Professor Şelariu's Supermathematics

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ABSTRACT

This article is a brief review of the book "Supermathematics. Bases", Vol. 1 and Vol. 2, 2nd edition, 2012, which represents a new field of research with many applications, initiated by Professor Mircea Eugen Şelariu. His work is unique in the world scientific literature, because it combines centric mathematics with eccentric mathematics.

INTRODUCTION

Supermathematics (SM) is a reunion of the familiar, ordinary mathematics, which was called in this paper centric mathematics (CM), to be distinguished from the new mathematics, called eccentric mathematics (EM). That is **SM = CM ∪ EM**.

For each point in the plane, which can be placed in an eccenter **E (e, ε)**, we can say that there is / there appears a new **EM**. Thus, an infinity of **EM** corresponds to a single **CM**; On the other hand, **CM = SM (e = 0)**;

Thus, **SM** indefinitely multiplies all known circular / trigonometric functions and introduces a host of new circular functions (*aex*, *bex*, *dex*, *rex*, etc.), much more important than the old ones and thereby, finally, indefinitely multiplies all known mathematical entities and introduces several new entities.

It was observed that **CM** is proper to **linear, perfect, ideal** systems and **EM** is proper to **nonlinear, real, imperfect** systems;

Therefore, with the apparition of **SM** the boundary between linear and nonlinear, between the ideal and the real, between perfection and imperfection disappeared;

SM marks out the linear eccentricity ***e*** and the angular one **ε**, the polar coordinates of the eccenter **E (e, ε)** as new dimensions of space: dimensions of its **formation** and **deformation**;

SM could have occured more than 300 years ago, if Euler, when defining trigonometric functions as direct circular functions, hadn't chosen **three superposed points**, which impoverished maths: **Pole E** of a half-line, **the center C** of the trigonometric circle (unit) and the origin **O** (0,0) of a reference point / right rectangular system;

SM occurred when pole **E** was expelled from center and called **eccenter**.

The following functions appear after the possible combination of the three points:

- **CCF centric circular** (**FSM - CC**)→ if **C≡ O ≡ E;**
- **FSM eccentric circular (FSM - EC)→** if **C ≡ O≠E;**
- **FSM elevated circular (FSM - ELC)→** if **C ≠O ≡ E;**
- **FSM exotic circular (FSM - EXC)→** if **C≠O ≠E.**

Among the **new entities,** there is also a host of new closed curves, occurring in the **continuous transformation** of the circle into a square (called quadrilobes / cvadrilobes), of the circle into a triangle (trilobes). In 3D, these continuous changes are of sphere into cube, of sphere into prism, of cone into pyramid, etc.

These continuous changes made possible the apparition of new 3D hybrid figures as: sphere-cube, cone-pyramid, pyramid-cone, etc.

In this work, by replacing the circle with a quadrilobe were defined the quadrilobe functions and by replacing it by a trilobe were defined the trilobe functions.

The book introduces new mathematical methods and techniques as well, such as:

- Integration through differential dividing;
- The hybrid analytic-numerical method → Determining K(k) with 15 accurate decimals;
- The method of moments separation → The kinetostatic method, extremely simple and exact, which reduces **d'Alambert method,** requiring the solution of some equations of equilibrium systems, to a simple elementary geometry problem;
- The eccentric circular movement of fixed and mobile point eccenter;
- The rigorous transformation of a polar diagram of pliancy into a circle;
- Solving some vibration systems of nonlinear static elastic features;
- Introduction of quadrilobic / cvadrilobic vibration systems.

DESCRIPTION OF WORK

Ch.1. INTRODUCTION

It is presented a short history of the SUPERMATHEMATICS discovery in connection with the research undertaken by the author at the University of Stuttgart, between 1969 - 1970, at the Institute and Department of Machine Tools of Prof. **Karel Tuffentsammer**, in the group of "Machine Tool Vibration".

Moreover, it is shown that the great mathematician **Leonhard Euler**, in defining trigonometric functions as circular functions, choosing <u>three superposed points</u> [**Origin O (0, 0)**, **circle center**, called at that time <u>trigonometric</u> circle **M** (0, 0), now renamed as <u>unit circle</u> and **the Pole** of a half-line **P(0,0)**] impoverished mathematics from the start. Mathematics itself remained extremely poor, with a single set of periodic functions (sinα, cosα, tanα, cotα, secα, cscα, etc.) and, therefore, generally, with unique mathematical entities (line, circle, square, sphere, cube, elliptic integrals, etc).

Through the mere expulsion of the pole **P** and called, therefore, **eccenter E(e, ε)** for any circle C(O, R) of radius R or marked by S (s, ε) for the unit circle CU(O, 1), for each point on the plane of the unit circle, in which a pole/eccenter S (s, ε) can be placed, a set of circular / trigonometric functions is obtained, called **eccentric**.

They were called **eccenters** because they were expelled from center O.

And on this basis, we obtain an infinite number of new mathematical entities, called **eccentric**, previously non-existent in mathematics (the crook line as an extension / generalization of the line; the eccentric circular or quadrilobes that complement the space between circle and square or, in other words, perform a continuous transformation of the circle into a perfect square, the eccentric sphere, which continually transforms the sphere into a perfect cube, cone-pyramid, sphere-cube, etc.).

The chapter ends with an overview of the main contributions that the new complements in mathematics, collectively called SUPERMATHEMATICS, bring in mathematics, informatics, mechanics, technology and other fields.

Ch. 2. DIVERSIFICATION OF PERIODIC FUNCTIONS

Seizing upon the existence of some "white spots" in mathematics, a number of great mathematicians have tried, in the past as well as today, and managed to partially rectify these shortcomings. Their efforts deserved to be reviewed, along with the discovery of supermathematics, even if they are not of the same broad reach, and some of them were incompletely presented, in a more sketchy way, were shaped by the author to a final form, compatible with mathematical programs.

It is about **Valeriu Alaci**'s quadratic functions and diamond functions, **M. Ovidiu Enulescu's** polygonal functions, **Malvinei Florica Baica** and **Mircea Cârdu**'s transtrigonometric functions, **Eugen Vişa**'s pseudo hyperbolic functions, all mathematics teachers and fellow-citizens with the author.

In the same city, Timişoara, on November 3, 1823, a young engineer officer at Timişoara garrison, **Ianos Bolyai**, (he was then 21), was sending his father, **Farkas Bolyai**, professor of mathematics at the college of Targu-Mures a touching letter. He wrote, among other things: "***From nothing I've created a new world***". It was the world of non-euclidean geometry.

Likewise, through the reunion of the ordinary centric mathematics (**CM**) with the new eccentric mathematics (**EM**) **the supermathematics** was created (**SM = CM $\cap$ EM**). It infinitely multiplies all **unique** entities of **CM** and, in addition, introduces new mathematical entities previously non-existent (cone-pyramid, sphere-cube, etc.).

In this case, it can be asserted that "**from nothing**" there were created new mathematical entities such as, for example, supermathematical eccentric circular functions (FSM-EC) eccentric amplitude aexθ and Aexα, beta eccentric bexθ and Bexα, radial eccentric rexθ and Rexα, eccentric derivatives dexθ and Dexα, cone-pyramids, square, triangular and other forms of cylinders, etc.

But it can also be asserted that from a single mathematical entity, which exists in **CM**, there were created infinite entities of the same kind in **EM** and, implicitly, in **SM**, or that **SM infinitely multiplies all CM entities**.

The involute functions of **George (Gogu) Constantinescu**, the creator of sonics, are particularly highlighted, the Romanian cosine Corα and the Romanian sine Sirα, which are unfortunately too little known like the inclined trigonometric functions of **Dr. Bihringer**, unfairly forgotten.

Ch.3. ADDITONS AND CORRECT REDEFINITIONS IN CENTRIC MATHEMATICS

Octavian Voinoiu's work, published by Nemira, "**INTRODUCTION IN SIGNADFORASIC MATHEMATICS**" revealed a number of mathematical entities, of first importance, wrongly introduced in mathematics, in centric mathematics (**CM**).

Supporter of **Sophocle**'s principle: "Errare humanum est, perseverare diabolicum", the author considered that, before presenting the new mathematical complements, it is strictly necessary to partially highlight and maybe correct the wrongly introduced entities, existing in **CM**.

In this respect, a simple example is the wrong definition of the sign of a fraction and, as a result, of the tangent as being the ratio tanα = $\frac{sin\propto}{cos\propto}$, while the correct definition is tanvα = $\frac{sin\propto}{Abs[cos\propto]}$, which has been called **Voinoiu** centric tangent. In this way, the new **FSM-EC Voinoiu** eccentric tangent texvθa could be "ab initio" properly defined, as the ratio between the eccentric sine sexθ and cosine cexθ, that is texvθ =$\frac{\text{sex}\theta}{Abs[\text{cex}\theta]}$.

Moreover, a number of entities that appeared in **EM** and consequently in **SM**, had no equivalents in **CM**. They are the most significant **FSM-EC**, the eccentric radial periodic function **rexθ**, a true "king" function and the eccentric derivative **dexθ,** which expresses alone the second order transfer function or the speed transmission ratio and / or all plane mechanisms turation ratio.

It was determined that the equivalents of these **FSM-EC** in **CM** are the centric radial functions **radα = $e^{i\alpha}$** and the eccentric derivative **derα = $e^{i(\alpha+\pi/2)}$**, which are exactly the **Euler-Cotes** functions or the phasors of centric radial directions, from the center O(0,0), respectively the phasor previously dephased with $\frac{\pi}{2}$ or the tangent phasor to the unit center in point W(α, 1), of polar coordinates, with the pole in origin O(0, 0).

At the end of this chapter, was presented a particularly important and original application on "The rigorous transformation of a polar diagram of pliancy into a circle", which comes to correct the incomplete studies on the most studied oscillating system in the scholarly literature.

Part 1

SUPERMATHEMATICAL ECCENTRIC CIRCULAR FUNCTIONS (FSM-EC)

It is known that in mathematics, the functions may be defined virtually on any closed or open plane curve, as well as direct functions and inverse function.Thus:

- On RIGHT TRIANGLE → Trigonometric functions
- On OBTUSE TRIANGLE →**Bihringer** inclined trigonometric functions
- On TRILOBES → **Şelariu** trilobe functions
- On CIRCLE → **Euler** circular functions
- On ELIPSE → **Jacobi** elliptic functions
- On SQUARE (rotated with $\frac{\pi}{4}$) → **Alaci** quadratic functions
- On RHOMBUS → **Alaci** diamond functions
- On CVADRILOBES → **Şelariu** cvadrilobe functions
- $\begin{cases} \text{On CVADRILOBES}\left(rotated\ with\ \frac{\pi}{4}\right) \rightarrow Transtrigonometric\ functions \\ On\ ASTROIDS \rightarrow Infratrigonometric\ functions \\ On\ SPIRALS\ \rightarrow Spiral\ paratrigonometric\ functions \end{cases}$

  → **MalvinaBaica - Mircea Cârdu**
- On POLYGON → **Enulescu** polygonal functions
- On LEMNISCATE → **Marcusevici** lemniscate functions

- On EVOLVENT → **Gogu Constantinescu** involute functions
- On HYPERBOLA ASYMPTOTES → **Eugen Vişa** pseudo-hyperbolic functions
- **On EQUILATERAL HYPERBOLA →** Hyperbolic functions

And there may be other such functions.

In this paper, were presented mainly the supermathematical functions (**FSM**) defined on the circle.

Part 1.1 SUPERMATHEMATICAL ECCENTRIC CIRCULAR FUNCTIONS OF ECCENTRIC VARIABLE

Euler's three superposed points (**Pole S (s, ε)** and the **center** of the unit circle **C (c, φ)** in **origin O** (0, 0) of a fixed point) may be separated in the following three ways, for each way of separation being proper other types of supermathematical functions (**SF**), as follows:

C(0,0) ≡ **O**(0, 0) ≡ **S(0,0)** → **CCF – Centric Circular Functions**

C(0,0) ≡ **O**(0, 0) ≠ **S(s,ε)** → **FSM-EC** → **Supermathematical Functions – Eccentric Circular**

C(c,φ) ≠ **O**(s, ε) ≡ **S(s,ε)** → **FSM-E$_L$C** → **Supermathematical Functions – Elevated Circular**

C(c,φ) ≠ **O**(0, 0) ≠ **S(s,ε)** → **FSM-ExC** → **Supermathematical Functions – Exotic Circular**

All supermathematical functions can be, in their turn, of eccentric variable **θ** and centric variable **α**. The first ones are continuous functions only for an eccenter **S** inside the unit circle / disk, that is a numerical linear eccentricity s ≤ 1.

The functions of centric variable are continuous for S placed anywhere in the plane of the unit circle, that is for s ∈ [0, ∞].

By intersecting the unit circle with a line ($\mathbf{d} = \mathbf{d}^+ \cap \mathbf{d}^-$) and not only with the positive half-line ($\mathbf{d}^+$), at the urge of some talented and genuine mathematicians as PhD. **Horst Clep**, **eccentric trigonometry** or **FSM-EC** was brought in agreement with the differential geometry that operates with lines. Therefore, all **FSM-EC** have two determinations: a **main** one, marked by index **1**, or without index, when other determinations are not used and no confusion may occur, resulting from the intersection of the unit circle with the positive half-line $\mathbf{d}^+$ and a **secondary** one, marked by index **2**, resulting from the intersection of the unit circle with the negative half-line $\mathbf{d}^-$.

For eccenter **S**, located outside the unit circle (**s** >1), four determinations appear, of which the intersection of the circle with $\mathbf{d}^+$ generates the first two, by indices **1** and **2**, and the intersection with $\mathbf{d}^-$, for indices 3 and 4, are obtained from the relations for determinations 1, respectively 2, for a variable **θ**, previously dephased with **π** , that is **θ** → **θ + π**.

In Part 1.1 of this work are mainly presented / approached **FSM-EC** of eccentric variable **θ**, with predilection for the numerical linear eccentricity **s ≤ 1** and angular eccentricity **ε = 0**.

There are reviewed and graphically defined, on the unit circle, the main **FSM-EC** which will be subject to a future approach.

Some **FSM-EC** are dependent on origin **O(0,0)** of the reference system / fixed point, while others are independent of it. The description of **FSM-EC** begins in Ch. 4 with a function that is independent of the origin of the polar or rectangular fixed point which underlies the susequent definition and other **FSM-EC**.

Ch. 4 RADIAL ECCENTRIC FUNCTION rex θ AND SOME OF ITS IMPORTANT MATHEMATICAL APPLICATIONS

FSM-EC which the work begins with is the radial eccentric function of eccentric variable **$rex_{1,2}\theta$**, the most important periodic function, a true "**king function**", as it was called by PhD. **Octav Em. Gheorghiu**, because it expresses the distance in plane between two points in polar coordinates: **$W_{1,2}$** on the unit circle UC(O, 1), at the intersection with line **d** to the **eccenter S (s, ε)**. Therefore, this function can express by itself the equations of all known plane curves, also called **centric** and of many new curves, which appeared along with **SM**, called **eccentric**.

Note: The expressions of $rex_{1,2}\theta$ are the solutions for algebraic equations of 2^{nd} degree that facilitate solving the inequalities of 2^{nd} degree.

Then, there are defined and summarized, with their applications, the following supermathematical functions.

Ch. 5 OTHER MATHEMATICAL AND TECHNICAL APPLICATIONS OF THE RADIAL ECCENTRIC FUNCTION Rex θ

No matter how exact, the determination of a calculus relation of the complete elliptic integral **K(k)** with at least 15 accurate decimals, which led to the new hybrid numerical-analytic methods of calculation (A version of the **Landen method** of the arithmetic-geometric mean is a sheer numerical method, which gives **numerical value**, while **the new method** (let's call it Şelariu) gives a simple analytic calculus relation).

Ch. 6 ECCENTRIC DERIVATIVE FUNCTION dex θ AND SOME MATHEMATICAL AND TECHNICAL APPLICATIONS

The expression of this function is the general expression of the movement ratio (speed, turation) of **ALL** known plane mechanisms.

It expresses the speed of a point on the circle in **eccentric circular motion (ECM)** a generalization of the centric circular motion.

Ch. 7 QUALITY ANALYSIS OF THE PROGRAMMED MOVEMENT WITH SUPERMATHEMATICAL FUNCTIONS

CH. 8 THE METHOD OF FORCES AND MOMENTS SEPARATION

It provides a simple and accurate solution for all mechanical systems required by plane forces or reducible to them (elastostatics) avoiding the need to solve some systems of equilibrium equations using **d'Alambert** method.

The 2^{nd} volume of **"SUPERMATHEMATICS. BASES"** continues with Ch. 12 entitled **"INTERGRALS AND ECCENTRIC ELLIPTIC FUNCTIONS"**. It is preceded by a table regarding **"THE ACTUAL SITUATION OF SUPRMATHEMATICS"** and **"THE LIST OF THE NEW MATHEMATICAL FUNCTIONS INTRODUCED BY THIS WORK"**, those introduced in mathematics, which the author called Centric Mathematics **(CM)** and in mathematics, in general, through the two volumes regarding supermathematics **(SM).** There are presented 60 new symbols for functions, introduced by the author in mathematics, through his work on supermathematics. And there were presented only the main functions, such as eccentric elliptic cosine and sine, **ceex**, **seex**, quadrilobe/(cvadrilobe) cosine and sine, **coq** and **siq**, but not the compound functions, such as tangent, cotangent, secant, cosecant. Yet, **Voinoiu tangent** tanvθ =$\frac{sin\theta}{cos\theta}$, the quadrilobe (cvadrilobe) tangent taqθ = $\frac{siq\theta}{coq\theta}$, etc., the derivative functions, as well as the the derivatives of the mentioned functions are presented.

And only this quantitative observation can reveal a lot of the qualities of this encyclopedic work, which is surprising and unique in the world literature, as it is its name of **SM**, from the moment of publication with this content, in 1978, and with this title, in 1993, as it results from the references attached to this paper.

From the first moment, the reader is impressed by the richness of the explanatory drawings, made with mathematical programs, using exactly **the supermathematical functions FSM** discovered by the author, as well as the numerous charts presenting the familes of new functions described in the work. For their intrinsic beauty, but also to complete the forms of the functions in a family, numerous families of **SM functions** in **3D** are also presented.

Here and now is where to quote **Ioan Ghiocel**, who prefaced the 2nd volume: "Do not wonder when Prof. M. E. Şelariu, under the pressure of inflection and folds of thoughts, brings together words that have not stood alongside from the foundation of the world, such as ***linear viscous damping circle, elevated functions, exotic functions, the line defined as a confluent of the crook line***, etc... !"

If, in the 1st vol., there were introduced particularly the eccentric circular supermathematical functions, abbreviated by the author as **FSM-EC**, of which we mention the functions **aex, bex, dex, cex, sex, rex, tex, ctex,** in the 2nd volume, **Ch. 12**, there were introduced new eccentric elliptic integrals of the first kind and of the second kind, generalizing the centric elliptic integrals, which they may represent, for numerical linear eccentricity **s = 0**, that is if the eccenter **S(s, ε)** overlaps the origin **O(0,0)** of the system of coordinates or of the fixed point **xOy**.

At the same time, there are presented eccentric elliptic, hyperbolic and parabolic functions, in terms of the classical known variables, but also in terms of arc of a unit circle, common tangent to the equilateral hyperbola, unit ellipse and to the parabola, in their peak. Finally, there are presented the centric elliptic, hyperbolic and quadratic functions, in terms of the arc of the unit circle previously mentioned, a unique case in the centric mathematics literature.

The author called them "***functions on cones with common peak***".

Chapter 13 is dedicated to the centric functions and to the eccentric **self-induced** ones, of the form sin[sin[sin[sin[sin[sin[…[sinx]]]]]]]]] or cex[cex[cex[cex[…[cex[θ]]]]]]]] and to the **induced** functions of the form [cos[sin[sin[tan[tan[cos[sin[cos[tan[...sin[x]]]]]] or cex[sex[sex[tex[tex[cex[sin[cos[tex[…sex[θ]]]]]].

There are also presented the derivatives of the induced and self-induced functions, centric and eccentric, as well as the derivatives of the **Voinoiu** centric and eccentric circular functions, initially presented in the first volume, as a necessary correction for the tangent and cotangent functions, wrongly introduced in mathematics, as the great Romanian mathematician **Octavian Voinoiu** demonstrated in his book "**INTRODUCTION IN SIGNADFORASIC MATHEMATICS**".

To make a difference between **Voinoiu** trigonometric functions it was necessary to determine the derivative of the function **Abs[f(x)]**, non-existent derivative in the scholarly literature. The author demonstrates (p. 73) that the derivative of this function is $\frac{d}{dx}[f(x)] = Sign[f(x)]\,\frac{d}{dx}[f(x)]$.

Chapter 14 is dedicated to eccentric hyperbolic functions. First the eccentric hyperbolas are presented and, especially, the eccentric rectangular hyperbola, as well as other centric and eccentric exponential function of the eccentric variable θ and the geometric definition of the centric and eccentric hyperbolic functions. Beside the classical hyperbolic functions, also known in centric mathematics **(CM)** such as cosine - **cexh** -, sine - **sexh** -, tangent - **texh** -, etc. eccentric hyperbolic, there are also presented functions which appeared at the same time with **FSM-EC,** as eccentric hyperbolic amplitude - **aexh** -, eccentric hyperbolic radial - **rexh** -, eccentric hyperbolic derivative - **dexh** - etc.

For the hyperbolic functions there were also presented the elevated hyperbolic cosine (celh) and sine (selh). In the conclusion of this chapter new geometric objects are presented. They are expressed with the help of these functions, newly introduced in mathematics.

Chapter 15 is dedicated to **FSM-EC** of centric variable α, marked by the author with capital letters (Aex, Bex, Cex, Dex, Rex, Sex, Tex, etc.) to be distinguished from those of eccentric variable θ (aex, bex, cex dex, rex, sex, tex, etc.). The chapter begins with the presentation of the explanatory drawings for defining **FSM-EC** in the case of an eccenter **S**(s, ε) placed inside the unit disk, i.e. inside the unit circle, and the case of the eccenter **S** placed outside it is presented separately.

FSM-EC bexθ and Bexα of numerical linear eccentricity s = 1, with their graphics in symmetrical sawteeth, respectively, asymmetric, were named by the author ***Octav Gheorghiu triangular functions*** in memory and honor of PhD **Octav Em. Gheorghiu**, successor of PhD **Alaci Valeriu** to the board of the Department of Mathematics at "Traian Vuia" Polytechnic Institute of Timisoara. Just as, in honor of the mathematician PhD Florentin Smarandache, the step functions, obtained with the help of **FSM-EC,** were called ***Smarandache step functions***.

In this chapter are outlined, without any doubt, the advantages of expressing some special periodic functions, triangular, quadratic, rectangular, step, etc. with the help of **FSM-CE**, which expresses them exactly, and with **FSM-EC** with only two simple terms, compared with their approximate expression by belaying in various series. Here as well, are presented the solutions of an undamped system of variable amplitudes, expressed by bexθ function, of the differential equation $\ddot{\Delta\varphi} + v_0^2 sin\varphi = \varphi_0 v_0^2 sin v_0 t$.

In Figure **15.28** you can find the drawings of the engine skotch yoke and engine slider crank and some **FSM-CE** that can be expressed by these mechanisms.

A new method of integration, which appeared due to **FSM-EC**, is presented in **Chapter 16**.

It is called ***"Method of integration through differential dividing"*** and it is based on dividing the variable θ in variables α and β, according to the **FSM-EC** known relationship: **θ = α + β**, which gives the differential **dθ** the possibility to divide, in its turn, in **dα** and **dβ**, i.e. **dθ = dα + dβ**.

In this way, a series of integrals, solvable by the residue theorem in the complex plane, can be solved directly and much easier, as illustrated through the applications in this chapter. One of the applications is completed together with PhD. Math. Florentin Smarandache and it was previously presented, separately, in an article.

Since at $\theta = \alpha = 0$ and for an angular eccentricity $\varepsilon = 0$, regardless of the numerical linear eccentricity value **s ∈ [-1, 1]** we obtain β = **bexθ** = arcsin [s.sin(θ - ε)] = 0 as well as for $\theta = \alpha = \pi$, the integration between limits 0 and π as well as between limits 0 and 2π result extremely conveniently. In this respect, the 8 applications presented in the paper are eloquent.

FSM-EC bexθ, described and noted in this chapter as **βsexθ** can also express the solutions of various nonlinear vibrating systems, subject of **Ch.17**.

There are presented the functions **bexθ = βsexθ** and **βcexθ** = arcsin[s.cos (θ - ε)] for an eccenter **S(s** ∈ [-1, +1], ε = 0) or **S(s** ∈ [0, +1] , ε = 0 V π), which is the same thing, as well as their derivatives as their geometric significance (**Fig.17.2**).

Since the wronskian matrix given by the solutions

$\begin{cases} x = \boldsymbol{\beta cex\theta} \\ y = \boldsymbol{\beta sex\theta} \end{cases}$, is different from zero, it results that the two solutions are linearly independent. The static elastic properties of these vibrating systems and the integral curves in the phase space are also presented.

Chapter 18 is dedicated to the **supermathematical functions** (centric, eccentric, elevated and exotic) on **cones,** as well as on centric cones, depending on the arc of the tangential circle to the peak of cones, and on eccentric cones, like a sort of prelude to **chapter 19**, on **the elliptic supermathematical functions of the arc of the circle**. On this occasion, are defined the **unit ellipses** on x, respectively on y, marked $\mathbf{U_x}$, respectively $\mathbf{U_y}$, so that the projections of the points on axis x, respectively y, inscribes itself in the interval [-1, +1].

Very voluminous, **Chapter 19** covers 42 pages (254...296), where the **supermathematical elliptic functions**, their properties, derivatives and the rotation speed of a point on the unit ellipses are defined. Besides the known elliptic functions in the centric mathematics - cosine cn(u, k) and sine sn(u, k) – are also presented here the new functions, such as eccentric elliptic amplitude, compared with the elliptic function **Jacobi** amplitude or amplitudinus - am(u, k) - and the eccentric elliptic derivative functions according to cosine →**dece**(α, k = s) and to sine →**dese**(α, k = s).

In **figure 19.12**, are presented Jacobi elliptic functions cn, sn dn, not on an ellipse, but on the unit circle, thanks to the new **FSM-EC**. The step elliptic functions were named by the author as **Smarandache** step elliptic functions, noted as **smce**(α, k) and **smse**(α,k), with their graphs presented in **figure 19.13**, along with the graphs of their derivatives.

In **paragraph 19.9** are presented the inter-trigonometric functions, defined on quadrilobes (cvadrilobes), which complement the space between **Alaci Valeriu** square and **Euler** unit circle, as well as the field between **Euler** centric circular functions and **Alaci Valeriu** quadratic trigonometric functions.

It is shown that the new closed curves called quadrilobes (cvadrilobes) by the author are equivalents of a unit 'ellipse' simultaneously on x and y (**Fig.19.19**).

With the help of these quadrilobe (cvadrilobe) functions were defined the continuous transformations of the circle into a perfect square, of the sphere into a perfect cube, as well as of the cone into a perfect pyramid with a square base. Their 3D images are presented in **figure 19.16**, being new (super)mathematical geometric objects.

In **paragraph 19.11** are presented the **supermathematical elliptic functions** as solutions of some nonlinear vibrating systems and **paragraph 19.12** is dedicated to the **elliptic functions of the arc of the circle**.

Paragraphs 19.13 and **19.14** refer to **SM centric hyperbolic functions**, respectively, **SM eccentric hyperbolic functions**, being also presented the cosine, sine and tangent functions and the new functions introduced by the author and called **Voinoiu** hyperbolic tangent.

Entitled **"Wormholes in mathematics"**, **Ch. 20** claims that they can be realised by means of some **hybrid FSM-EC**. In author's opinion, the wormhole would be a possible faster way of connection, between centric circular mathematics and elliptic mathematics, which is the author’s lifetime dream, unfortunately not completely realized yet. There are presented two rewardable "breakthroughs": **Neville Theta C** represented exactly by means of **FSM-EC** eccentric cosine **cexθ** (**fig. 20.2, a** and **fig. 20.2, b**) and expressing the **Jacobi Zeta** elliptic function by means of the modified **FSM-EC** sine [bexθ] (**fig. 20.3**).

Paragraph 20.3 presents other **special hybrid mathematical functions**.

Chapter 21 refers to **eccentric analytic trigonometric functions** of real variable (R-analytic **§ 21.2**) and centric **(§ 21.3**). **Paragraph 21.4** is dedicated to **eccentric analytic circular functions of eccentric variable** dependent on the origin of the reference point (cos, sin, tan, etc.), and **§ 21.5** to those independent of the origin of the coordinate axes system (bex, dex, rex, aex, etc.). **Paragraph 21.10** deals with **double analytic FSM-EC**.

Chapter 22 refers to **FSM-EC of complex variable (C - analytic)** and it is richly illustrated, especially in 3D, as well as **§ 22.3** regarding the various mathematical objects represented by **FSM-EC** and **FSM-AEC**, ending with the mathematical representation of some technical parts and systems.

Instead of afterword, **Ch.23** refers to **"The dark matter of the mathematical universe"** where are presented the eccentric irrational numbers, the **eccentricity** as a **new** hidden **dimension of the space**, the **mathematical hybridization**, the eccentric real numbers and **eccentric trigonometric system**, compared with the centric one, to emphasize the definite advantages of the first, which is a continuous system, while the centric one is discreet. Hence the big advantages of curves and technical surfaces approximation, besides the fact that, along with the appartion of supermathematics, a whole range of surfaces, previously considered non-mathematical, became (super)mathematical surfaces and, therefore, they can be exactly represented using the new functions of **Mircea Eugen Şelariu**’s supermathematics.

CONCLUSION

The innovative force of Professor Mircea Eugen Șelariu’s supermathematics recommends it as an internationally valuable theory, which opens new branches of research with lots of applications.

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