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ON SOME SMARANDACHE PROBLEMS

Edited by M. Perez

1. PROPOSED PROBLEM

Let $n \geq 2$. As a generalization of the integer part of a number one defines the Inferior Smarandache Prime Part as: $ISPP(n)$ is the largest prime less than or equal to n. For example: $ISPP(9) = 7$ because $7 < 9 < 11$, also $ISPP(13) = 13$. Similarly the Superior Smarandache Prime Part is defined as: $SSPP(n)$ is smallest prime greater than or equal to n. For example: $SSPP(9) = 11$ because $7 < 9 < 11$, also $SSPP(13) = 13$. Questions:
1) Show that a number p is prime if and only if

$$ISPP(p) = SSPP(p).$$

2) Let $k > 0$ be a given integer. Solve the diophantine equation:

$$ISPP(x) + SSPP(x) = k.$$

Solution by Hans Gunter, Koln (Germany)

The Inferior Smarandache Prime Part, $ISPP(n)$, does not exist for $n < 2$.
1) The first question is obvious (Carlos Rivera).
2) The second question:

a) If $k = 2p$ and p =prime (i.e., k is the double of a prime), then the Smarandache diophantine equation

$$ISPP(x) + SSPP(x) = 2p$$

has one solution only: $x = p$ (Carlos Rivera).

b) If k is equal to the sum of two consecutive primes, $k = p(n) + p(n+1)$, where $p(m)$ is the m-th prime, then the above Smarandache diophantine equation has many solutions: all the integers between $p(n)$ and $p(n+1)$ [of course, the extremes $p(n)$ and $p(n+1)$ are excluded]. Except the case $k = 5 = 2+3$, when this equation has no solution. The sub-cases when this equation has one solution only is when $p(n)$ and $p(n+1)$ are twin primes, i.e. $p(n+1)-p(n) = 2$, and then the solution is $p(n)+1$. For example: $ISPP(x)+SSPP(x) = 24$ has the only solution $x = 12$ because $11 < 12 < 13$ and $24 = 11 + 13$ (Teresinha DaCosta).

Let's consider an example:

$$ISPP(x) + SSPP(x) = 100,$$

because 100=47+53 (two consecutive primes), then $x = 48, 49, 50, 51$, and 52 (all the integers between 47 and 53).

$$ISPP(48) + SSPP(48) = 47 + 53 = 100.$$

Another example:

$$ISPP(x) + SSPP(x) = 99$$

has no solution, because if $x = 47$ then

$$ISPP(47) + SSPP(47) = 47 + 47 < 99,$$

and if $x = 48$ then

$$ISPP(48) + SSPP(48) = 47 + 53 = 100 > 99.$$

If $x \leq 47$ then

$$ISPP(x) + SSPP(x) < 99,$$

while if $x \geq 48$ then

$$ISPP(x) + SSPP(x) > 99.$$

c) If k is not equal to the double of a prime, or k is not equal to the sum of two consecutive primes, then the above Smarandache diophantine equation has no solution.

A remark: We can consider the equation more general: Find the real number x (not necessarily integer number) such that

$$ISPP(x) + SSPP(x) = k,$$

where $k > 0$.

Example: Then if $k = 100$ then x is any real number in the open interval (47, 53), therefore infinitely many real solutions. While integer solutions are only five: 48, 49, 50, 51, 52.

A criterion of primality: The integers p and $p + 2$ are twin primes if and only if the diophantine smarandacheian equation

$$ISPP(x) + SSPP(x) = 2p + 2$$

has only the solution $x = p + 1$.

References

[1] C. Dumitrescu and V. Seleacu, "Some Notions And Questions In Nimber Theory", Sequences, 37 - 38, http://www.gallup.unm.edu/ smarandache/SNAQINT.txt

[2] T. Tabirca and S. Tabirca, "A New Equation For The Load Balance Scheduling Based on Smarandache f-Inferior Part Function", http://www.gallup.unm.edu/ smarandache/tabirca-sm-inf-part.pdf [The Smarandache f-Inferior Part Function is a greater generalization of ISPP.]

2. PROPOSED PROBLEM

Prove that in the infinite Smarandache Prime Base 1,2,3,5,7,11,13,... (defined as all prime numbers proceeded by 1) any positive integer can be uniquely written with only two digits: 0 and 1 (a linear combination of distinct primes and integer 1, whose coefficients are 0 and 1 only).

Unsolved question: What is the integer with the largest number of digits 1 in this base?

Solution by Maria T. Marcos, Manila, Philippines

For example: 12 is between 11 and 13 then 12=11+1 in SPB. or

$$12 = 1 \times 11 + 0 \times 7 + 0 \times 5 + 0 \times 3 + 0 \times 2 + 1 \times 1 = 100001$$

in SPB. Similarly as

$$402 = 4 \times 100 + 0 \times 10 + 4 \times 1 = 402$$

in base 10 (the infinite base 10 is:

$$1, 10, 100, 1000, 10000, 100000, \ldots).$$

$$0 = 0 \text{ in SPB}$$

$$1 = 1 \text{ in SPB}$$

$$2 = 1 \times 2 + 0 \times 1 = 10 \text{ in SPB}$$

$$3 = 1 \times 3 + 0 \times 2 + 0 \times 1 = 100 \text{ in SPB}$$

$$4 = 1 \times 3 + 0 \times 2 + 1 \times 1 = 101 \text{ in SPB}$$

$$5 = 3 + 2 = 1 \times 3 + 1 \times 2 + 0 \times 1 = 110 \text{ in SPB}$$

$$15 = 13 + 2 = 1 \times 13 + 0 \times 11 + 0 \times 7 + 0 \times 5 + 0 \times 3 + 1 \times 2 + 0 \times 1 = 1000010 \text{ in SPB}$$

This base is a particular case of the Smarandache general base - see [3].

Let's convert backwards: If 1001 is a number in the SPB, then this is in base ten:

$$1 \times 5 + 0 \times 3 + 0 \times 2 + 1 \times 1 = 5 + 0 + 0 + 1 = 6.$$

We do not get digits greater than 1 because of Chebyshev's theorem.

It is only a unique writing.

10 = 7+3, that is it. We do not decompose 3 anymore because 3 belongs to the Smarandache prime base.

$11 = 7 + 4 = 7 + 3 + 1$, because 4 did not belong to the SPB we had to decompose 4 as well.

11 has a unique representation: $11 = 7 + 3 + 1$.

The rule is:

- any number n is between $p(k)$ and $p(k + 1)$ mandatory:

$$p(k) \leq n < p(k + 1),$$

where $p(k)$ is the k-th prime; I mean any number is between two consecutive primes.

For another example:
27 is between 23 and 29, thus 27=23+4, but 4 is between 3 and 5 therefore 4=3+1, therefore 27=23+3+1 in the SPB (a unique representation).

Not allowed to say that $27 = 19 + 8$ because 27 is not between 19 and 29 but between 23 and 29.

The proof that all digits are 0 or 1 relies on the Chebyshev's theorem that between a number n and $2n$ there is at least a prime. Thus, between a prime q and $2q$ there is as least a prime. Thus $2p(k) > p(k+1)$ where $p(k)$ means the k-th prime.

References

[1] Dumitrescu, C., Seleacu, V., "Some notions and questions in number theory", Xiquan Publ. Hse., Glendale, 1994, Sections #47-51; http://www.gallup.unm.edu/ smarandache/snaqint.txt

[2] Grebenikova, Irina, "Some Bases of Numerations", ¡Abstracts of Papers Presented at the American Mathematical Society¿, Vol. 17, No. 3, Issue 105, 1996, p. 588.

[3] Smarandache Bases, http://www.gallup.unm.edu/ smarandache/bases.txt

3. PROPOSED PROBLEM

Let p be a positive prime, and $S(n)$ the Smarandache Function, defined as the smallest integer such that $S(n)!$ be divisible by n. The factorial of m is the product of all integers from 1 to m. Prove that

$$S(p^p) = p^2.$$

Solution by Alecu Stuparu, 0945 Balcesti, Valcea, Romania

Because p is prime and $S(p^p)$ must be divisible by p, one gets that $S(p^p) = p$, or $2p$, or $3p$, etc.

More, $S(p^p)$ must be divisible by p^p, therefore

$$S(p^p) = p \cong p, \text{ or } p \cong (p+1), \text{ or } p \cong (p+2), \text{ etc.}$$

But the smallest one is $p \cong p$ [because $p \cong (p-1)!$ is not divisible by p^p, but by p^{p-1}]. Therefore

$$S(p^p) = p^2.$$

4. PROPOSED PROBLEM

Let $S3f(n)$ be the triple Smarandache function, i.e. the smallest integer m such that $m!!!$ is divisible by n. Here $m!!!$ is the triple factorial, i.e. $m!!! = m(m-3)(m-6)...$ the product of all such positive non-zero integers. For example $8!!! = 8(8-3)(8-6) = 8(5)(2) = 80$. $S3f(10) = 5$ because $5!!! = 5(5-3) = 5(2) = 10$, which is divisible by 10, and it is the smallest one with this property. $S3f(30) = 15, S3f(9) = 6. S3f(21) = 21$.

Question: Prove that if n is divisible by 3 then $S3f(n)$ is also divisible by 3.

Solution by K. L. Ramsharan, Madras, India

Let $S3f(n) = m$.

$S3f(n)!!! = m!!!$ has to be divisible by n according to the definition of this function, i.e. m has to be a multiple of 3, because n is a multiple of 3. In m is not a multiple of 3, then no factor of $m!!! = m(m-3)(m-6)...$ will be a multiple of 3, therefore $m!!!$ would not be divisible by n. Absurd.

5. PROPOSED PROBLEM

Let $Sdf(n)$ represent the Smarandache double factorial function, i.e. the smallest positive integer such that $Sdf(n)!!$ is divisible by n, where double factorial $m!! = 1 \times 3 \times 5 \times ... \times m$ if m is odd, and $m!! = 2 \times 4 \times 6 \times ... \times m$ if m is even. Solve the diophantine equation $Sdf(x) = p$, when p is prime. How many solutions are there?

Solution by Carlos Gustavo Moreira, Rio de Janeiro, Brazil

For the equation $Sdf(x) = p$ =prime, the number of solutions is $\geq 2^k$, where $k = (p-3)/2$. The general solution of the equation $Sdf(x) = p$ =prime is $p \times m$, where m is any divisor of $(p-2)!!$.

Let us consider the example for the Smarandache double factorial function $Sdf(x) = 17$. The solutions are $17 \times m$, where m is any divisor of $(17-2)!!$ which is equal to $3 \times 5 \times 7 \times 9 \times 11 \times 13 \times 15 = (3^4) \times (5^2) \times 7 \times 11 \times 13$ which has $(4+1) \times (2+1) \times (1+1) \times (1+1) \times (1+1) = 120$ divisor, therefore 120 solutions $< 2^7 = 128$.

The number of solutions is not $2^7 = 128$ because some solutions were counted twice, for example: $17 \times 3 \times 5$ is the same as 17×15 or $17 \times 3 \times 15$ is the same as $17 \times 5 \times 9$.

Comment by Gilbert Johnson,
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How to determine the solutions and how to find a superior limit for the number of solutions.

Using the definition of Sdf, we find that: $p!!$ is divisible by x, and p is the smallest positive integer with this property. Because p is prime, x should be a multiple of p (otherwise p would not be the smallest positive integer with that property). $p!!$ is a multiple of x.

a) If $p = 2$, then $x = 2$.

b) If $p > 2$, then p is odd and $p!! = 1 \times 3 \times 5 \times \ldots \times p = Mx$ (multiple of x).

Solutions are formed by all combinations of p, times none, one, or more factors from 3, 5, ..., $p - 2$.

Let $(p-3)/2 = k$ and rCs represent combinations of s elements taken by r.

So:

- for one factor: p, we have 1 solution: $x = p$; i.e. $0Ck$ solution;
- for two factors:

$$p \times 3, p \times 5, .., p \times (p-2),$$

we have k solutions:

$$x = p \times 3, p \times 5, \ldots, p \times (p-2);$$

i.e. $1Ck$ solutions;

- for three factors:

$$p \times 3 \times 5, p \times 3 \times 7, \ldots, p \times 3 \times (p-2); p \times 5 \times 7, \ldots, p \times 5 \times (p-2); \ldots, p \times (p-4) \times (p-2),$$

we have $2Ck$ solutions; etc. and so on: - for k factors:

$$p \times 3 \times 5 \times \ldots \times (p-2),$$

we have kCk solutions.

Thus, the general solution has the form:

$$x = p \times c_1 \times c_2 \times \ldots \times c_j,$$

with all c_j distinct integers and belonging to $\{3, 5, \ldots, p-2\}$, $0 \leq j \leq k$, and $k = (p-3)/2$. The smallest solution is $x = p$, the largest solution is $x = p!!$.

The total number of solutions is less than or equal to $0Ck + 1Ck + 2Ck + \ldots + kCk = 2k$, where $k = (p-3)/2$.

Therefore, the number of solutions of this equation is equal to the number of divisors of $(p-2)!!$.