Smarandache's Cevians Theorem (II)

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Abstract.
In this paper we present the Smarandache's Cevians Theorem (II) in the geometry of the triangle.

Smarandache's Cevians Theorem (II)
In a triangle ΔABC we draw the Cevians AA_1, BB_1, CC_1 that intersect in P. Then:

$$\frac{PA}{PA_1}\cdot\frac{PB}{PB_1}\cdot\frac{PC}{PC_1}=\frac{AB\cdot BC\cdot CA}{A_1B\cdot B_1C\cdot C_1A}$$

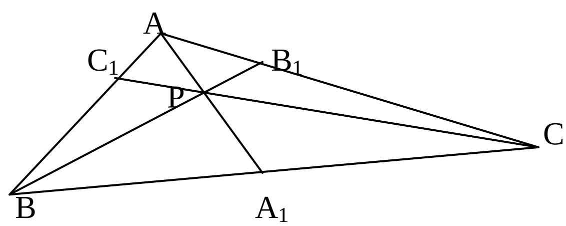
Solution 6.
In the triangle $\Delta\ ABC$ we apply the Ceva's theorem:

$$AC_1\cdot BA_1\cdot CB_1=-AB_1\cdot CA_1\cdot BC_1 \qquad (1)$$

In the triangle $\Delta\ AA_1B$, cut by the transversal CC_1, we'll apply the Menelaus' theorem:

$$AC_1\cdot BC\cdot A_1P=AP\cdot A_1C\cdot BC_1 \qquad (2)$$

In the triangle $\Delta\ BB_1C$, cut by the transversal AA_1, we apply again the Menelaus' theorem:



$$BA_1\cdot CA\cdot B_1P=BP\cdot B_1A\cdot CA_1 \qquad (3)$$

We apply one more time the Menelaus' theorem in the triangle $\Delta\ CC_1A$ cut by the transversal BB_1:

$$AB\cdot C_1P\cdot CB_1=AB_1\cdot CP\cdot C_1B \qquad (4)$$

We divide each relation (2), (3), and (4) by relation (1), and we obtain:

$$\frac{PA}{PA_1}=\frac{BC}{BA_1}\cdot\frac{B_1A}{B_1C} \qquad (5)$$

$$\frac{PB}{PB_1} = \frac{CA}{CB_1} \cdot \frac{C_1B}{C_1A} \qquad (6)$$

$$\frac{PC}{PC_1} = \frac{AB}{AC_1} \cdot \frac{A_1C}{A_1B} \qquad (7)$$

Multiplying (5) by (6) and by (7), we have:

$$\frac{PA}{PA_1} \cdot \frac{PB}{PB_1} \cdot \frac{PC}{PC_1} = \frac{AB \cdot BC \cdot CA}{A_1B \cdot B_1C \cdot C_1A} \cdot \frac{AB_1 \cdot BC_1 \cdot CA_1}{A_1B \cdot B_1C \cdot C_1A}$$

but the last fraction is equal to 1 in conformity to Ceva's theorem.

Unsolved Problem related to the Smarandache's Cevians Theorem (II).

Is it possible to generalize this problem for a polygon?

References:

[1] F. Smarandache, *Eight Solved and Eight Open Problems in Elementary Geometry*, in arXiv.org.

[2] F. Smarandache, *Problèmes avec et sans… problèmes!*, pp. 49 & 54-60, Somipress, Fés, Morocoo, 1983.