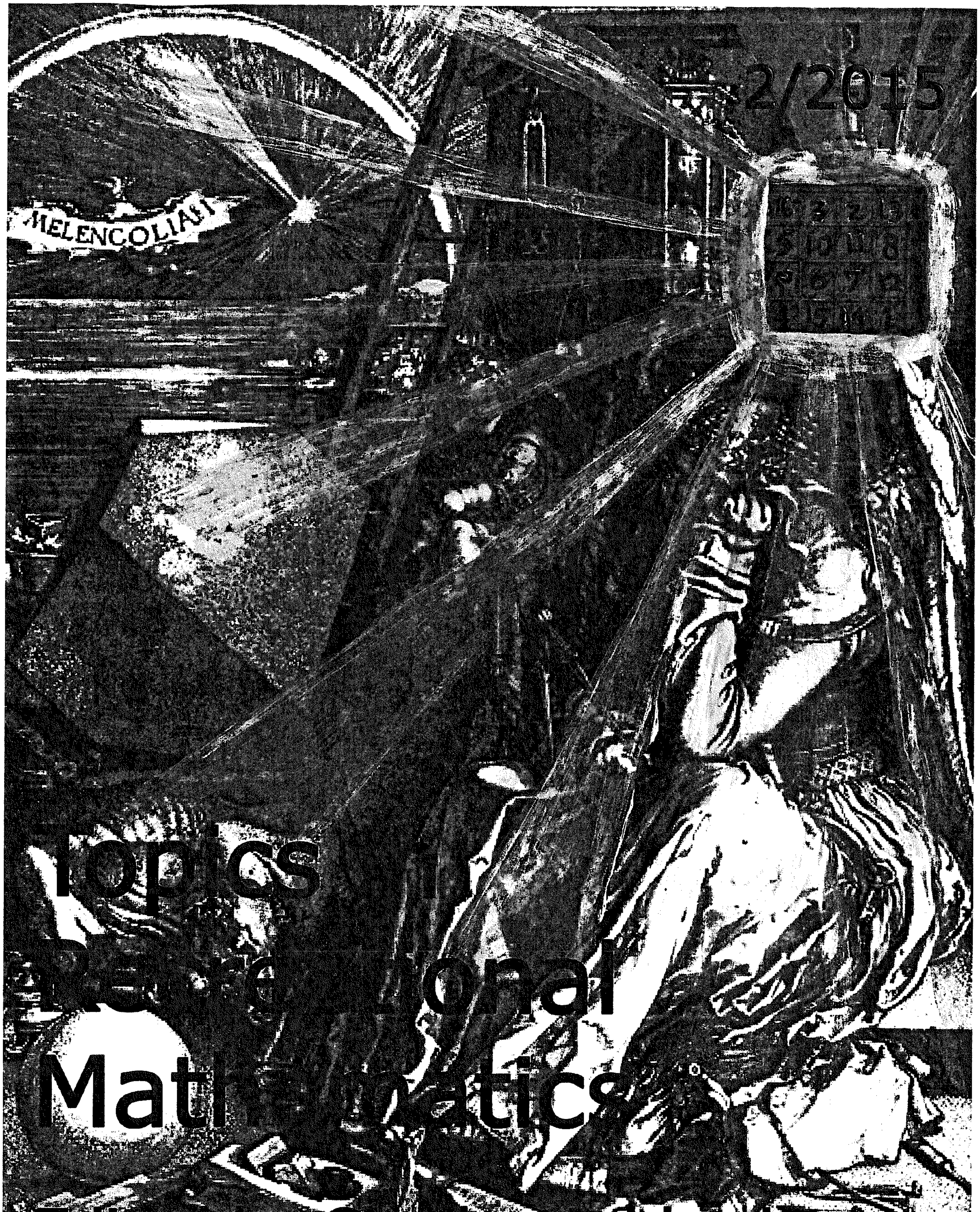

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Contents

AN EXAMPLE OF A SMARANDACHE GEOMETRY

Professor Ion Patrascu

National College Fraţii Buzeşti

Craiova, Romania

Abstract

For centuries it was thought that the geometry codified by Euclid and based on the parallel postulate

Given any line l and a point p not on l, one and only one line can be drawn through p parallel to l.

was the only geometry that existed. This idea was overturned when two other geometries based on a different number of parallel lines being drawn through p were discovered. In a hyperbolic geometry it is possible to construct infinitely many lines parallel to l passing through p and in an elliptic geometry it is not possible to construct any lines through p parallel to l.

This paper gives an example of a geometry where more than one form of the parallel axiom is valid within the geometry.

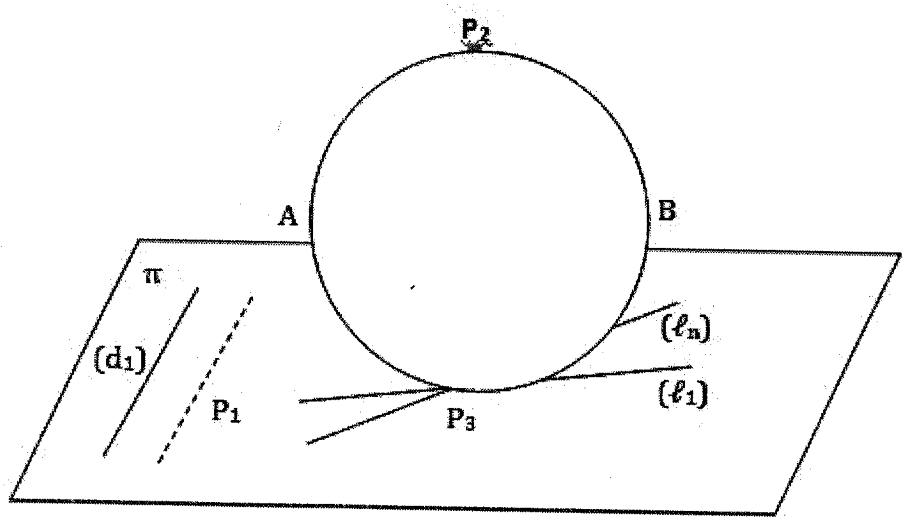
A Smarandache geometry is a geometry in which at least one of the five fundamental axioms is either validated and invalidated, or only invalidated, but in multiple ways (in the same geometric space).

Consider the parallel or the fifth axiom of Euclidean geometry.

Through a point outside a line one can only draw one parallel to that line.

We can build a model of Smarandache geometry where the axiom of parallels is validated for some lines and points, and invalidated for other points and lines.

Figure 1



Consider a plane (π) and a sphere S of center O that is tangent to the plane (π) in the point P_3. The line (d_1) and the point P_1 belong to the plane (π).

The concepts of "line" and "point" in the plane (π) are the classical ones. On the sphere, the "line" is a big circle of the sphere, and the "point" is any classical point on the surface of the sphere.

Two lines are called parallel if they have no common point. Hence, the parallel axiom has three different forms in this Smarandache geometry model:

1. Through the point P_1 one can draw only one parallel to the line (d_1) [as in the Euclidean geometry].

2. Through the point P_2 one cannot draw any parallel to the line AB because a great circle of the sphere passing through P_2 will intersect the great circle AB [as in the non-Euclidean elliptical geometry].

3. Through the point P_3 belonging to the plane (π) and to the sphere S, one can draw an infinity of lines $(\ell_1), \ldots, (\ell_n), \ldots$, all contained in the plane (π), which do not intersect the line AB, so they are parallel to the line AB [as in the non-Euclidean hyperbolic geometry].

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