p(x) = x^2 + 1$ is reducible over Z_p, then $C(Z_p)$ is just a ring (logical extension).

(c) If n is not a prime and $p(x) = x^2 + 1$ is irreducible over Z_n, then $C(Z_n)$ is an algebraic extension ring of the ring Z_n with degree two.

(d) If n is not a prime and $p(x) = x^2 + 1$ is reducible over Z_n, then $C(Z_n)$ is a logical extension of the ring Z_n.

Proof:

(a) Suppose that $p(x) = x^2 + 1$ is irreducible over Z_p, then it has no roots in Z_p, thus i_F is an algebraic element over Z_p, and by classical algebraic result, we get that C(Z_p) is an algebraic extension field of the field Z_p with degree equal to deg(p) which is two.

(b) i_F is a root of $p(x) = x^2 + 1$, but p(x) has a root in Z_p, because it is reducible, hence $C(Z_p)$ is just a ring (logical extension). [$C(Z_p)$ is not a field because there are $a, b \in Z_p$ such that $a^2 + b^2 \equiv 0(mod\ p)$, where $b = 1\ and\ a\ is\ the\ root\ of\ p(x) in\ Z_p$].

(c) It holds by a similar argument of section (a).

(d) It holds by a similar argument of (b).

The following theorem suggests a classification of the ring $C(< Z_n \cup I >)$.

Theorem:

Let $C(< Z_n \cup I >)$ be the neutrosophic complex modulo integers ring. Then

$C(< Z_n \cup I >) \cong \mathrm{C}(\mathrm{Z_n}) \times \mathrm{C}(\mathrm{Z_n})$.

Proof:

Firstly, we prove that $C(< Z_n \cup I >) = [\mathrm{C}(\mathrm{Z_n})](\mathrm{I})$, where $[\mathrm{C}(\mathrm{Z_n})](\mathrm{I})$ is the neutrosophic ring generated by I and $\mathrm{C}(\mathrm{Z_n})$.

Let $x = a + bi + cI + diI \in C(< Z_n \cup I >)$, then $x = (a + bi) + I(c + di) \in [\mathrm{C}(\mathrm{Z_n})](\mathrm{I})$, hence $C(< Z_n \cup I >) \leq [\mathrm{C}(\mathrm{Z_n})](\mathrm{I})$. Conversely, let $x = (a + bi) + (c + di)I \in [\mathrm{C}(\mathrm{Z_n})](\mathrm{I})$. It is clear that

$x \in C(< Z_n \cup I >)$. This implies that $C(< Z_n \cup I >) = [\mathrm{C}(\mathrm{Z_n})](\mathrm{I})$.

By the classification theorem of neutrosophic rings in [5], we find that $C(< Z_n \cup I >) = [\mathrm{C}(\mathrm{Z_n})](\mathrm{I}) \cong \mathrm{C}(\mathrm{Z_n}) \times \mathrm{C}(\mathrm{Z_n})$.

Problem (24): Is $C(< Z_{19} \cup I >)$ a field?.

The answer is no, since I is not invertible.

The group of units problem and other open questions

In this section, we determine the necessary and sufficient condition for the invertibility of neutrosophic complex numbers modulo integers.

First of all, we characterize the algebraic structure of $C(Z_n)$ as an isomorphic image of a matrices subring of size 2×2.

Theorem:

Let $C(Z_n)$ be the ring of neutrosophic complex numbers modulo integers. Then $C(Z_n)$ is isomorphic to a sub ring of $M_{2\times 2}(Z_n) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in Z_n\}$.

Proof:

Let $S = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix}; a, b \in Z_n\}$ be a subring of $M_{2\times2}(Z_n)$, we define

$f: C(Z_n) \to S; f(a + bi_F) = (\begin{smallmatrix} a & b \\ -b & a \end{smallmatrix})$, it is easy to see that f is a well defined bijective map.

Let $x = a + bi_F, y = c + di_F$ be two arbitrary elements in $C(Z_n)$, we have

$f(x + y) = \begin{pmatrix} a + c & b + d \\ -b - d & a + c \end{pmatrix} = \begin{pmatrix} a & b \\ -b & b \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = f(x) + f(y).$

$f(x.y) = \begin{pmatrix} ac - bd & ad + bc \\ -ad - bc & ac - bd \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} . \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = f(x).f(y)$. Thus f is a ring isomorphism.

Now, we can find the condition of invertibility, as an easy result from Theorem.

Theorem:

Let $C(Z_n)$ be the ring of neutrosophic complex numbers modulo integers, $x = a + bi_F$ be an arbitrary elements in $C(Z_n)$. Then x is invertible if and only if $a^2 + b^2 \neq 0 \; and \; a^2 + b^2$ is invertible in Z_n.

Proof:

Since $C(Z_n) \cong S$, then x is invertible in $C(Z_n)$ if and only if $f(x) = (\begin{smallmatrix} a & b \\ -b & a \end{smallmatrix})$ is invertible in S.

It is well known that the matrix $(\begin{smallmatrix} a & b \\ -b & a \end{smallmatrix})$ is invertible if and only if its inverse matrix is an element from S. Hence we have the following

(a) $\det\left[\begin{pmatrix} a & b \\ -b & a \end{pmatrix}\right] = a^2 + b^2 \neq 0.$

(b) $\det\left[\begin{pmatrix} a & b \\ -b & a \end{pmatrix}\right] = a^2 + b^2$ is invertible in Z_n, so the inverse matrix can be defined.

Thus, our proof is complete.

The condition (b) is sufficient, that is because if $a^2 + b^2$ is invertible in Z_n, then $a^2 + b^2 \neq 0$.

Example:

Consider the ring $C(Z_5) = \{a + bi_F; a, b \in Z_5\}$. The group of units in $C(Z_5)$ is equal to

$U = \{1,2,3,4, i_F, 2i_F, 3i_F, 4i_F, 1 + i_F, 1 + 4i_F, 2 + 2i_F, 2 + 3i_F, 3 + 2i_F, 3 + 3i_F, 4 + i_F, 4 + 4i_F\}$.

Example :

Consider the ring $C(Z_4) = \{a + bi_F; a, b \in Z_4\}$. The group of units in $C(Z_4)$ is equal to

$U = \{1,3, i_F, 3i_F, 1 + 2i_F, 2 + i_F, 2 + 3i_F, 3 + 2i_F\}$.

Example:

Consider the ring $C(Z_6) = \{a + bi_F; a, b \in Z_6\}$. The group of units in $C(Z_6)$ is equal to

$U = \{1,5, i_F, 5i_F, 1 + 2i_F, 1 + 4i_F, 2 + i_F, 2 + 3i_F, 2 + 5i_F, 3 + 2i_F, 3 + 4i_F, 4 + i_F, 4 + 3i_F, 4 + 5i_F, 5 + 2i_F, 5 + 4i_F\}$.

Now, we introduce the algebraic structure of the group of units in the ring $C(< Z_n \cup I >)$.

Theorem:

The group of units in the ring $C(< Z_n \cup I >)$, has the following property

$U(C(< Z_n \cup I >)) \cong U(C(Z_n)) \times U(C(Z_n))$.

The proof holds directly from the fact that $C(< Z_n \cup I >) \cong \mathrm{C}(\mathrm{Z_n}) \times \mathrm{C}(\mathrm{Z_n})$.

Remark:

A very interesting and hard problem is still open. This problem can be summarized as follows:

Describe the algebraic structure of the group of units in the ring $C(Z_n)$.

Although we have found the necessary and sufficient condition of any element in $C(Z_n)$ to be a unit, but the classification of this group as a direct product of cyclic groups is still unknown.

Remark:

As a result of Theorem 4.2, we can find zero divisors in $C(Z_n)$. Every element $x = a + bi_F \in C(Z_n)$ is a zero divisor if and only if its isomorphic image $f(x) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is a zero divisor in the ring S.

Any matrix with form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is a zero divisor if and only if its determinant is a zero divisor in Z_n, thus the necessary and sufficient condition for any element $x = a + bi_F \in C(Z_n)$ to be a zero divisor is $a^2 + b^2$is a zero divisor in Z_n. Now, we are able to solve another open problem.

Problem (50): Find Zero divisors and units in $C(Z_{24})$.

To solve the problem we shall determine the zero divisors in Z_{24} firstly.

We have 3,8,6,4,12,2 are zero divisors, that is because $3.8 = 6.4 = 12.2 = 0$. And $-3 = 21, -8 = 16, -6 = 18, -4 = 20, -2 = 22$ are zero divisors clearly. Also, the product of any two zero divisors is a zero divisor.

According to our discussion, zero divisors in $C(Z_{24})$ are

3,8,4,6,12,2,21,16,18,20,22, 15. The rest of zero divisors in $C(Z_{24})$ are elements with form $a + bi_F$, where $a^2 + b^2 \in \{3,8,4,6,12,2,21,16,18,20,22,15\}$.

To determine the units in $C(Z_{24})$, we shall determine units in Z_{24}. We have $U(Z_{24}) = \{1,5,7,11,13,17,19,23\}$. The other units in $C(Z_{24})$ are the elements with form $x = a + bi_F;\ a^2 + b^2 \in U(Z_{24})$.

The following theorems helps us in finding ideals of the ring $C(Z_n)$, and $C(< Z_n \cup I >)$.

Theorem:

Let $C(Z_n)$ be a neutrosophic complex modulo integers ring, $S = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix}; a, b \in Z_n\}$ be its corresponding isomorphic subring. Let $I_{H_j} = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix}; a, b \in H_j\}$, where $(H_j, +)$ is a subgroup of Z_n. We have

(a) Ideals of $C(Z_n)$ are exactly the isomorphic image of the sets I_{H_j}.

(b) If $(H_j, +, .)$ is a maximal ideal in $(Z_n, +, .)$, then I_{H_j} is a maximal ideal in $C(Z_n)$.

Proof:

Firstly, we shall determine the structure of additive subgroups in S. Let A,B be two subsets of Z_n, and $M = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix}; a \in A, b \in B\}$. Let $x = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}, y = \begin{pmatrix} c & d \\ -d & c \end{pmatrix}$ be two arbitrary elements in M.

(M,+) is a subgroup of S if and only if $x - y \in M$, which is equivalent to $a - b \in A, c - d \in B$, hence A,B are subgroups of Z_n.

Now, we prove that M is an ideal in S if and only if $A = B$.

Since A,B are subgroups of Z_n, we find that $(A.+,.), (B,+,.)$ are ideals in the ring $(Z_n, +, .)$.

Firstly, we assume that A=B. Let $x = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \in M \; and \; r = \begin{pmatrix} c & d \\ -d & c \end{pmatrix} \in S$, we have

$x.r = \begin{pmatrix} ac - bd & ad + bc \\ -ad - bc & ac - bd \end{pmatrix}$. We have

$ac - bd \in A$, that is because $ac \in A \; (A \; is \; an \; ideal \; in \; Z_n)$ and $bd \in A \; (for \; the \; same \; reason)$. This implies that $x.r \in M$ and M is an ideal in S. Conversely, we suppose that M is an ideal in S, hence for any two elements Let $x = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \in M \; and \; r = \begin{pmatrix} c & d \\ -d & c \end{pmatrix} \in S$, we have

$x.r = \begin{pmatrix} ac - bd & ad + bc \\ -ad - bc & ac - bd \end{pmatrix} \in M$, this implies that $ac - bd \in A$ and $ad + bc \in B$.

We know that A,B are ideals in Z_n, hence $ac \in A \; (because \; a \in A, c \in Z_n)$ and $bc \in B (because \; b \in B \; and \; c \in Z_n)$. This means that $-bd \in A$ and $ad \in B$ for all $b \in B, a \in A, d \in Z_n$, we put $d = 1$ to find that $a \in B \; and \; b \in A$. Thus $A = B$.

According to Theorem , we have $C(Z_n) \cong S$, hence all ideals in $C(Z_n)$ are exactly the isomorphic image of the ideals in S. hence the proof is complete.

(b) Suppose that $I_{H_j} = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix}; a, b \in H_j\}$ is a maximal ideal in S, hence it is easy to see that H_j is a maximal ideal in Z_n.

Remark:

Every ideal in $C(Z_n)$ has the form $f^{-1}\left(I_{H_j}\right) = \{a + bi_F; a, b \in H_j\}$, where H_j is a subgroup of Z_n.

Theorem:

Ideals in $C(< Z_n \cup I >)$ are equal to the isomorphic image of the set $J = \{I_{H_j} \times I_{H_s};\ H_j, H_s \leq Z_n\}$. Also, maximal ideals in $C(< Z_n \cup I >)$ are equal to the isomorphic image of the set $J = \{I_{H_j} \times I_{H_s};\ H_j, H_s \leq Z_n\ and\ I_{H_j}, I_{H_s}\ are\ maximal\}$.

Proof:

According to Theorem , we have $C(< Z_n \cup I >) \cong \mathrm{C(Z_n)} \times \mathrm{C(Z_n)}$. the isomorphism between them is defined in [5] as follows:

$f: C(< Z_n \cup I >) \to \mathrm{C(Z_n)} \times \mathrm{C(Z_n)}$; $f(a + bI) = (a, a + b); a, b \in C(Z_n)$. The inverse isomorphism is $f^{-1}: C(Z_n) \times C(Z_n) \to C(< Z_n \cup I >);\ f^{-1}(a, b) = a + (b - a)I; a, b \in C(Z_n)$.

According to Remark 4.11, ideals in $C(Z_n)$ has the form $\{a + bi_F; a, b \in H_j\}$, where H_j is a subgroup of Z_n, hence ideals in $\mathrm{C(Z_n)} \times \mathrm{C(Z_n)}$ has the form $I = \{(a + bi_F, c + di_F); a, b \in H_j\ and\ c, d \in H_s\}$, where H_j, H_s are two subgroups of Z_n. Thus ideals in $C(< Z_n \cup I >)$ has the form

$f^{-1}(I) = \{(a + bi_F) + [(c + di_F) - (a + bi_F)]I; a, b \in H_j\ and\ c, d \in H_s\} = \mathrm{I_{H_j}} + \left(\mathrm{I_{H_s}} - \mathrm{I_{H_j}}\right)\mathrm{I}$, where H_j, H_s are two subgroups of Z_n.

Also, maximal ideals in $C(< Z_n \cup I >)$ has the form $f^{-1}(I) = \{(a + bi_F) + [(c + di_F) - (a + bi_F)]I; a, b \in H_j\ and\ c, d \in H_s\}$, where H_j, H_s are two maximal ideals of Z_n.

Problem (28): Find ideals in $\mathrm{C}(\langle Z_6 \cup \mathrm{I}\rangle)$.

Subgroups (Ideals) of Z_6 are $A = \{0\}, B = \{0,2,4\}, C = \{0,3\}, D = \{0,1,2,3,4,5\}$.

Ideals of C(Z_6) are $X = I_A = \{0\}, Y = I_B = \{0,2,4,2i_F, 4i_F, 2+2i_F, 2+4i_F, 4+4i_F, 4+2i_F\}$,

$Z = I_C = \{0,3,3i_F, 3+3i_F\}, T = I_D = \mathrm{C}(Z_6)$.

Ideals of C(⟨Z_6 ∪ I⟩) are the sets with form $M + (N - M)I; M, N \in \{X, Y, Z, T\}$.

Problem (29): Find maximum ideals of C(⟨Z_{18} ∪ I⟩).

First of all, we shall find maximum ideals in Z_{18}. They are $A = \{0,2,4,6,8,10,12,14,16\}$,

$B = \{0,3,6,9,12,15\}, C = Z_{18}$.

Maximal ideals in $C(Z_{18})$ are $I_A = \{a + bi_F; a, b \in A\}, I_B = \{c + di_F; c, d \in B), I_C = C(Z_{18})$.

Hence, maximal ideals in C(⟨Z_{18} ∪ I⟩) are P= $I_A + (I_B - I_A)I = I_A + I_C I, Q = I_B + (I_A - I_B)I =$

$I_B + I_C I,$

$R = I_C + (I_A - I_C)I = I_C + (I_B - I_C)I = I_C + I_C I = \mathrm{C}(\langle Z_{18} \cup \mathrm{I}\rangle)$.

Find an ideal I in C(Z_{128}) so that C(Z_{128})/I is a field..***Problem (51):***

We have $J =< 2 >$ is a maximal ideal in Z_{128}. Hence $I_J = \{a + bi_F; a, b \in J\}$ is a maximal ideal in C(Z_{128}), thus C(Z_{128})/I_J is a field with order 4.

Problem (52): Does there exist an ideal I in C(Z_{49}) so that C(Z_{49})/I is a field?.

It is sufficient to find a maximal ideal in Z_{49}. We have $J =< 7 >$ is maximal in Z_{49}, hence $I_J = \{a + bi_F; a, b \in J\}$ is maximal in C(Z_{49}), and C(Z_{49})/ I_J is a field with order 49.

Problem (55): Find a necessary and sufficient condition for a complex modulo integers ring $S = C(Z_n)$ to have ideal I such that $C(Z_n)/_I$ is never a field.

The answer is depending on finding a non maximal ideal in $C(Z_n)$, since if I is a maximal ideal in $C(Z_n)$, we get a field $C(Z_n)/_I$.

We have the following cases:

(a) If n is a prime and $P(x) = x^2 + 1$ is irreducible over Z_n, then $C(Z_n)$ is a field and it has no proper ideals. (The only maximal ideal is I={0}). Thus the problem is not solvable in this case.

(b) If n is a prime and $P(x) = x^2 + 1$ is reducible over Z_n, then $C(Z_n)$ is a finite ring with n^2 elements. Thus every proper ideal I in $C(Z_n)$ has exactly n elements (because I is a subgroup under addition and then its order divides the order of $C(Z_n)$ by classical Lagrange's theorem).

Now, $C(Z_n)/_I$ is a ring with n elements (n is a prime), thus it is a field. Hence the problem is not solvable in this case.

(c) If n is not a prime and there is an integer s with property $s \neq gcd(s,n) = a \geq 2$, we define the following principal ideal $I =< s >$, where s is an integer with property $s \neq gcd(s,n) = a \geq 2$. It is clear that $I < J =< a >\neq C(Z_n)$, hence I is not maximal and $C(Z_n)/_I$ is never a field.

(d) If n is not a prime, but a prime power $n = p^n$. For $n = 2$, there is $< p >$ as the unique proper ideal and it is a maximal ideal in Z_n, hence $I_{<p>} = \{a + bi_F; a,b \in< p >\}$ is maximal in $C(Z_n)$, hence $C(Z_n)/_I$ is a field and the problem is not solvable in this case.

For $n \geq 3$, there is a non maximal ideal $< p^2 >$ in Z_n, hence $I_{<p^2>} = \{a + bi_F; a,b \in< p^2 >\}$ is non maximal in $C(Z_n)$, hence $C(Z_n)/_I$ is never a field.

(e) If n is not a prime and not a prime power, and there is not any integer s with property $s \neq gcd(s,n) = a \geq 2$, then $< s >$ is maximal in Z_n, , hence $I_{<s>} = \{a + bi_F; a,b \in< s >\}$ is maximal in $C(Z_n)$, hence $C(Z_n)/_I$ is a field, and the problem is not solvable in this case. (All ideals are maximal in this case).

Conclusion

In this paper, we have classified the ring of finite neutrosophic complex numbers as irect product of two rings. On the other hand, we have presented solutions for 12 open problems suggested by Smarandache and Kandasamy in [20].

As a future research direction, we aim to solve all Smarandache-Kandasamy open problems.

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