Available online at www.qu.edu.iq/journalcm
JOURNAL OF AL-QADISIYAH FOR COMPUTER SCIENCE AND MATHEMATICS
ISSN:2521-3504(online) ISSN:2074-0204(print)

The Dynamical Properties of Modified of Bogdanov Map

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ARTICLE INFO

Article history:
Received: 26 /03/2021
Rrevised form: 05 /04/2021
Accepted : 15 /04/2021
Available online: 07 /05/2021

Keywords:

Sensitive dependence on initial conditions,
Lyapunov exponent,
Topological entropy.

ABSTRACT

In the work, we study the general properties of modified of the Bogdanov map in the form $F_{a,m,k}$.

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ y+ay+k\,cosx-kx+mxy \end{pmatrix}$$

We prove it has sensitive dependence on initial conditions and positive Lyapunov exponent with some conditions . So we give estimate of topological entropy of modified Bogdanov map .

MSC. 41A25; 41A35; 41A36.

DOI : https://doi.org/10.29304/jqcm.2021.13.2.790

1. Introduction

Chaos theory is a branch of mathematics concern with studying the properties of deterministic systems that depend on their behavior on a set of elementary conditions,

making their study somewhat complex using traditional mathematical tools. Mathematicians use chaos theory to model these systems in ways Different in order to arrive at a specific mathematical description of it and its behavior depending on all possible initial conditions. The Bogdanov map provides a good approximation to the dynamics of the Poincaré map of periodically forced oscillators, first considered by Bogdanov [5]. Vicente

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Communicated by: *Dr. Rana Jumaa Surayh aljanabi.*

Aboites & etal, studied the dynamic behavior among periodic orbits high periodicity and chaos of Bogdanov map were observed through bifurcation[6]

The Bogdanov map is 2-dimension and discrete dynamical system, its form $F_{a,m,k}$ $\binom{x}{y} = \binom{x+y}{y+ay+kx^2-kx+mxy}$, we changed this form of Bogdanov map to modified Bogdanov map by replacing x^2 to $\cos(x)$ and denoted to $F^1_{a,m,k}$. We find important chaotic properties of it. One of this properties sensitive dependence on initial condition and Lyapunov expanents.

2. Basic Definitions

Any pair $\binom{k}{h}$ for which f $\binom{k}{h} = k,\ g\binom{k}{h} = h$ is called fixed point of 2-D dynamical system [3]. Let V be a subset of R^2 and $v_0 = \begin{bmatrix} x \\ y \end{bmatrix}$ be any element in v Consider $F_{a,m,k}$:V→R^2 a map. Furthermore [4], assume that the first partials of the coordinate maps $F_{a,m,k(1)}$ and $F_{a,m,k(2)}$ of F exist at v_0 is the linear map D $F_{a,m,k}$ (v_0) defined on R^2 by D$F_{a,m,k}$ (v_0)=$\begin{pmatrix} \frac{\partial F^1_{a,m,k}1(v_0)}{\partial x} & \frac{\partial F^1_{a,m,k}1(v_0)}{\partial y} \\ \frac{\partial F^1_{a,m,k}2(v_0)}{\partial x} & \frac{\partial F^1_{a,m,k}2(v_0)}{\partial y} \end{pmatrix}$, For all v_0 in V the determine of D$F_{a,m,k}$ (v_0) is said the Jacobin of $F_{a,m,k}$ at v_0 and is denoted by J=det D$F_{a,m,k}$ (v_0), if $|JF_{a,m,k}(v_0)|$<1then $F_{a,m,k}$ (v_0) is area contracting at v_0 and if $|JF_{a,m,k}(v_0)| > 1$ then $F^1_{a,m,k}$ is area expanding at v_0 [1].

3. Some properties of modified Bogdanov map

In the section, we study modified Bogdanov map from fixed point, diffeomorphism (one- to -one ,onto, invertible, c^∞).Also we determined the contracting and expanding area.

Proposition(3-1)

If k≠0 Then $F_{a,m,k}$ has unique fixed point

Proof

By definition of a fixed point

$F_{a,m,k}$ $\binom{x}{y} = \binom{x+y}{y+ay+k\ \cos(x)-kx+mxy}$, y=0 so k $\cos(x)$ =kx then x=0 therefore p=$\binom{0}{0}$ is unique fixed point.

Proposition(3-2)

Let $F: R^2 \to R^2$ be the $F_{a,m,k}$ then the Jacobin of $F_{a,m,k}$ is $1+a+k$

Proof

$DF_{a,m,k}\binom{x}{y} = \begin{pmatrix} 1 & 1 \\ -k\sin(x)-k+my & 1+a+mx \end{pmatrix}$ $so\ JF_{a,m,k} = det\begin{pmatrix} 1 & 1 \\ -k & 1+a \end{pmatrix} = 1+a+k$

Proposition (3-3)

$F_{a,m,k}$ is area contracting map if $|1+a+k|<1$ and If $|1+a+k|>1$ then $F_{a,m,k}$ is area expanding.

Proof

by proposition (3-2) then $|1+a+k|<1$ hence $k<a<2+k$

therefore the $F_{a,m,k}\begin{pmatrix}x\\y\end{pmatrix}$ is an area contracting map and if $2+k<a<k$ the $F_{a,m,k}$ is an area expanding.

Proposition(3-4)

The eigenvalues of D $F_{a,m,k}\begin{pmatrix}x\\y\end{pmatrix}$ $at\ fixed\ point\ is\ \lambda_{1,2}=\frac{-(a+b)\pm\sqrt{(a^2+8a-4k)}}{2}$

Proof

$Det(DF_{a,m,k}(v)-I\lambda)=det\begin{pmatrix}1 & 1\\ -k & 1+a\end{pmatrix}=(1+a)(k)=0$ then $\lambda_{1,2}=\frac{-(a+b)\pm\sqrt{(a^2+8a-4k)}}{2}$.

Proposition (3-5)

Let $F:R^2\rightarrow R^2$ be modified bogdanov map, then $F_{a,m,k}$ is diffeomorphism.

Proof

1. Let $T(x,y)=(x+y,y+ay+kcosx-kx+mxy)$

$T(1,0)=(1,k\cos(1)-k)$

T(0, 1)=(1, 1+a+k)

Then , $F_{a,m,k}\begin{pmatrix}1 & kcos(1)-k\\ 1 & 1+a+k\end{pmatrix}$ $from\begin{pmatrix}1 & kcos(1)-k\\ 0 & 2a+3k-2kcos(1)\end{pmatrix}$

$F_{a,m,k}$ has a pivot position in every Colum then $F_{a,m,k}$ is one to one and has a pivot position in every row then $F_{a,m,k}$ is onto.

2. $F_{a,m,k}$ is C^∞ : $F_{a,m,k}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x+y\\ y+ay+kcos(x)-kx+mxy\end{pmatrix}$

Note that $\frac{\partial f1(x,y)}{\partial x}=1,\quad \frac{\partial^2 f1(x,y)}{\partial x^2}=0\ldots\ldots\ldots.\frac{\partial^n f1(x,y)}{\partial x^n}=0\ for\ all\ n\in Nand\ n\geq 2$

$$\frac{\partial f1(x,y)}{\partial y}=1,\quad \frac{\partial^2 f1(x,y)}{\partial y^2}=0\ldots\ldots\ldots.\frac{\partial^n f1(x,y)}{\partial x^n}=0\ \ for\ all\ n\in Nand\ n\geq 2$$

$$\frac{\partial f2(x,y)}{\partial x}=-ksin-k+my\ (x),\frac{\partial^2 f2(x,y)}{\partial x^2}=kcos(x)\quad for\ all\ n\in N$$

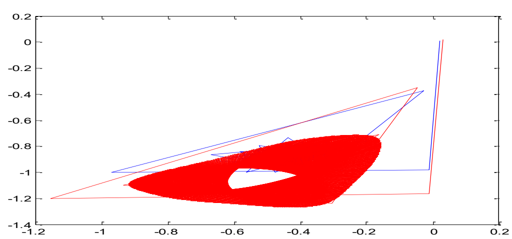
$$\frac{\partial f2(x,y)}{\partial y}=1+a+mx,\frac{\partial^2 f2(x,y)}{\partial y^2}=0\ldots\ldots\ldots.\frac{\partial^n f2(x,y)}{\partial x^n}=0\ for\ all\ n\in Nand\ n\geq 2$$

Then the partial derivatives continuous and exist then $F_{a,m,k}$ is c^∞

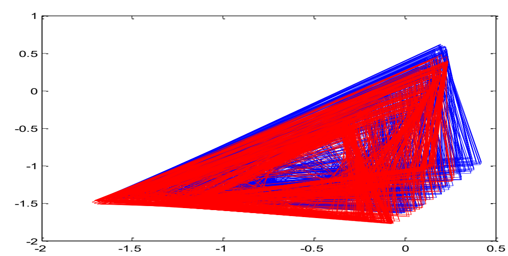
4. Sensitive Dependence on initial conditions (S.D.I)

Definition (4-1)

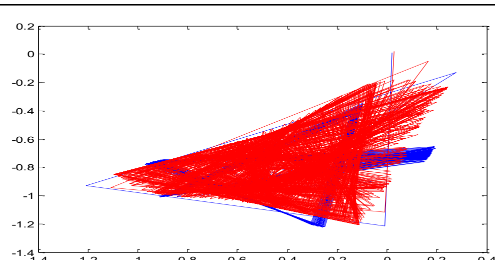
Let (X, k) be a metric space. A map F: (X, k) →(X, k) is said to be sensitive dependence on initial conditions (S. D. I) if there exit $\epsilon > 0 \ni$ for any $x_0 \in X$ and any open set U⊆ X containing x_0 there exists $y_0 \in U$ and $n \in Z^{+}$ *such that* $d\,(f^n(x_0), f^n(y_0)) >\in$ that is $\exists \in > 0,\ \forall\, x > 0, \exists\, y \in B_0(x),\ \exists\, n \in N,$ $d\big(f^n(x_0),\ f^n(y_0)\big) >\in$.



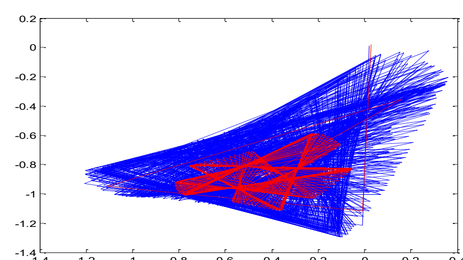
x1 (1) = 0.02 ;y1 (1) = 0.01;a1=-1.00005;m1=-1.20004;k1=-1;and x2 (1) = 0.03 ;y2 (1) = 0.02;a2=-1.000004;m2=-1.100009;k2=-1.2;



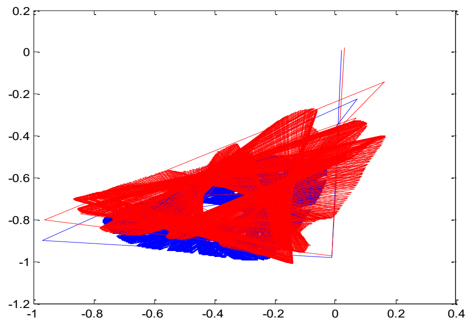
x1 (1) = 0.02 ;y1 (1) = 0.01;a1=-1.00005;m1=1.20004;k1=-1.4;andx2 (1) = 0.03 ;y2 (1) = 0.02;a2= 1.000004;m2=-1.100009;k2=-1.5;



x1 (1) = 0.02 ;y1 (1) = 0.01;a1=-1.25;m1=-1.4003;k1=-1.24;and x2 (1) = 0.03 ;y2 (1) = 0.02;a2=-1.18;m2=-1.5004;k2=-1.15;



x1 (1) = 0.02 ;y1 (1) = 0.01;a1=-1.25;m1=-1.501;k1=-1.24;and x2 (1) = 0.03 ;y2 (1) = 0.02;a2=-1.18;m2=-1.203;k2=-1.1




```
x1 (1) = 0.02 ;y1 (1) = 0.01;a1=-1.10000;
m1=-1.40000; k1=1.0000024;andx2 (1) = 0.03;
y2 (1) = 0.02; a2=-1.20000; m2=-1.60000;k2=-
1.0000015;
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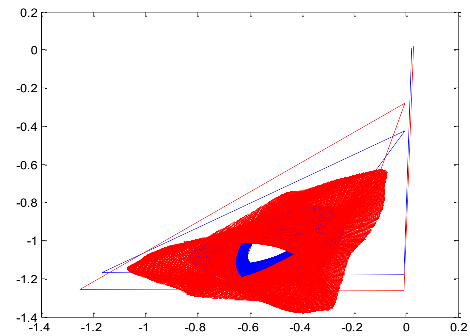


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x1 (1) = 0.02 ;y1 (1) = 0.01;a1=-1.025; m1=-
1.04; k1=-1.2;  and x2 (1) = 0.03 ;
y2(1)=0.02; a2=-1.034; m2=-1.09; k2=-1.3;
```

5.Lyapunov exponent

The Lyapunov play role important in the field of calculus and integration and control theory, as it has many life applications in the field of Physics, Medicine, Engineering, Space Science and other Scientific fields.

Proposition(5-1)

For all $\binom{x}{y}$ then $F_{a,m,k}$ $\binom{x}{y}$ has positive Lyapunov exponent .

Proof

$X=\binom{x}{y} \in R^2$, the Lyapunov exponent of $F_{a,m,k}$ is given by

$X_1(\binom{x}{y}, v1)=\lim_{n\to\infty}\frac{1}{n}\ln\left\|DF^n_{a,m,k}\binom{x}{y}, v1\right\|$ by (3-4) we have $F_{a,m,k}$ has two eigenvalues $\ni |\lambda_1| = \frac{1}{|\lambda_1|}$. If $|\lambda_1| < 1$ then $X_1(\binom{x}{y}, v1)=\lim_{n\to\infty}\frac{1}{n}\ln\left\|(DF^n_{a,m,k}\binom{x}{y}, v1)^n\right\| > \ln\left\|\frac{-(a+b)+\sqrt{(a\text{^}2+8a-4k)}}{2}\right\|$, By hypothesis $L_1>0$ so if $|\lambda_1| < 1$ then $\lim_{n\to\infty}\frac{1}{n}\ln\left\|(DF^n_{a,m,k}\binom{x}{y}, v1)^n\right\| < \ln\left\|\frac{-(a+b)-\sqrt{(a\text{^}2+8a-4k)}}{2}\right\|$, SO

Lv= max {x1(x,v1),x2(x,v2)} hence the Lyapunov exponent of modified Bogdanov map is positive .

Remark (5-2)

If the Lyapunov exponents is positive , then the sensitive dependence on initial condition exists. Therefore $F_{a,m,k}$ has sensitive dependence on initial condition and Lyapunov exponent.

6. Topological entropy

In 1965, Adler, Konheim and McAndrew introduced the concept of topological entropy. Their definition was modeled after the definition of the Kolmogorov- Sinai, put The second definition of before Dinaburg and Rufus Bowen and who explained the meaning of a topological entropy for a system given by an iterated map. Topological entropy is the measure of the complexity of the system and the topological entropy is a non-negative real number[5].

We give estimate of topological entropy of continuous map .

We recall the theorem (3.35) in [2] by

Theorem(6-1)

Let $f: R^n \to R^n$ be a continuous map then

$$h_{top}(f) \geq \log|\lambda|$$

where λ is the largest eigenvalue of D f(v), where $v \in R^n$.

We give estimate of topological entropy of modified Bogdanov map

Proposition (6-2)

If $|\lambda_1| > |\lambda_2|$ therefore $h_{top}(F_{a,m,k}) \geq \log|\lambda_1|$

Proof

By proposition (3-4)and by hypothesis hence

$h_{top}(F_{a,m,k})\,) \geq \log|\lambda_1|$ then $h_{top}((F_{a,m,k}) \geq \log\left|\frac{-(a+b)+\sqrt{(a^2+8a-4k)}}{2}\right|$

Remark(6-3)

In the same way if $|\lambda_2| > |\lambda_1|$ then $h_{top}((F_{a,m,k}) \geq \log|\lambda_2|$.

We recall the theorem (3.35) in [2] by :

Theorem (6.4)

Let $f: R^n \to R^n$ be

$$h_{top}\,(F_{a,m,k}(x)) \leq \log max_{x \in R^n}\, max_{L \in T_{XR^n}} \left|\det\left(DF_{a,m,k}(x)\middle|L\right)\right|$$

Proposition(6-5)

The upper estimate of topological entropy of modified Bogdanov map.

Proof

by theorem (3.35) on [2] we get

$$\mathrm{Htop}(\mathrm{Fa,m,k}) \le \log \max_{x\in R^2} \max_{L\in T\,xR^n} \left|\det\left(\left(D\left(\mathrm{F}_{\mathrm{a,m,k}}(\mathrm{x})\right)\middle|L\right)\right|\right. \le \log \max_{x\in R^2} \max_{L\in T\,xR^n} |1+a+k| \le \log|1+a+k|$$

References

[1] Gulick D., ”Encounters with Chaos”, McGraw-Hill, Inc., New York,1992.

[2] Katok S. R., "The Estimation from above for the Topological Entropy of a Diffeomorphism. Global Theory of Dynamical Systems", Lect. Notes Math. 819, pp. 258–264, 1980.

[3] Rasolk. A. H., "On Henon map", College of Education University of Bablyon, January, 2006.

[4] Bogdanov, R.I., "Bifurcation of the limit cycle of a family of plane vector fields" , Trudy Sem. Petrovsk. 2, 23. In Russian: the English translation is Sel. Math. Sov. Vol.1, No.4, PP.373-88,1981.

[5] Simonsen Jakob Grue.," On The Computability of The Topological, Discrete Mathematics and Theoretical Computer Science", Vol.8, (2006), pp83-96.

[6] Vicente Aboites &etal.,"Bogdanov map for Modelling a phase- conjugated Ring Resonator",vol.21(4),pp384.

[7] Mohsin Al-juboori, A. (2017). A novel Approach to improve biometric authentication using Steerable-Locality Sensitive Discriminant Analysis. Journal of Al-Qadisiyah for Computer Science and Mathematics, 9(1), 61-70. Retrieved from https://qu.edu.iq/journalcm/index.php/journalcm/article/view/16.

[8] Jabbar Kadhim, I., & Kadhim Jebur, S. (2017). Study of Some Dynamical Concepts in General Topological spaces. Journal of Al-Qadisiyah for Computer Science and Mathematics, 9(1), 12-22. Retrieved from https://qu.edu.iq/journalcm/index.php/journalcm/article/view/9.

[9] Arif, G., Wuhaib, S., & Rashad, M. (2020). Infected Intermediate Predator and Harvest in Food Chain. Journal of Al-Qadisiyah for Computer Science and Mathematics, 12(1), Math Page 120 -. https://doi.org/10.29304/jqcm.2020.12.1.683.

[10] Gaze Salih Al-Khafajy, D., & Abdulridha Mutar, M. (2017). On DM- Compact Smarandache Topological Semigroups. Journal of Al-Qadisiyah for Computer Science and Mathematics, 8(2), 25-33. Retrieved from https://qu.edu.iq/journalcm/index.php/journalcm/article/view/30.

[11] Mahdy Ali, S. (2017). Redefine Fuzzy Topological Vector Space by using Michalek’s Fuzzy Topological Space. Journal of Al-Qadisiyah for Computer Science and Mathematics, 7(2), 61-65. Retrieved from https://qu.edu.iq/journalcm/index.php/journalcm/article/view/64.

[12] AL-Taï,¢aiiA., A.AL-Mayahi, N., & Youssif Hussein, B. (2017). On Equivalent Martingale Measures on Lp-space. Journal of Al-Qadisiyah for Computer Science and Mathematics, 2(1), 6-13. Retrieved from https://qu.edu.iq/journalcm/index.php/journalcm/article/view/207.

[13] Abdul khalik Alkhafaji, M., & Himza Almyaly, A. (2017). Countability Axioms in Smooth Fuzzy Topological Spaces. Journal of Al-Qadisiyah for Computer Science and Mathematics, 7(1), 41-52. Retrieved from https://qu.edu.iq/journalcm/index.php/journalcm/article/view/85.

[14] MAHDI ALI, S. (2017). A new kind of Fuzzy Topological Vector Spaces. Journal of Al-Qadisiyah for Computer Science and Mathematics, 3(2), 63-72. Retrieved from https://qu.edu.iq/journalcm/index.php/journalcm/article/view/275.

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